1.2.1 Find the dot product of $\vec{\boldsymbol{a}} = 2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + 2\hat{\boldsymbol{k}}$.

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{a} \cdot \vec{b} = (2 \times 2) + (3 \times 1) + (-1 \times 2)$$

$$= 4 + 3 - 2$$

$$= 5$$

1.2.2 Find the dot product of $\vec{F} = 0.5\,\mathrm{N}\hat{\imath} + 1.2\,\mathrm{N}\hat{\jmath} + 1.5\,\mathrm{N}\hat{k}$ and $\hat{\lambda} = -0.8\hat{\imath} + 0.6\hat{\jmath}$.

$$\vec{F} = (0.5\ \mathrm{N})\hat{i} + (1.2\mathrm{N})\hat{j} + (1.5\ \mathrm{N})\hat{k}$$

$$\hat{\lambda} = (-0.8\hat{i} + \hat{j})\ m$$

$$\vec{F}\cdot\hat{\lambda} = (0.5\mathrm{N} \times (-0.8m)) + (1.2\mathrm{N} \times 1m) + (1.5\mathrm{N} \times 0m)$$

$$= -0.4\ \mathrm{Nm} + 1.2\mathrm{Nm}$$

$$= 0.8\,\mathrm{N\,m}$$

1.2.3 Find the dot product $\vec{F} \cdot \vec{r}$ where $\vec{F} = (5\hat{\imath} + 4\hat{\jmath})\,\mathrm{N}$ and $\vec{r} = (-0.8\hat{\imath} + \hat{\jmath})\,\mathrm{m}$. Draw the two vectors and justify your answer for the dot product.

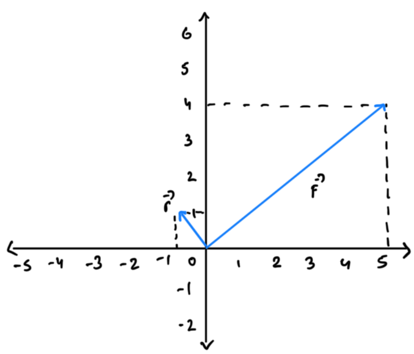
$\vec{F} = (5\hat{i} + 4\hat{j})\,N$

$\vec{r} = (-0.8\hat{i} + \hat{j})\,m$

$\vec{F} \cdot \vec{r} = (5\,N \times (-0.8)m) + (4N \times 1m)$

$= -4\,Nm + 4\,Nm$

$= 0\,Nm$



Since $\vec{F}$ and $\vec{r}$ are perpendicular to each other, the projection of one on the other would be at the point they intesect with the length of 0.

1.2.4 Two vectors, $\vec{a} = -4\sqrt{3}\hat{\imath} + 12\hat{\jmath}$ and $\vec{b} = \hat{\imath} - \sqrt{3}\hat{\jmath}$ are given. Find the dot product of the two vectors. How is $\vec{a}\cdot\vec{b}$ related to $|\vec{a}||\vec{b}|$ in this case?

$\vec{a} = -4\sqrt{3}\,\hat{i} + 12\hat{j}$

$\vec{b} = \hat{i} - \sqrt{3}\,\hat{j}$

Find dot product,

$\vec{a}\cdot\vec{b} = (-4\sqrt{3}\times 1) + (12\times(-\sqrt{3}))$

$= -16\sqrt{3}$

Find Product of Magnitudes,

$|\vec{a}| = \sqrt{(-4\sqrt{3})^2 + 12^2}$

$= 8\sqrt{3}$

$|\vec{b}| = \sqrt{1^2 + (\sqrt{3})^2}$

$= 2$

$|\vec{a}|\,|\vec{b}| = 8\sqrt{3}\times 2$

$= 16\sqrt{3} = \vec{a}\cdot\vec{b}$

Hence $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|$

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1.2.5 Find the dot product of two vectors $\vec{F} = 10\,\text{lbf}\,\hat{\imath} - 20\,\text{lbf}\,\hat{\jmath}$ and $\hat{\lambda} = 0.8\hat{\imath} + 0.6\hat{\jmath}$. Sketch $\vec{F}$ and $\hat{\lambda}$ and show what their dot product represents .

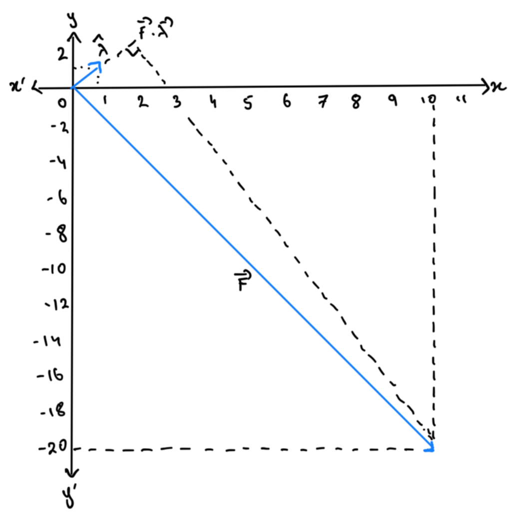
$$\vec{F} = (10\,\text{lbf})\hat{\imath} - (20\,\text{lbf})\hat{\jmath}$$

$$\hat{\lambda} = 0.8\hat{\imath} + 0.6\hat{\jmath}$$

$$\vec{F}\cdot\vec{\lambda} = (10\,\text{lbf} \times 0.8) + (-20\,\text{lbf} \times 0.6)$$

$$= 8\,\text{lbf} - 12\,\text{lbf}$$

$$= -4\,\text{lbf}$$



1.2.6 The position vector of a point A is $\vec{r}_A = 30\,\text{cm}\hat{\imath}$. Find the dot product of $\vec{r}_A$ with $\hat{\lambda} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$.

