SAMPLE 1.25 **Moment of a force:** A force $\vec{F} = 1\,\text{N}\hat{\imath} + 20\,\text{N}\hat{\jmath}$ acts at point A of an object pinned at O as shown in the figure. The distance OA = 2 m. Find the moment of the force about the pin at point O.

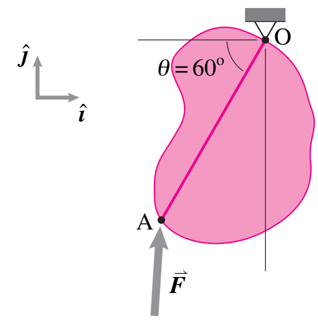

Figure 1.61

Solution The given force acts through point A on the body. Therefore, we can compute its moment about O as follows.

$$
\begin{aligned}
\vec{M}_{\text{O}} &= \vec{r}_{\text{OA}} \times \vec{F} \\
&= \underbrace{(-2\,\text{m} \cdot \cos 60^{\circ}\hat{\imath} - 2\,\text{m} \cdot \sin 60^{\circ}\,\hat{\jmath})}_{\vec{r}_{\text{OA}}} \times \underbrace{(1\,\text{N}\hat{\imath} + 20\,\text{N}\,\hat{\jmath})}_{\vec{F}} \\
&= (-1\,\text{m}\hat{\imath} - \sqrt{3}\,\text{m}\,\hat{\jmath}) \times (1\,\text{N}\hat{\imath} + 20\,\text{N}\,\hat{\jmath}) \\
&= -20\,\text{N}\cdot\text{m}\hat{k} + 1.73\,\text{N}\cdot\text{m}\hat{k} \\
&= -18.27\,\text{N}\cdot\text{m}\hat{k}.
\end{aligned}
$$

$$\boxed{\vec{M}_{\text{O}} = -18.27\,\text{N}\cdot\text{m}\hat{k}}$$

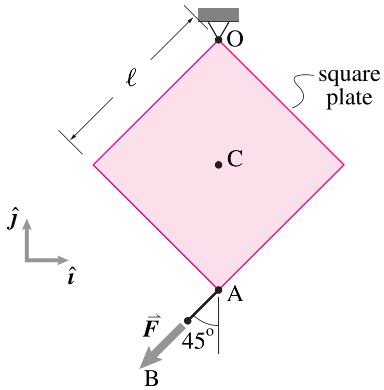

Figure 1.62

SAMPLE 1.26 A 2 m×2 m square plate hangs from one of its corners as shown in the figure. At the diagonally opposite end, a force of 50 N is applied by pulling on the string AB. Find the moment of the applied force about the center C of the plate using

1. The component of the force perpendicular to $\vec{r}_{A/C}$,
2. The lever arm (the perpendicular distance from C to the force vector), and
3. The vectors $\vec{F}$ and $\vec{r}_{A/C}$.

Solution

1. To find the moment about point C, we need to find the component of $\vec{F}$ perpendicular to CA. From the figure, we see that the desired component is $F\sin\theta$ where $\theta = 45°$. Therefore,

$$M_{/C} = |\vec{F}|\sin\theta|\vec{r}_{A/C}| = 50\,\text{N}\cdot\frac{1}{\sqrt{2}}\cdot\sqrt{2}\,\text{m} = 50\,\text{N·m}.$$

The direction of this moment is obtained by curling the fingers of the right hand from $\vec{r}_{A/C}$ towards $\vec{F}$ which points into the page, *i.e.*, $-\hat{k}$ direction. Thus, $\vec{M}_{/C} = -50\,\text{N·m}\hat{k}$.

$$\boxed{\vec{M}_{/C} = -50\,\text{N·m}\hat{k}}$$

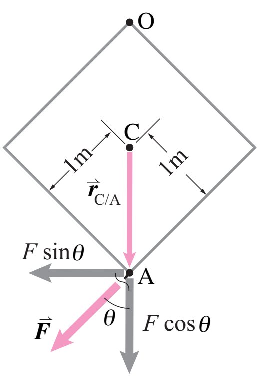

Figure 1.63: The moment $M_{/C} = F\sin\theta\cdot|\vec{r}_{A/C}|$ and acts clockwise.[Note: this is *not* a free-body diagram. It is just an illustration of the force at A. Not shown are the reaction at O and the weight.

2. The lever arm is the perpendicular distance from point C to the line of action of $\vec{F}$. This perpendicular distance is $d = |\vec{r}_{A/C}|\sin\theta = \sqrt{2}\,\text{m}\cdot(1/\sqrt{2}) = 1\,\text{m}$ (see fig. 1.63). Therefore, the moment of $\vec{F}$ about point C is

$$\begin{aligned}\vec{M}_{/C} &= Fd(-\hat{k})\\ &= -(50\,\text{N})\cdot(1\,\text{m})\hat{k} = -50\,\text{N·m}\hat{k}.\end{aligned}$$

where the direction of the moment is evident from the right hand rule as pointed out above.

$$\boxed{\vec{M}_{/C} = -50\,\text{N·m}\hat{k}}$$

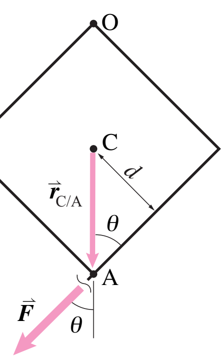

Figure 1.64: The moment $M_{/C} = Fd$ and acts clockwise.

