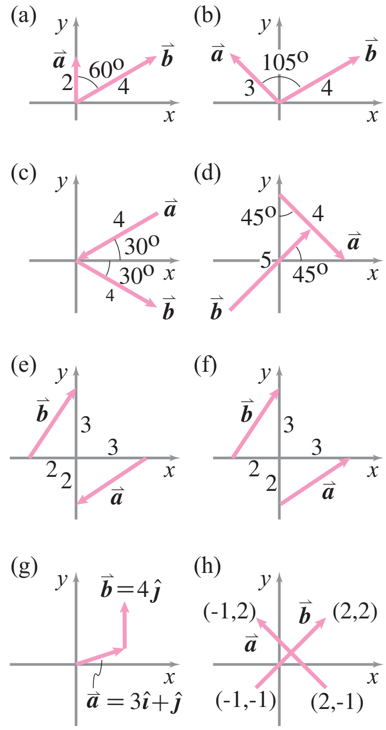
1.3.1 Find the cross product of the two vectors shown in the figures below from the information given in the figures.



Problem 1.1

(a) $|\vec{a}| = 2$

$|\vec{b}| = 4$

$\theta = 60^\circ$

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$= \cancel{2} \times 4 \times \frac{\sqrt{3}}{\cancel{2}} \times \hat{n}$

$= 4\sqrt{3}\,\hat{k}$

(b) $|\vec{a}| = 3$

$|\vec{b}| = 4$

$\theta = 105^\circ$

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$= 3 \times 4 \times 0.966\,\hat{n}$

$= 11.59\,\hat{k}$

(c) $|\vec{a}| = 4$

$|\vec{b}| = 4$

$\theta = 60^\circ$

$\vec{a} \times \vec{b} = 4 \times 4 \times \cos 60 \times \hat{n}$

$= 8\,\hat{k}$

(d) $|\vec{a}| = 4$

$|\vec{b}| = 5$

$\theta = 180 - (45 \cdot 45)$

$= 90^\circ$

$\vec{a} \times \vec{b} = 4 \times 5 \times \cos 90 \times \hat{n}$

$= 20\,\hat{k}$

(e) $\vec{a} = -3\hat{i} - 2\hat{j}$

$\vec{b} = -2\hat{i} + 3\hat{j}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ -3 & -2 & 0 \\ -2 & 3 & 0 \end{vmatrix}$$

$= (0-0)\hat{i} - (0-0)\hat{j} + (-9-4)\hat{k}$

$= -13\hat{k}$

(f) $\vec{a} = 3\hat{i} - 2\hat{j}$

$\vec{b} = -2\hat{i} + 3\hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 3 & -2 & 0 \\ -2 & 3 & 0 \end{vmatrix}$$

$= (0-0)\hat{i} - (0-0)\hat{j} + (9-4)\hat{k}$

$= 5\hat{k}$

(g) $\vec{a} = 3\hat{i} + \hat{j}$

$\vec{b} = 4\hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$= (0-0)\hat{i} - (0-0)\hat{j} + (12-0)\hat{k}$

$= 12\hat{k}$

(h) $\vec{a} = (-1-2)\hat{i} + (2-(-1))\hat{j}$

$= -3\hat{i} + 3\hat{j}$

$\vec{b} = (2-(-1))\hat{i} + (2-(-1))\hat{j}$

$= 3\hat{i} + 3\hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

$= (0-0)\hat{i} - (0-0)\hat{j} + (-9-9)\hat{k}$

$= -18\hat{k}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.3.2 Vector algebra. For each equation below state whether:

1. The equation is nonsense. If so, why?
2. Is always true. Why? Give an example.
3. Is never true. Why? Give an example.
4. Is sometimes true. Give examples both ways.