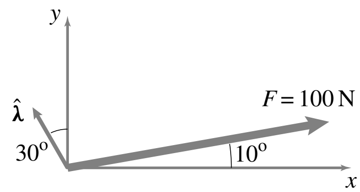
$$\vec{r}_A = (30\text{cm})\hat{\imath}$$

$$\hat{\lambda} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$$

$$\vec{r}_A \cdot \hat{\lambda} = \left(30\text{cm} \times \frac{\sqrt{3}}{2}\right) + \left(0\text{cm} \times \frac{1}{2}\right)$$

$$= 15\sqrt{3}\ \text{cm}$$

1.2.7 From the figure below, find the component of force $\vec{F}$ in the direction of $\hat{\lambda}$.



Problem 1.7

$|\vec{F}| = 100N$

$|\hat{\lambda}| = 1$

Find angle between $\vec{F}$ and $\hat{\lambda}$,

$\theta_{F\lambda} = (90-10) + 30$

$= 110°$

$\vec{F}\cdot\hat{\lambda} = |\vec{F}|\,|\vec{\lambda}|\cos\theta_{F\lambda}$

$= 100\,N \times 1 \times \cos 110$

$= -34.2N$

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1.2.8 Find the angle between $\vec{F}_1 = 2\,\mathrm{N}\hat{\imath} + 5\,\mathrm{N}\hat{\jmath}$ and $\vec{F}_2 = -2\,\mathrm{N}\hat{\imath} + 6\,\mathrm{N}\hat{\jmath}$.

$$\vec{F}_1 = 2\mathrm{N}\hat{\imath} + 5\mathrm{N}\hat{\jmath}$$

$$\vec{F}_2 = -2\mathrm{N}\hat{\imath} + 6\mathrm{N}\hat{\jmath}$$

Find Dot Product,

$$\vec{F}_1 \cdot \vec{F}_2 = (2\mathrm{N} \times (-2\mathrm{N})) + (5\mathrm{N} \times 6\mathrm{N})$$

$$= -4\mathrm{N}^2 + 30\mathrm{N}^2$$

$$= 26\mathrm{N}^2$$

Find magnitude of $\vec{F}_1$ and $\vec{F}_2$,

$$|\vec{F}_1| = \sqrt{(2\mathrm{N})^2 + (5\mathrm{N})^2}$$

$$= \sqrt{39}\ \mathrm{N}$$

$$|\vec{F}_2| = \sqrt{(-2\mathrm{N})^2 + (6\mathrm{N})^2}$$

$$= 2\sqrt{10}\ \mathrm{N}$$

$$\vec{F}_1 \cdot \vec{F}_2 = |\vec{F}_1|\,|\vec{F}_2| \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1|\,|\vec{F}_2|}\right)$$

$$= \cos^{-1}\left[\frac{\cancel{26}\ 13}{\cancel{2}\sqrt{390}}\right]$$

$$= 48.83^\circ$$

1.2.9 Given $\vec{\omega} = 2\,\text{rad/s}\,\hat{\imath} + 3\,\text{rad/s}\,\hat{\jmath}$, $\vec{H}_1 = (20\hat{\imath} + 30\hat{\jmath})\,\text{kg}\,\text{m}^2/\text{s}$ and $\vec{H}_2 = (10\hat{\imath} + 15\hat{\jmath} + 6\hat{k})\,\text{kg}\,\text{m}^2/\text{s}$, find (a) the angle between $\vec{\omega}$ and $\vec{H}_1$ and (b) the angle between $\vec{\omega}$ and $\vec{H}_2$.

$$\vec{\omega} = (2\ \text{rad/s})\hat{i} + (3\ \text{rad/s})\hat{j}$$

$$\vec{H}_1 = (20\hat{i} + 30\hat{j})\ \text{kg m}^2/\text{s}$$

$$\vec{H}_2 = (10\hat{i} + 15\hat{j} + 6\hat{k})\ \text{kg m}^2/\text{s}$$

1. $$\vec{H}_1 \cdot \vec{\omega} = (20\text{kg m}^2/\text{s} \times 2\ \text{rad/s}) + (30\text{kg m}^2/\text{s} \times 3\ \text{rad/s})$$

$$= 40\ \text{kg m}^2\text{rad/s}^2 + 90\ \text{kg m}^2\ \text{rad/s}^2$$

$$= 130\ \text{kg m}^2\ \text{rad/s}^2$$

$$|\vec{H}_1| = \sqrt{(20\,\text{kg m}^2/\text{s})^2 + (30\ \text{kg m}^2/\text{s})^2}$$

$$= 10\sqrt{13}\ \ \text{kg m}^2/\text{s}$$

$$|\vec{\omega}| = \sqrt{(2\,\text{rad/s})^2 + (3\ \text{rad/s})^2}$$

$$= \sqrt{13}\ \ \text{rad/s}$$

Find angle between $\vec{H}_1$ and $\vec{\omega}$,

$$\theta = \cos^{-1}\left[\frac{\vec{H}_1 \cdot \vec{\omega}}{|\vec{H}_1||\vec{\omega}|}\right]$$

$$= \cos^{-1}\left[\frac{130\ \text{kg m}^2\ \text{rad/s}^2}{10\sqrt{13}\ \text{kg m}^2/\text{s} \times \sqrt{13}\ \text{rad s}^{-1}}\right]$$

$$= 0^\circ$$

2. $$\vec{H}_2 \cdot \vec{\omega} = (10\ \text{kg m}^2/\text{s} \times 2\ \text{rad/s}) + (15\ \text{kg m}^2/\text{s} \times 3\ \text{rad/s}) + (6\text{kg m}^2/\text{s} \times 0\ \text{rad/s})$$

$$= (20 + 45)\ \text{kg m}^2\ \text{rad/s}^2$$

$$= 65\ \text{kg m}^2\ \text{rad/s}^2$$

$$|\vec{H}_2| = \sqrt{(10\ \text{kg m}^2/\text{s})^2 + (15\ \text{kg m}^2/\text{s})^2 + (6\ \text{kg m}^2/\text{s})^2}$$

$= 19\ \mathrm{kg\,m^2/s}$

Find angle between $\vec{H_2}$ and $\vec{\omega}$,

$$\theta = \cos^{-1}\left[\frac{\vec{H_2}\cdot\vec{\omega}}{|\vec{H_2}||\vec{\omega}|}\right]$$

$$= \cos^{-1}\left[\frac{65\ \mathrm{kg\,m^2\,rad/s^2}}{19\ \mathrm{kg\,m^2/s} \times \sqrt{13}\ \mathrm{rad/s}}\right]$$

$$= 18.4^\circ$$

1.2.10 The unit normal to a surface is given as $\hat{\boldsymbol{n}} = 0.74\hat{\boldsymbol{\imath}} + 0.67\hat{\boldsymbol{\jmath}}$. If the weight of a block on this surface acts in the $-\hat{\boldsymbol{\jmath}}$ direction, find the angle that a 1000 N normal force makes with the direction of weight of the block.