3. The moment $\vec{M}_C$ is calculated from $\vec{F}$ and $\vec{r}_{A/C}$ by carrying out the cross product $\vec{r}_{A/C}\times\vec{F}$ in a straightforward manner. For this calculation, we first need to find the vectors $\vec{r}_{A/C}$ and $\vec{F}$:

$$\begin{aligned}\vec{r}_{A/C} &= -CA\,\hat{\jmath} = -\frac{\ell}{\sqrt{2}}\hat{\jmath} \quad (\text{since OA} = 2\,\text{CA} = \sqrt{2}\ell)\\ \vec{F} &= F(-\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) = -F(\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}).\end{aligned}$$

Hence,

$$\begin{aligned}\vec{M}_{/C} &= \vec{r}_{A/C}\times\vec{F}\\ &= -\frac{\ell}{\sqrt{2}}\hat{\jmath}\times[-F(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})]\\ &= -\frac{\ell}{\sqrt{2}}F\cos\theta\hat{k}\\ &= -\frac{2\,\text{m}}{\sqrt{2}}\cdot 50\,\text{N}\cdot\cos 45°\hat{k} = -50\,\text{N·m}\hat{k}.\end{aligned}$$

$$\boxed{\vec{M}_{/C} = -50\,\text{N·m}\hat{k}}$$

SAMPLE 1.27 **Moment about an axis:** A vertical force of unknown magnitude F acts at point B of a triangular plate ABC shown in the figure. Find the moment of the force about the edge CA of the plate.

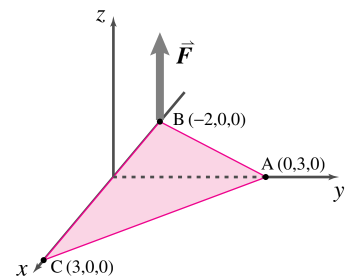

Figure 1.65

Solution The moment of a force $\vec{F}$ about an axis x-x is given by

$$M_{xx} = \hat{\lambda}_{\mathrm{xx}} \cdot (\vec{r} \times \vec{F})$$

where $\hat{\lambda}_{\mathrm{xx}}$ is a unit vector along the axis x-x, $\vec{r}$ is a position vector from any point on the axis to the applied force. In this problem, the given axis is CA. Therefore, we can take $\vec{r}$ to be $\vec{r}_{\mathrm{AB}}$ or $\vec{r}_{\mathrm{CB}}$. Here,

$$\hat{\lambda}_{\mathrm{CA}} = \frac{\vec{r}_{\mathrm{CA}}}{|\vec{r}_{\mathrm{CA}}|} = \frac{3(-\hat{\imath} + \hat{\jmath})}{\sqrt{9+9}} = -\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}.$$

Now, moment about point A is

$$\begin{aligned} \vec{M}_{\mathrm{A}} &= \vec{r}_{\mathrm{AB}} \times \vec{F} \\ &= (-2\hat{\imath} - 3\hat{\jmath}) \times F\hat{k} = 2F\hat{\jmath} - 3F\hat{\imath}. \end{aligned}$$

Therefore, the moment about CA is

$$\begin{aligned} M_{\mathrm{CA}} &= \hat{\lambda}_{\mathrm{CA}} \cdot (\vec{r}_{\mathrm{AB}} \times \vec{F}) = \hat{\lambda}_{\mathrm{CA}} \cdot \vec{M}_{\mathrm{A}} \\ &= (-\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}) \cdot (-3F\hat{\imath} + 2F\hat{\jmath}) \\ &= (\frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}})F = \frac{5}{\sqrt{2}}F. \end{aligned}$$

$$\boxed{M_{CA} = \frac{5}{\sqrt{2}}F}$$

Comments: Note that the sign of M_{CA} depends on whether we use $\hat{\lambda}_{\mathrm{CA}}$ or $\hat{\lambda}_{\mathrm{AC}}$ in our calculation. Because $\hat{\lambda}_{\mathrm{CA}} = -\hat{\lambda}_{\mathrm{AC}}$, M_{AC} will turn out to be $-\frac{5}{\sqrt{2}}F$. Since moment is a vector, it has a definite direction. Naturally, $\vec{M}_{\mathrm{CA}} = M_{\mathrm{CA}}\hat{\lambda}_{\mathrm{CA}} = -M_{\mathrm{CA}}\hat{\lambda}_{\mathrm{AC}}$. Thus, $M_{\mathrm{AC}} = -M_{\mathrm{CA}}$.