You may use trivial examples.

a) $\vec{B} \times \vec{C} = \vec{C} \times \vec{B}$

b) $\vec{B} \times \vec{C} = \vec{C} \cdot \vec{B}$

c) $\vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$

d) $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

(a) 4. If $C = B$ or $B = 0$ or $C = 0$

$\vec{B} = 3\hat{i} + 4\hat{j}$

$\vec{C} = 3\hat{i} + 4\hat{j}$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0$$

$$\vec{C} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0$$

$\vec{B} = 3\hat{i} + 4\hat{j}$

$\vec{C} = 4\hat{i} + 3\hat{j}$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 4 & 3 & 0 \end{vmatrix} = -7\hat{k}$$

(b) 3. $\vec{B} \times \vec{C}$ gives a vector, while $\vec{B} \cdot \vec{C}$ gives a scalar

$\vec{B} = 3\hat{i} + 4\hat{j}$

$\vec{C} = 4\hat{i} + 3\hat{j}$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 4 & 3 & 0 \end{vmatrix} = -7\hat{k}$$

$\vec{B} \cdot \vec{C} = 24$

(c) 3. Vector Multiplication is not associative.

$\vec{A} = 3\hat{i} + 4\hat{j}$

$\vec{B} = 4\hat{i} + 3\hat{j}$

$\vec{C} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 4 & 3 & 0 \end{vmatrix} = -7\hat{k}$$

$\vec{C} \cdot (\vec{A} \times \vec{B}) = -7$

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$$\vec{C} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 3 & 4 & 0 \end{vmatrix} = -3\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{B} \cdot (\vec{C} \times \vec{A}) = -12 + 9$$
$$= -3$$

(d) 2. Vector Algebra is Distributive

$$\vec{A} = 3\hat{i} + 4\hat{j}$$
$$\vec{B} = 4\hat{i} + 3\hat{j}$$
$$\vec{C} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{i} - 4\hat{j} + \hat{k}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 3 & -4 & 1 \end{vmatrix} = 4\hat{i} - 3\hat{j} - 24\hat{k}$$

$$\vec{A} \cdot \vec{C} = 7$$

$$\vec{A} \cdot \vec{B} = 24$$

$$(\vec{A} \cdot \vec{C})\vec{B} = 28\hat{i} + 21\hat{k}$$

$$(\vec{A} \cdot \vec{B})\vec{C} = 24\hat{i} + 24\hat{j} + 24\hat{k}$$

$$(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} = 4\hat{i} - 3\hat{j} - 24\hat{k} = LHS$$

1.3.3 What do you get when you cross a vector and a scalar?

Answer:
No partial credit.

A cross product requires two vectors to calculate, hence you cannot cross a vector with a scalar. You can multiply a scalar with a vector where each component of the vector is multiplied with the scalar.

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1.3.4 Carry out the following cross products in different ways and determine which method takes the least amount of time for you.

a) $\vec{\boldsymbol{r}} = 2.0\,\text{ft}\hat{\boldsymbol{\imath}} + 3.0\,\text{ft}\hat{\boldsymbol{\jmath}} - 1.5\,\text{ft}\hat{\boldsymbol{k}}$; $\vec{\boldsymbol{F}} = -0.3\,\text{lbf}\hat{\boldsymbol{\imath}} - 1.0\,\text{lbf}\hat{\boldsymbol{k}}$; $\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} =$?

b) $\vec{\boldsymbol{r}} = (-\hat{\boldsymbol{\imath}} + 2.0\hat{\boldsymbol{\jmath}} + 0.4\hat{\boldsymbol{k}})\,\text{m}$; $\vec{\boldsymbol{L}} = (3.5\hat{\boldsymbol{\jmath}} - 2.0\hat{\boldsymbol{k}})\,\text{kg m/s}$; $\vec{\boldsymbol{r}} \times \vec{\boldsymbol{L}} =$?

c) $\vec{\omega} = (\hat{\boldsymbol{\imath}} - 1.5\hat{\boldsymbol{\jmath}})\,\text{rad/s}$; $\vec{\boldsymbol{r}} = (10\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}} + 3\hat{\boldsymbol{k}})\in$; $\vec{\omega} \times \vec{\boldsymbol{r}} =$?