$\hat{n} = 0.74\hat{i} + 0.67\hat{j}$

$\vec{F} = -1000\,N\,\hat{j}$

Find Dot Product and magnitudes,

$\vec{F} \cdot \vec{n} = (-1000N \times 0.74) + (0N \times 0.67)$

$= -7400N$

$|\vec{F}| = 10000N$

$|\hat{n}| = 1$

$\theta = \cos^{-1}\left[\frac{\vec{F} \cdot \hat{n}}{|\vec{F}||\hat{n}|}\right]$

$= \cos^{-1}\left[\frac{7400N}{1000N \times 1}\right]$

$= 42.27^{\circ}$

1.2.11 Vector algebra. For each equation below state whether:

1. The equation is nonsense. If so, why?
2. Is always true. Why? Give an example.
3. Is never true. Why? Give an example.
4. Is sometimes true. Give examples both ways.

You may use trivial examples.

a) $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$

b) $\vec{\boldsymbol{A}} + b = b + \vec{\boldsymbol{A}}$

c) $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$

d) $\vec{\boldsymbol{B}}/\vec{\boldsymbol{C}} = B/C$

e) $b/\vec{\boldsymbol{A}} = b/A$

f) $\vec{\boldsymbol{A}} = (\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}})\vec{\boldsymbol{B}} + (\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}})\vec{\boldsymbol{C}} + (\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{D}})\vec{\boldsymbol{D}}$

(a) 2. Vector addition is commutative.

$$\vec{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{A} + \vec{B} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

$$\vec{B} + \vec{A} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

Hence, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(b) 2. A scalar can't be added to a vector, hence the equation still holds.

$$\vec{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad b = 4$$

$$\vec{A} + b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 4$$

$$b + \vec{A} = 4 + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $\vec{A} + b = b + \vec{A}$

(c) 2. Dot Products are Commutative, hence the equation holds true.

$$\vec{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{A} \cdot \vec{B} = (1 \times 3) + (2 \times 2) + (3 \times 1) = 10$$

$$\vec{B} \cdot \vec{A} = (3 \times 1) + (2 \times 2) + (1 \times 3) = 10$$

Hence, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

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(d) 1. There is no such things as vector division, it holds no geometric interpretation.

(e) 1. It is not possible to divide a scalar by a vector.

(f) 4. It is true iff $\vec{B}, \vec{C}, \vec{D}$ are orthonormal vectors.

$$\vec{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{C} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{D} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{A} = [(1\times1) + (2\times0) + (3\times0)]\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + [(1\times0) + (2\times1) + (3\times0)]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + [(1\times0) + (2\times0) + (3\times1)]\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

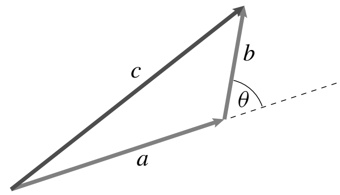
$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \vec{A}$$

1.2.12 Use the dot product to show 'the law of cosines'; i. e.,

$$c^2 = a^2 + b^2 + 2ab\cos\theta.$$

(Hint: $\vec{\boldsymbol{c}} = \vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$; also, $\vec{\boldsymbol{c}} \cdot \vec{\boldsymbol{c}} = \vec{\boldsymbol{c}} \cdot \vec{\boldsymbol{c}}$)



Problem 1.12

To Prove: $c^2 = a^2 + b^2 + 2ab\cos\theta$

$\vec{c} = \vec{a} + \vec{b}$

Write c^2 as $\vec{c} \times \vec{c}$

$c^2 = (\vec{a} + \vec{b})(\vec{a} + \vec{b})$

$= a^2 + b^2 + 2\times\vec{a}\cdot\vec{b}$

$= a^2 + b^2 + 2\times|\vec{a}||\vec{b}|\times\cos\theta$

$= a^2 + b^2 + 2ab\cos\theta$

= RHS

Hence, Proved.

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1.2.13 Find the direction cosines of $\vec{F} = 3\,\mathrm{N}\hat{\imath} - 4\,\mathrm{N}\hat{\jmath} + 5\,\mathrm{N}\hat{k}$.

$\vec{F} = (3\hat{\imath} - 4\hat{\jmath} + 5\hat{k})\,\mathrm{N}$

$|\vec{F}| = \sqrt{(3\,\mathrm{N})^2 + (-4\,\mathrm{N})^2 + (5\,\mathrm{N})^2}$

$= 5\sqrt{2}$

$\hat{\lambda} = \frac{3}{5\sqrt{2}}\,\hat{\imath} - \frac{4}{5\sqrt{2}}\,\hat{\jmath} + \frac{1}{\sqrt{2}}\,\hat{k}$

Find angle with x-axis,

$\theta = \cos^{-1}\left[\frac{\hat{\imath}\cdot\hat{\lambda}}{|\hat{\imath}|\,|\hat{\lambda}|}\right]$

$= \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$

$= 64.89^\circ$

Find angle with y-axis,

$\phi = \cos^{-1}\left[\frac{\hat{\jmath}\cdot\hat{\lambda}}{|\hat{\jmath}|\,|\hat{\lambda}|}\right]$

$= \cos^{-1}\left(\frac{4}{5\sqrt{2}}\right)$

$= 55.55^\circ$

Find angle with z-axis,

$\psi = \cos^{-1}\left[\frac{\hat{k}\cdot\hat{\lambda}}{|\hat{k}|\,|\hat{\lambda}|}\right]$

$= \cos^{-1}\left[1/\sqrt{2}\right]$

$= 45^\circ$

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1.2.14 A force acting on a bead of mass m is given as $\vec{F} = -20\,\mathrm{lbf}\,\hat{\imath} + 22\,\mathrm{lbf}\,\hat{\jmath} + 12\,\mathrm{lbf}\,\hat{k}$. What is the angle between the force and the z-axis?

$$\vec{F} = (-20\hat{\imath} + 22\hat{\jmath} + 12\hat{k})\,\mathrm{lbf}$$

$$|\vec{F}| = \sqrt{(-20\,\mathrm{lbf})^2 + (22\,\mathrm{lbf})^2 + (12\,\mathrm{lbf})^2}$$

$$= 2\sqrt{257}\,\mathrm{lbf}$$

Find angle with z-axis,

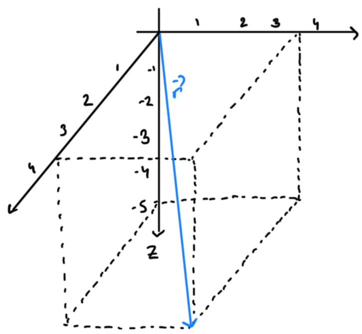
$$\psi = \cos^{-1}\left[\frac{\hat{k}\cdot\vec{F}}{|\hat{k}|\,|\vec{F}|}\right]$$

$$= \cos^{-1}\left[\frac{12\,\mathrm{lbf}}{2\sqrt{257}\,\mathrm{lbf}}\right]$$

$$= 68.02^\circ$$

1.2.15 (a) Draw the vector $\vec{r} = 3.5 \in \hat{\imath} + 3.5 \in \hat{\jmath} - 4.95 \in \hat{k}$. (b) Find the angle this vector makes with the z-axis. (c) Find the angle this vector makes with the x-y plane.

(a) $\vec{r} = 3.5 \text{ in } \hat{\imath} + 3.5 \text{ in } \hat{\jmath} - 4.95 \text{ in } \hat{k}$



(b) $\psi = \cos^{-1}\left[\dfrac{\hat{k}\cdot\vec{r}}{|\hat{k}||\vec{r}|}\right]$

$\hat{k}\cdot\vec{r} = -4.95 \text{ in}$

$|\hat{k}| = 1$

$|\vec{r}| = \sqrt{(3.5 \text{ in})^2 + (3.5 \text{ in})^2 + (-4.95 \text{ in})^2}$

$= 7 \text{ in}$

Find angle with z-axis,

$\psi = \cos^{-1}\left[\dfrac{-4.95 \text{ in}}{1 \times 7 \text{ in}}\right]$

$= 135^\circ$

(c) Let the x-y plane be denoted by $\hat{\lambda} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$

$\hat{\lambda}\cdot\vec{r} = \dfrac{3.5}{\sqrt{2}} + \dfrac{3.5}{\sqrt{2}}$

$= \dfrac{7}{\sqrt{2}}$

$|\hat{\lambda}| = 1$

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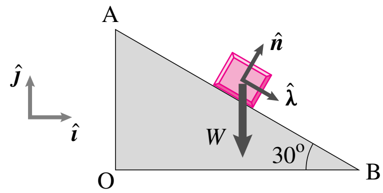
$$\theta = \cos^{-1}\left\{\frac{\hat{\lambda}\cdot\vec{r}}{|\hat{\lambda}||\vec{r}|}\right\}$$

$$= \cos^{-1}\left[\frac{\cancel{\pi}}{\sqrt{2}}\times\frac{1}{\cancel{\pi}}\right]$$

$$= 45^\circ$$

1.2.16 In the figure shown, $\hat{\boldsymbol{\lambda}}$ and $\hat{\boldsymbol{n}}$ are unit vectors parallel and perpendicular to the surface AB, respectively. A force $\vec{\boldsymbol{W}} = -50\,\mathrm{N}\,\hat{\boldsymbol{\jmath}}$ acts on the block. Find the components of $\vec{\boldsymbol{W}}$ along $\hat{\boldsymbol{\lambda}}$ and $\hat{\boldsymbol{n}}$.



Problem 1.16

Find unit vector in the direction of $\vec{\lambda}$,

$$\vec{\lambda} = \cos 330\,\hat{i} + \sin 330\,\hat{j}$$
$$= \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$|\vec{\lambda}| = 1$$

$$\hat{\lambda} = \frac{\vec{\lambda}}{|\vec{\lambda}|}$$

$$= \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$\vec{W} = -50\,N\hat{j}$$

Component of $\vec{W}$ along $\hat{\lambda}$: $\vec{W}\cdot\hat{\lambda}$

$$= \cancel{\left(0N \times \frac{\sqrt{3}}{2}\right)}^{0} + \left(\overset{25}{-50}N \times -\frac{1}{2}\right)$$

$$= 25N$$

$$\hat{n} = \cos 60\,\hat{i} + \sin 60\,\hat{j}$$

$$= \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$$

Component of $\vec{W}$ along $\hat{n}$: $\vec{W}\cdot\hat{n}$

$$= \left(0N \times \frac{1}{2}\right) + \left(-50N \times \frac{\sqrt{3}}{2}\right)$$