(a) $\vec{r} = (2\,\text{ft})\hat{i} + (3\,\text{ft})\hat{j} + (-1.5\,\text{ft})\hat{k}$

$\vec{F} = (-0.3\,\text{lbf})\hat{i} + (-1\,\text{lbf})\hat{k}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\,\text{ft} & 3\,\text{ft} & -1.5\,\text{ft} \\ -0.3\,\text{lbf} & 0 & -1\,\text{lbf} \end{vmatrix}$$

$$= -3\,\text{lbf ft}\,\hat{i} + 2.45\,\text{lbf ft}\,\hat{j} + 0.9\,\text{lbf ft}\,\hat{k}$$

$$= (-3\hat{i} + 2.45\hat{j} + 0.9\hat{k})\,\text{lbf ft}$$

(b) $\vec{r} = (-\hat{i} + 2\hat{j} + 0.4\hat{k})\,\text{m}$

$\vec{L} = (3.5\hat{j} - 2\hat{k})\,\text{kg m/s}$

$$\vec{r} \times \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1\,\text{m} & 2\,\text{m} & 0.4\,\text{m} \\ 0 & 3.5\,\text{kg m/s} & -2\,\text{kg m/s} \end{vmatrix}$$

$$= (-5.4\hat{i} - 2\hat{j} - 3.5\hat{k})\,\text{kg m}^2/\text{s}$$

(c) $\vec{\omega} = (\hat{i} - 1.5\hat{j})\,\text{rad/s}$

$\vec{r} = (10\hat{i} - 2\hat{j} + 3\hat{k})\,\text{in}$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1\,\text{rad/s} & -1.5\,\text{rad/s} & 0 \\ 10\,\text{in} & -2\,\text{in} & 3\,\text{in} \end{vmatrix}$$

$$= (-4.5\hat{i} - 3\hat{j} + 13\hat{k})\,\text{rad in/s}$$

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1.3.5 Cross Product program Write a program that will calculate cross products. The input to the function should be the components of the two vectors and the output should be the components of the cross product. As a model, here is a function file that calculates dot products in pseudo code.

```
%program definition
z(1)=a(1)*b(1);
z(2)=a(2)*b(2);
z(3)=a(3)*b(3);
w=z(1)+z(2)+z(3);
```

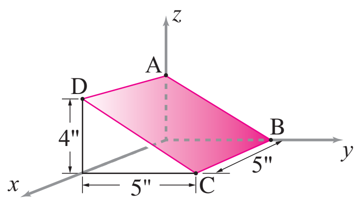
Code:

```
% Example Vectors %
v1 = [3;4;0];
v2 = [4;3;0];
cross(v1,v2)

% Function to Calculate Cross Product%
function w = cross(v1,v2)
    w = zeros(3,1);
    w(1) = (v1(2)*v2(3))-(v1(3)*v2(2));
    w(2) = -(v1(1)*v2(3))-(v1(3)*v2(1));
    w(3) = (v1(1)*v2(2))-(v1(2)*v2(1));
end
```

The obtained cross product is : $-7\hat{k}$

1.3.6 Find a unit vector normal to the surface ABCD shown in the figure.



Problem 1.6

$\vec{r}_A = 4\text{in}\,\hat{k}$

$\vec{r}_B = 5\text{in}\,\hat{j}$

$\vec{r}_D = 5\text{in}\,\hat{i} + 4\text{in}\,\hat{k}$

$\vec{r}_{AB} = 5\text{in}\,\hat{j} - 4\text{in}\,\hat{k}$

$\vec{r}_{AD} = 5\text{in}\,\hat{i}$

Find Cross Product,

$$\vec{r}_{AB} \times \vec{r}_{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & -4 \\ 5 & 0 & 0 \end{vmatrix} \text{in}^2$$

$$= -20\,\text{in}^2\,\hat{j} - 25\,\text{in}^2\,\hat{k}$$

Find magnitude of Cross Product,

$$|\vec{r}_{AB} \times \vec{r}_{AD}| = \sqrt{(-20\,\text{in}^2)^2 + (-25\,\text{in}^2)^2}$$

$$= 5\sqrt{41}\,\text{in}^2$$

Find unit vector in the direction of the cross product,

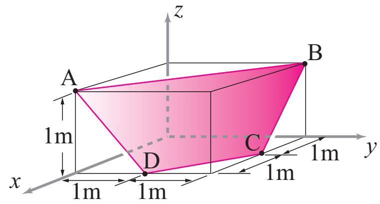
$$\text{Unit Vector} = \frac{\vec{r}_{AB} \times \vec{r}_{AD}}{|\vec{r}_{AB} \times \vec{r}_{AD}|}$$

$$= \frac{-4}{\sqrt{41}}\hat{j} - \frac{5}{\sqrt{41}}\hat{k}$$

$$= \frac{-1}{\sqrt{41}}(4\hat{j} + 5\hat{k})$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.3.7 If the magnitude of a force $\vec{N}$ normal to the surface ABCD in the figure is 1000 N, write $\vec{N}$ as a vector.

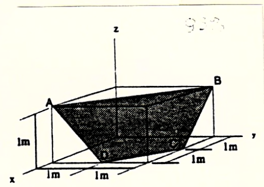


Problem 1.7

Answer:
$\vec{N} = \frac{1000\,\text{N}}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$.

Page 5

1.40 If the magnitude of a force $\underline{\mathbf{N}}$ normal to the surface ABCD in the figure is 1000 N, write $\underline{\mathbf{N}}$ as a vector.



probfig 1.40: (Filename:efig1.2.12)

Find Vector Normal to a surface by taking two vectors on the surface (non-collinear, non-parallel) and forming their cross product.

$\underline{r}_{DC} = (-\hat{\imath} + \hat{\jmath})\,m \qquad \underline{r}_{DA} = (-\hat{\jmath} + \hat{k})\,m$

$$\underline{r}_N = \underline{r}_{DC} \times \underline{r}_{DA} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1m & 1m & 0 \\ 0 & -1m & 1m \end{vmatrix} = (\hat{\imath} + \hat{\jmath} + \hat{k})\,m$$

$$\underline{\lambda}_N = \frac{\underline{r}_N}{|\underline{r}_N|} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$$

$$\boxed{\underline{N} = \frac{1000\,N}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})}$$

$\vec{r}_A = (2\hat{i} + \hat{k})m$

$\vec{r}_B = (2\hat{j} + \hat{k})m$

$\vec{r}_D = (2\hat{i} + \hat{j})m$

$\vec{r}_{AB} = 2(-\hat{i} + \hat{j})m$

$\vec{r}_{AD} = (\hat{j} - \hat{k})m$

Find cross product and its magnitude,

$$\vec{r}_{AB} \times \vec{r}_{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & 0 \\ 0 & 1 & -1 \end{vmatrix} m^2$$

$$= -2\hat{i} - 2\hat{j} - 2\hat{k}$$

$|\vec{r}_{AB} \times \vec{r}_{AD}| = 2\sqrt{3}\ m^2$

$$\hat{\lambda} = \frac{\vec{r}_{AB} \times \vec{r}_{AD}}{|\vec{r}_{AB} \times \vec{r}_{AD}|}$$

$$= \frac{-1}{2\sqrt{3}} (\hat{i} + \hat{j} + \hat{k})$$

$\vec{N} = 1000N \times \hat{\lambda}$

$$= \frac{-500N}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k})$$

1.3.8 The equation of a surface is given as $z = 2x - y$. Find a unit vector $\hat{\boldsymbol{n}}$ normal to the surface.

Take any two vectors that fit the equation.

$$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix}$$

$$= 2\hat{i} - 2\hat{j} - 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{2^2 + (-2)^2 + (-2)^2}$$

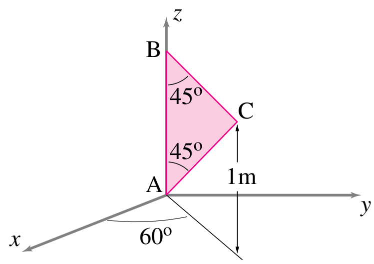
$$= 2\sqrt{3}$$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$= \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.3.9 In the figure, a triangular plate ACB, attached to rod AB, rotates about the z-axis. At the instant shown, the plate makes an angle of 60° with the x-axis. Find and draw a vector normal to the surface ACB.