$$= -25\sqrt{3}N$$

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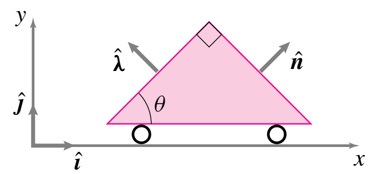
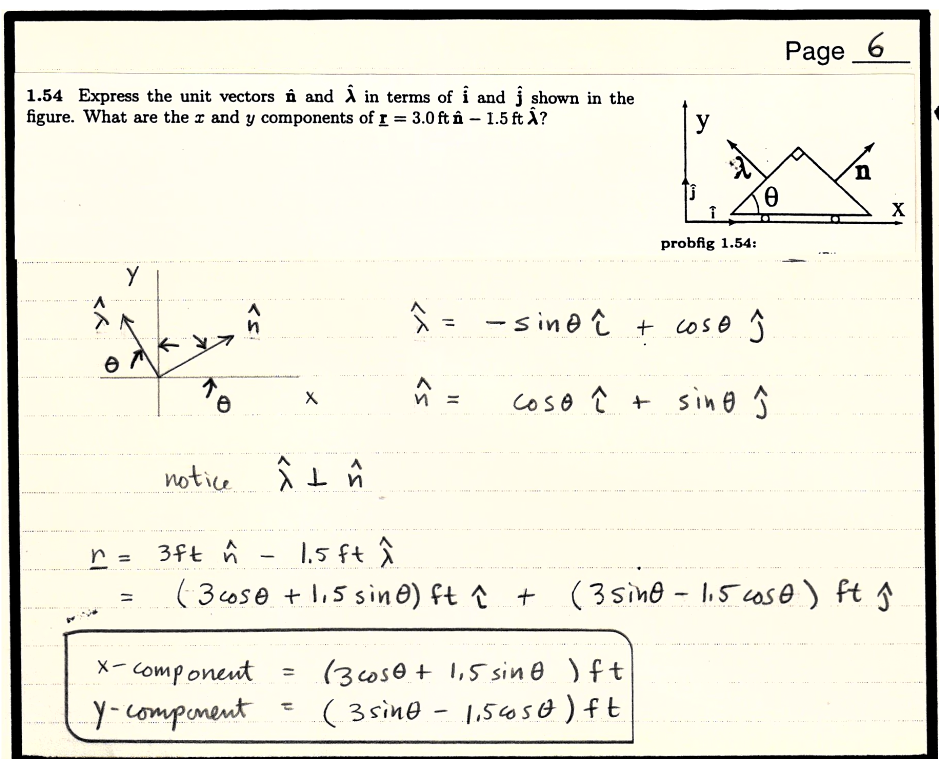
1.2.17 Express the unit vectors $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{\lambda}}$ in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ shown in the figure. What are the x and y components of $\vec{\boldsymbol{r}} = 3.0\,\mathrm{ft}\hat{\boldsymbol{n}} - 1.5\,\mathrm{ft}\hat{\boldsymbol{\lambda}}$?



Problem 1.17

Answer:
$r_x = \vec{\boldsymbol{r}}\cdot\hat{\boldsymbol{\imath}} = (3\cos\theta + 1.5\sin\theta)\,\mathrm{ft}$, $r_y = \vec{\boldsymbol{r}}\cdot\hat{\boldsymbol{\jmath}} = (3\sin\theta - 1.5\cos\theta)\,\mathrm{ft}$.



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1.54 Express the unit vectors $\hat{\mathbf{n}}$ and $\hat{\boldsymbol{\lambda}}$ in terms of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ shown in the figure. What are the x and y components of $\underline{\mathbf{r}} = 3.0\,\mathrm{ft}\,\hat{\mathbf{n}} - 1.5\,\mathrm{ft}\,\hat{\boldsymbol{\lambda}}$?

probfig 1.54:

$\hat{\lambda} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$\hat{n} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

notice $\hat{\lambda} \perp \hat{n}$

$$\underline{r} = 3\,\mathrm{ft}\,\hat{n} - 1.5\,\mathrm{ft}\,\hat{\lambda} = (3\cos\theta + 1.5\sin\theta)\,\mathrm{ft}\,\hat{\imath} + (3\sin\theta - 1.5\cos\theta)\,\mathrm{ft}\,\hat{\jmath}$$

x-component = $(3\cos\theta + 1.5\sin\theta)\,\mathrm{ft}$
y-component = $(3\sin\theta - 1.5\cos\theta)\,\mathrm{ft}$

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$$\vec{r} = 3.0\,ft\,\hat{n} - 1.5\,ft\,\hat{\lambda}$$

$$\hat{n} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\lambda} = \cos(\theta + 90)\,\hat{i} + \sin(\theta + 90)\,\hat{j}$$

$$= -\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

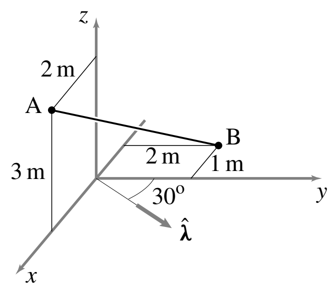
$$\vec{r} = 3.0ft\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) - 1.5ft\,(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= (4.5\cos\theta\,\hat{i} + 1.5\sin\theta\,\hat{j})\,ft$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.2.18 Find the projection of vector $\vec{r}_{AB}$ (you have to first find this position vector)

1. on the y-axis, and
2. in the $\hat{\lambda}$ direction.



Problem 1.18

$\vec{r}_A = (3m)\hat{i} + (2m)\hat{k}$

$\vec{r}_B = (-1m)\hat{i} + (2m)\hat{j}$

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$= (-4m)\hat{i} + (2m)\hat{j} - (2m)\hat{k}$

1. Find Dot Product,

$\vec{r}_{AB} \cdot \hat{j} = (-4m \times 0) + (2m \times 1) + (-2m \times 0)$

$= 2m$

2. $\hat{\lambda} = \sin 30 \hat{i} + \cos 30 \hat{j}$

$= \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

Find Dot Product,

$\vec{r}_{AB} \cdot \hat{\lambda} = \left(-4m \times \frac{1}{2}\right) + \left(2m \times \frac{\sqrt{3}}{2}\right) + \left(-2m \times 0\right)$

$= (-2 + \sqrt{3})m$

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1.2.19 The net force acting on a particle is $\vec{F} = 2\,\mathrm{N}\hat{\imath} + 10\,\mathrm{N}\hat{\jmath}$. Find the components of this force in another coordinate system with basis vectors $\hat{\imath}' = -\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$ and $\hat{\jmath}' = -\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}$. For $\theta = 30^\circ$, sketch the vector $\vec{F}$ and show its components in the two coordinate systems.