Problem 1.9

$$\vec{r}_B = 2m\,\hat{k}$$
$$\vec{r}_A = 0$$
$$\vec{r}_{AB} = 2m\,\hat{k}$$

Find position vector of C,

$$\vec{r}_C = \frac{1m}{\sin 45}\,\hat{j} + \left(\frac{1m}{\sin 45 \times \sin 60} \times \cos 60\right)\hat{i} + 1m\,\hat{k}$$

$$= \sqrt{2}\,m\,\hat{j} + \left(2\cancel{\sqrt{2}} \times \frac{3}{\cancel{\sqrt{2}}}\right) m\hat{i} + 1m\hat{k}$$

$$= 3m\hat{i} + \sqrt{2}\,m\hat{j} + 1m\hat{k}$$

$$\vec{r}_{AC} = (3\hat{i} + \sqrt{2}\,\hat{j} + \hat{k})\,m$$

$$\vec{r}_{AB} \times \vec{r}_{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 3 & \sqrt{2} & 1 \end{vmatrix} m^2$$

$$= (-2\sqrt{2}\,\hat{i} + 6\hat{j})\,m^2$$

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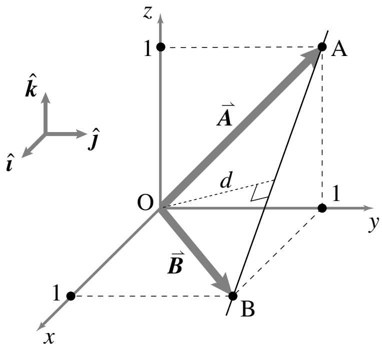
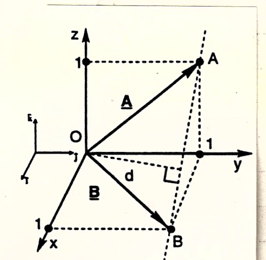
1.3.10 What is the distance d between the origin and the line AB shown? (You may write your solution in terms of $\vec{A}$ and $\vec{B}$ before doing any arithmetic).



Problem 1.10

Answer:
$d = \sqrt{\frac{3}{2}}$.

1.55 What is the distance d between the origin and the line AB shown? (You may write your solution in terms of **A** and **B** before doing any arithmetic).



probfig 1.55:

Note: Triangle ABO has area $\frac{1}{2}|\underline{A}\times\underline{B}| = \frac{1}{2} d\, |\underline{r}_{AB}|$

$$d = \frac{|\underline{A}\times\underline{B}|}{|\underline{r}_{AB}|} = \frac{|(\hat{\jmath}+\hat{k})\times(\hat{\imath}+\hat{\jmath})|}{\sqrt{2}} = \frac{|\hat{\imath}-\hat{\jmath}+\hat{k}|}{\sqrt{2}}$$

$$\boxed{d = \sqrt{\frac{3}{2}}}$$

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$\vec{A} = \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + \hat{j}$

$\vec{r}_{AB} = \hat{i} - \hat{k}$

Find unit vector in the direction of the cross product,

$$\hat{n} = \frac{\hat{k} \times \vec{r}_{AB}}{|\hat{k} \times \vec{r}_{AB}|}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix} \times \frac{1}{|\vec{r}_A \times \vec{r}_{BC}|}$$

$$= (\hat{j}) \times \frac{1}{1}$$

$$= \hat{j}$$

$d = \vec{B} \cdot \hat{n}$

$= 1$

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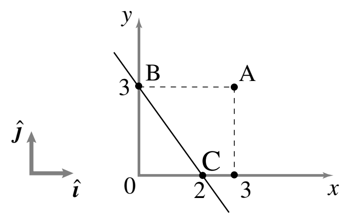
1.3.11 What is the perpendicular distance between point A and line BC shown? (There are at least 3 ways to do this using various vector products, how many ways can you find?)



Problem 1.11

$\vec{r}_A = 3\hat{i} + 3\hat{j}$

$\vec{r}_{BC} = 2\hat{i} - 3\hat{j}$

$\vec{r}_{AC} = -\hat{i} - 3\hat{j}$

Find unit vector in the direction of the cross product,

$$\hat{n} = \frac{\hat{k} \times \vec{r}_{BC}}{|\hat{k} \times \vec{r}_{BC}|}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 2 & -3 & 0 \end{vmatrix} \times \frac{1}{|\vec{r}_A \times \vec{r}_{BC}|}$$

$$= (3\hat{i} + 2\hat{j}) \times \frac{1}{\sqrt{13}}$$

$$= \frac{1}{\sqrt{13}}(3\hat{i} + 2\hat{j})$$

$$d = \vec{r}_{AC} \cdot \hat{n}$$

$$= -1 \times \frac{3}{\sqrt{13}} + -3 \times \frac{2}{\sqrt{13}}$$

$$= \frac{-9}{\sqrt{13}}$$

$$= -2.49$$

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1.3.12 Given a force, $\vec{F}_1 = (-3\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ N acting at a point P whose position is given by $\vec{r}_{P/O} = (4\hat{\imath} - 2\hat{\jmath} + 7\hat{k})$ m, what is the moment about an axis through the origin O with direction $\hat{\lambda} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath}$?