$\vec{F} = 2N\hat{\imath} + 10N\hat{\jmath}$

$\hat{\imath}' = -\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\jmath}' = -\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}$

Find $\hat{\imath}$ in terms of $\hat{\imath}'$ and $\hat{\jmath}'$.

$\cos\theta\,\hat{\imath}' = -\cos^2\theta\,\hat{\imath} + \sin\theta\cos\theta\,\hat{\jmath} \quad - (i)$

$\sin\theta\,\hat{\jmath}' = -\sin^2\theta\,\hat{\imath} - \sin\theta\cos\theta\,\hat{\jmath} \quad - (ii)$

$(i) + (ii)$

$\cos\theta\,\hat{\imath}' + \sin\theta\,\hat{\jmath}' = -(\cos^2\theta + \sin^2\theta)\,\hat{\imath}$

$\hat{\imath} = -(\cos\theta\,\hat{\imath}' + \sin\theta\,\hat{\jmath}')$

$\sin\theta\,\hat{\imath}' = -\cos\theta\sin\theta\,\hat{\imath} + \sin^2\theta\,\hat{\jmath} \quad - (iii)$

$\cos\theta\,\hat{\jmath}' = -\cos\theta\sin\theta\,\hat{\imath} - \cos^2\theta\,\hat{\jmath} \quad - (iv)$

$(iii) - (iv)$

$\sin\theta\,\hat{\imath}' - \cos\theta\,\hat{\jmath}' = (\sin^2\theta + \cos^2\theta)\,\hat{\jmath}$

$\hat{\jmath} = \sin\theta\,\hat{\imath}' - \cos\theta\,\hat{\jmath}'$

Plug in $\hat{\imath}$ and $\hat{\jmath}$ into $\vec{F}$,

$\vec{F} = -2N(\cos\theta\,\hat{\imath}' + \sin\theta\,\hat{\jmath}') + 10N(\sin\theta\,\hat{\imath}' - \cos\theta\,\hat{\jmath}')$

$= (10N\sin\theta - 2N\cos\theta)\,\hat{\imath}' - (10N\cos\theta + 2N\sin\theta)\,\hat{\jmath}'$

$\theta = 30^\circ$

$\vec{F} = (5N - \sqrt{3}\,N)\,\hat{\imath}' - (5\sqrt{3}\,N + 1N)\,\hat{\jmath}'$

$= (3.26N)\,\hat{\imath}' - (9.66)\,\hat{\jmath}'$

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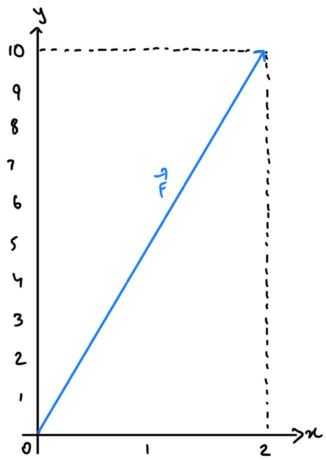
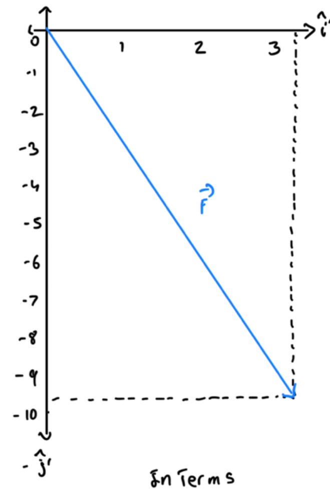
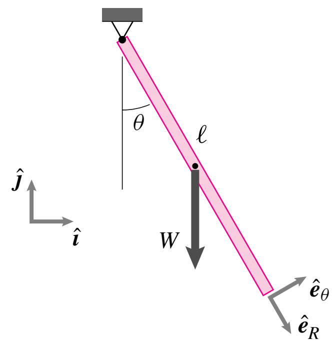


In Terms of $\hat{\imath}$ and $\hat{\jmath}$



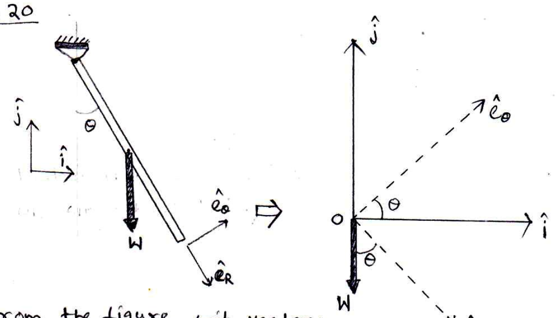
In Terms of $\hat{\imath}$ and $\hat{\jmath}$

1.2.20 Find the unit vectors $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ with the geometry shown in the figure. What are the components of $\vec{\boldsymbol{W}}$ along $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$?