$\vec{F} = (-3\hat{\imath} + 2\hat{\jmath} + 5\hat{k})\,N$

$\vec{r}_{P/O} = (4\hat{\imath} - 2\hat{\jmath} + 7\hat{k})\,m$

Find Moment,

$$\vec{M}_{/O} = \vec{r}_{P/O} \times \vec{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 2 & 5 \\ 4 & -2 & 7 \end{vmatrix} \text{Nm}$$

$$= (-7\hat{\imath} + 41\hat{\jmath} - 2\hat{k})\,\text{Nm}$$

$$\hat{\lambda} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath}$$

$$\hat{\lambda} \cdot \vec{M}_{/O} = \left(\frac{-14}{\sqrt{5}} + \frac{41}{\sqrt{5}}\right)\text{Nm}$$

$$= 12.07\ \text{Nm}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.3.13 A, B, and C are located by position vectors $\vec{r}_A = (1,2,3)$, $\vec{r}_B = (4,5,6)$, and $\vec{r}_C = (7,8,10)$.

a) Use the vector dot product to find the angle BAC (A is at the vertex of this angle).

b) Use the vector cross product to find the angle BCA (C is at the vertex of this angle).

c) Find a unit vector perpendicular to the plane ABC.

d) How far is the infinite line defined by AB from the origin? (That is, how close is the closest point on this line to the origin?)

e) Is the origin co-planar with the points A, B, and C?

(a) $\vec{r}_A = 1\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r}_B = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{r}_C = 7\hat{i} + 8\hat{j} + 10\hat{k}$

$\vec{r}_{AB} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{r}_{AC} = 6\hat{i} + 6\hat{j} + 7\hat{k}$

Find angle between $\vec{r}_{AB}$ and $\vec{r}_{AC}$,

$$\theta_{BAC} = \cos^{-1}\left[\frac{\vec{r}_{AB}\cdot\vec{r}_{AC}}{|\vec{r}_{AB}|\,|\vec{r}_{AC}|}\right]$$

$$= \cos^{-1}\left[\frac{18+18+21}{3\sqrt{3}\times 11}\right]$$

$$= \cos^{-1}\left[\frac{57}{33\sqrt{3}}\right]$$

$$= 4.25^\circ$$

(b) $\vec{r}_{CB} = -3\hat{i} - 3\hat{j} - 4\hat{k}$

$\vec{r}_{CA} = -6\hat{i} - 6\hat{j} - 7\hat{k}$

$$\theta_{BCA} = \cos^{-1}\left[\frac{\vec{r}_{CB}\cdot\vec{r}_{CA}}{|\vec{r}_{CB}|\,|\vec{r}_{CA}|}\right]$$

$$= \cos^{-1}\left[\frac{18+18+28}{\sqrt{34}\times 11}\right]$$

$$= \cos^{-1}\left[\frac{64}{11\sqrt{34}}\right]$$

$$= 3.79$$

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(c) $\vec{r}_{AC} \times \vec{r}_{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ 6 & 6 & 7 \end{vmatrix}$

$= 3\hat{i} - 3\hat{j}$

$\hat{n} = \dfrac{\vec{r}_{AC} \times \vec{r}_{AB}}{|\vec{r}_{AC} \times \vec{r}_{AB}|}$

$= \dfrac{1}{3\sqrt{2}} (\hat{i} - \hat{j})$

(d) $d = \vec{r}_B \cdot \hat{n}$

$= \dfrac{4}{3\sqrt{2}} - \dfrac{5}{3\sqrt{2}}$

$= \dfrac{-1}{3\sqrt{2}}$

$= -0.235$

(e) For any vector space, spanned by two vectors, it must pass through the origin.

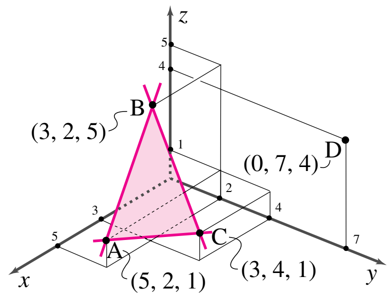
1.3.14 Points A, B, and C in the figure define an infinite plane.

a) Find a unit normal vector to the plane.

b) Find the perpendicular distance from point D to this infinite plane (not necessarily inside the triangle ABC).

c) What are the coordinates of the point on the plane closest to point D?

d) Is this point on or off the triangle used to define the plane?



Problem 1.14

Answer:
(a) $\hat{\boldsymbol{n}} = \frac{1}{3}(2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}})$.
(b) $d = 1$.
(c) $\frac{1}{3}(-2, 19, 11)$.

$\vec r_A = 5\hat i + 2\hat j + \hat k$

$\vec r_B = 3\hat i + 2\hat j + 5\hat k$

$\vec r_C = 3\hat i + 4\hat j + \hat k$

(a) $\vec r_{AB} = -2\hat i + 4\hat k$

$\vec r_{AC} = -2\hat i + 2\hat j$

$$\vec r_{AB} \times \vec r_{AC} = \begin{vmatrix} \hat i & \hat j & \hat k \\ -2 & 0 & 4 \\ -2 & 2 & 0 \end{vmatrix}$$

$$= -8\hat i - 8\hat j - 4\hat k$$

$$\hat n = -\tfrac{2}{3}\hat i - \tfrac{2}{3}\hat i - \tfrac{1}{3}\hat k$$

(b) $\vec r_D = 7\hat j + 4\hat k$

Find distance between D and ABC,

$$d = \hat n \cdot \vec r_D$$

$$= \left(-\tfrac{2}{3} \times 0\right) + \left(-\tfrac{2}{3} \times 7\right) + \left(-\tfrac{1}{3} \times 4\right)$$

$$= -6$$

(c) Closest Point $= \vec r_D - d\hat n$

$$= 7\hat j + 4\hat k + 4\hat i + 4\hat j + 8\hat k$$

$$= 4\hat i + 11\hat j + 12\hat k$$

(d) It lies outside the triange, as the co-ordinates are larger than that of the vertices.

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1.3.15 What point on the line that goes through the points (1,2,3) and (7,12,15) is closest to the origin?

$$\vec{r}_A = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{r}_B = 7\hat{i} + 12\hat{j} + 15\hat{k}$$

$$\vec{r}_A \times \vec{r}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 7 & 12 & 15 \end{vmatrix}$$

$$= -6\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{n} = \frac{-\hat{i}}{\sqrt{6}} + \frac{\hat{j}}{\sqrt{6}} - \frac{1}{3\sqrt{6}}\hat{k}$$

Find Distance to Origin,

$$d = \vec{r}_0 \cdot \hat{n}$$

$$= 0$$

Closest Point: $\vec{r}_0 - d\hat{n}$

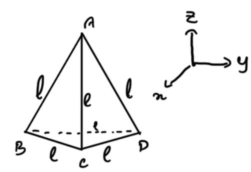
$$= (0,0,0)$$

1.3.16 A regular tetrahedron is a triangular-based pyramid where all 6 edges have the same length ℓ. What is the perpendicular distance between a pair of non-touching edges? (There are many ways to solve this problem).

Answer:
$\ell/\sqrt{2}$

$B = (0,0,0)$
$C = (\frac{\sqrt{3}\ell}{2}, \ell/2, 0)$
$D = (0, \ell, 0)$
$A = (\sqrt{3}\ell/4, \ell/4, \sqrt{3}\ell/2)$

Using Properties of Equilateral Triangles



$$\vec{r}_{BC} = \left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)\ell$$

$$\vec{r}_{DC} = \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)\ell$$

Find Cross Product,

$$\vec{r}_{DC} \times \vec{r}_{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sqrt{3}/2 & 1 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \end{vmatrix}$$

$$= -\frac{3\sqrt{3}}{4}\hat{k}$$

$$\hat{n} = \hat{k}$$

Perpendicular Distance: $\hat{n} \cdot \vec{r}_A$

$$= \frac{\sqrt{3}\ell}{2}$$

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1.3.17 Why did the chicken cross the road?

Answer:
To get chicken road sin theta.

To get chicken road sin theta.