Problem 1.20

2.2.20



From the figure unit vectors $\hat{e}_\theta$ and $\hat{e}_r$ can be written in terms of $\hat{i}$ & $\hat{j}$.

$$\boxed{\begin{aligned}\hat{e}_\theta &= \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \\ \hat{e}_R &= \sin\theta\,\hat{i} - \cos\theta\,\hat{j}\end{aligned}}$$

component of W along $\boxed{\hat{e}_R = W\cos\theta}$

" " $\boxed{\hat{e}_\theta = -W\sin\theta}$

Thus W can be written as

$$W = (W\cos\theta)\hat{e}_R - (W\sin\theta)\hat{e}_\theta$$

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$\vec{e}_R = \cos(270+\theta)\hat{i} + \sin(270+\theta)\hat{j}$

$= \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

$\hat{e}_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\vec{\omega} = -|\vec{\omega}|\hat{j}$

Find Dot Product,

$\vec{\omega}\cdot\hat{e}_R = (\cancel{0 \times \cos\theta})^{0} + (-|\vec{\omega}| \times (-\sin\theta))$

$= |\vec{\omega}|\sin\theta$

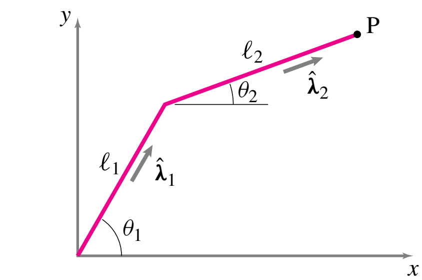
$\vec{\omega}\cdot\vec{e}_\theta = (\cancel{0 \times \cos\theta})^{0} + (-|\vec{\omega}| \times \sin\theta))$

$= -|\vec{\omega}|\sin\theta$

1.2.21 Write the position vector of point P in terms of $\hat{\boldsymbol{\lambda}}_1$ and $\hat{\boldsymbol{\lambda}}_2$ and

1. find the y-component of $\vec{\boldsymbol{r}}_\text{P}$,

2. find the component of $\vec{\boldsymbol{r}}_\text{P}$ along $\hat{\boldsymbol{\lambda}}_1$.



Problem 1.21

$$\vec{r}_P = l_1 \hat{\lambda}_1 + l_2 \hat{\lambda}_2$$

$$\hat{\lambda}_1 = \cos\theta_1 \hat{i} + \sin\theta_1 \hat{j}$$

$$\hat{\lambda}_2 = \cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}$$

$$\vec{r}_P = (l_1 \cos\theta_1 + l_2 \cos\theta_2)\hat{i} + (l_1 \sin\theta_1 + l_2 \sin\theta_2)\hat{j}$$

1. $\vec{r}_P \cdot \hat{j} = (l_1 \sin\theta_1 + l_2 \sin\theta_2)$

2. $\vec{r}_P \cdot \hat{\lambda}_1 = (l_1 \cos\theta_1 + l_2 \cos\theta_2)\, l_1 \cos\theta_1 + (l_1 \sin\theta_1 + l_2 \sin\theta_2)\, l_1 \sin\theta_1$

$$= l_1(\cos^2\theta_1 + \sin^2\theta_1) + l_1 l_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2)$$

$$= l_1 + l_1 l_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2)$$

1.2.22
Let $\vec{F}_1 = 30\,\mathrm{N}\hat{\imath} + 40\,\mathrm{N}\hat{\jmath} - 10\,\mathrm{N}\hat{k}, \vec{F}_2 = -20\,\mathrm{N}\hat{\jmath} + 2\,\mathrm{N}\hat{k}$, and $\vec{F}_3 = F_{3_x}\hat{\imath} + F_{3_y}\hat{\jmath} - F_{3_z}\hat{k}$. If the sum of all these forces must equal zero, find the required scalar equations to solve for the components of $\vec{F}_3$.

2.2.22

$$\vec{F}_1 = 30\mathrm{N}\,\hat{i} + 40\mathrm{N}\,\hat{j} - 10\mathrm{N}\,\hat{k}$$
$$\vec{F}_2 = -20\mathrm{N}\,\hat{j} + 2\mathrm{N}\,\hat{k}$$
$$\vec{F}_3 = F_{3x}\hat{i} + F_{3y}\hat{j} - F_{3z}\hat{k}$$

Sum of all 3 forces are zero

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

$$\Rightarrow (30\mathrm{N} + F_{3x})\,\hat{i} + (40\mathrm{N} - 20\mathrm{N} + F_{3y})\,\hat{j} + (-10\mathrm{N} + 2\mathrm{N} - F_{3z})\,\hat{k} = \vec{0}$$

For above to be true each component must be equal to zero,

$$30\mathrm{N} + F_{3x} = 0 \Rightarrow \boxed{F_{3x} = -30\mathrm{N}}$$
$$40\mathrm{N} - 20\mathrm{N} + F_{3y} = 0 \Rightarrow \boxed{F_{3y} = -20\mathrm{N}}$$
$$-10\mathrm{N} + 2\mathrm{N} - F_{3z} = 0 \Rightarrow \boxed{F_{3z} = -8\mathrm{N}}$$

Thus $\boxed{\vec{F}_3 = -30\mathrm{N}\,\hat{i} - 20\mathrm{N}\,\hat{j} + 8\mathrm{N}\,\hat{k}}$

$\vec{F}_1 = 30\text{N}\,\hat{i} + 40\text{N}\,\hat{j} - 10\text{N}\,\hat{k}$

$\vec{F}_2 = -20\hat{j} + 2\text{N}\,\hat{k}$

$\vec{F}_3 = F_{3x}\,\hat{i} + F_{3y}\,\hat{j} + F_{3z}\,\hat{k}$

Add Vectors and Equate to $\vec{0}$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

$30\text{N}\,\hat{i} + 40\text{N}\,\hat{j} - 10\text{N}\,\hat{k} - 20\text{N}\,\hat{j} + 2\text{N}\,\hat{k} + \vec{F}_3 = \vec{0}$

$\vec{F}_3 = -30\text{N}\,\hat{i} - 20\text{N}\,\hat{j} + 8\text{N}\,\hat{k}$

$\Rightarrow F_{3x} = -30\text{N}, \; F_{3y} = -20\text{N}, \; F_{3z} = 8\text{N}$

1.2.23 A force $\vec{F}$ is directed from point A(3,2,0) to point B(0,2,4). If the x-component of the force is 120 N, find the y- and z-components of $\vec{F}$.

$\vec{r}_A = 3\hat{i} + 2\hat{j}$

$\vec{r}_B = 2\hat{j} + 4\hat{k}$

Subtract Position vector of A from B,

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$= -3\hat{i} + 4\hat{k}$

$\vec{F} = \vec{r}_{AB} \times \frac{\overset{40}{\cancel{120}}\,N}{-\cancel{3}}$

$= -40N \times \vec{r}_{AB}$

$= 120N\hat{i} - 160N\hat{k}$

$F_y = 0$, $F_z = -160N$

1.2.24 A vector equation for the sum of forces results in the following equation:

$$\frac{F}{2}(\hat{\imath} - \sqrt{3}\hat{\jmath}) + \frac{R}{5}(3\hat{\imath} + 6\hat{\jmath}) = 25\,\mathrm{N}\hat{\lambda}$$

where $\hat{\lambda} = 0.30\hat{\imath} - 0.954\hat{\jmath}$. Find two scalar equations by dotting both sides of the equation first with $\hat{\lambda}$ and then separately with a vector orthogonal to $\hat{\lambda}$.

$$\frac{F}{2}(\hat{i} - \sqrt{3}\hat{j}) + \frac{R}{5}(3\hat{i} + 6\hat{j}) = 25\,N\hat{\lambda}$$

$$\left(\frac{F}{2} + \frac{3R}{5}\right)\hat{i} + \left(-\frac{F\sqrt{3}}{2} + \frac{6R}{5}\right)\hat{j} = 25N\hat{\lambda}$$

$$\hat{\lambda} = 0.3\hat{i} - 0.954\hat{j}$$

Dot both sides by $\hat{\lambda}$,

$$\frac{3}{10}\left(\frac{F}{2} + \frac{3R}{5}\right) + (-0.954)\left(\frac{-F\sqrt{3}}{2} + \frac{6R}{5}\right) = 25\left(0.3^2 + (0.954)^2\right)$$

$$\frac{F}{2}\left(\frac{3}{10} + (0.954 \times \sqrt{3})\right) + \frac{3R}{5}\left(\frac{3}{10} + (2 \times (-0.954))\right) = 25$$

$$\hat{\lambda}' \cdot \hat{\lambda} = 0 \quad [\text{Perpendicular}]$$

Let $\hat{\lambda}'$ be $x\hat{i} + y\hat{j}$

$$0.3x - 0.954y = 0$$

$$x = \frac{0.954}{0.3}y$$

$$1 = \sqrt{x^2 + y^2}$$

$$1 = \left(\left(\frac{0.954}{0.3}\right)^2 + 1\right)y^2$$

$$y = \left(\left(\frac{0.954}{0.3}\right)^2 + 1\right)^{-1/2}$$

$$= 0.299 \sim 0.3$$

$$x = 0.954$$

$$\hat{\lambda}' = 0.954\hat{i} + 0.3\hat{j}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\left(\frac{F}{2} + \frac{3R}{5}\right)\hat{\imath} + \left(-\frac{F\sqrt{3}}{2} + \frac{6R}{5}\right)\hat{\jmath} = 0 \cdot \hat{\lambda}'$$

Dot with $\hat{\lambda}'$,

$$0.954\left(\frac{F}{2} + \frac{3R}{5}\right) + \frac{3}{10}\left(-\frac{F\sqrt{3}}{2} + \frac{6R}{5}\right) = 0$$

$$\frac{F}{2}\left(0.954 - \frac{3\sqrt{3}}{10}\right) + \frac{3R}{5}\left(0.954 + \frac{3}{25}\right) = 0$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.2.25 Write a computer program (or use a canned program) to find the dot product of two 3-D vectors. Test the program by computing the dot products $\hat{\imath} \cdot \hat{\imath}$, $\hat{\imath} \cdot \hat{\jmath}$, and $\hat{\jmath} \cdot \hat{k}$. Now use the program to find the projections of $\vec{F} = (2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})\,\text{N}$ along the line $\vec{r}_{AB} = (0.5\hat{\imath} - 0.2\hat{\jmath} + 0.1\hat{k})\,\text{m}$.

Code :

```
i = [1;0;0];
j = [0;1;0];
k = [0;0;1];

% Testing Program %
disp("Start Function Test")
dot(i,i)
dot(i,j)
dot(j,k)
disp("Function Test Completed")

% Main Question %
F = [2;2;-3];
rAB = [0.5;-0.2;0.1];
disp(" ");
disp("Answer: ")
dot(F,rAB)

function scalar = dot(v1,v2)
    scalar = 0;
    for i = 1:3
        scalar = scalar + (v1(i)*v2(i));
    end
end
```

Output of Code

Start Function Test
1
0
0
Function Test Completed
Answer : 0.3

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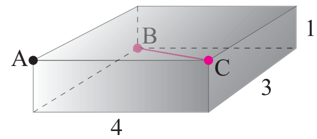
1.2.26 What is the shortest distance between the point A and the diagonal BC of the parallelepiped shown? (Use vector methods and don't use cross product.)



Problem 1.26

$\vec{r}_A = 0\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{r}_B = 3\hat{j} - \hat{k}$

$\vec{r}_C = 4\hat{i}$

$\vec{r}_{BC} = \vec{r}_B - \vec{r}_C$

$= 4i - 3\hat{j} + \hat{k}$

Coordinates for any point on BC : $1z = -3y = 4x = \lambda$

$\Rightarrow x = \frac{1}{4}\lambda, \quad y = -\frac{1}{3}\lambda, \quad z = \lambda$

Vetor from A to any point on BC : $(\frac{1}{4}\lambda - 0)\hat{i} + (-\frac{1}{3}\lambda - 0)\hat{j} + (\lambda - 0)\hat{k}$

$\vec{r}_{BC} \cdot \vec{r}_{BC/A} = 0$

$(4 \times \frac{1}{4}\lambda) + (-3 \times -\frac{1}{3}\lambda) + (1 \times \lambda) = 0$

$3\lambda = 0$

$\lambda = \frac{1}{3}$

Point on BC closest to A: $\frac{3}{4}\hat{i} - \hat{j} + \frac{1}{3}\hat{k}$

Shortest Distance $= \sqrt{(3/4)^2 + (1)^2 + (1/3)^2}$

$= \frac{\sqrt{241}}{12}$