1.1 Vector notation and vector addition

1.1.1 Draw the vector $\vec{r} = (5\text{ m})\hat{\imath} + (5\text{ m})\hat{\jmath}$.

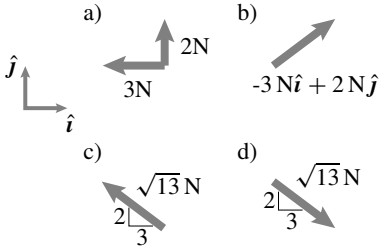
1.1.2 A vector $\vec{a}$ is 2 m long and points northwest at an angle 60° from the north. Draw the vector.

1.1.3 The position vector of a point B measures 3 m and is directed at 40° CCW from the negative x-axis. Show the position vector.

1.1.4 Draw a force vector that is given as $\vec{F} = 2\text{ N}\hat{\imath} + 2\text{ N}\hat{\jmath} + 1\text{ N}\hat{k}$.

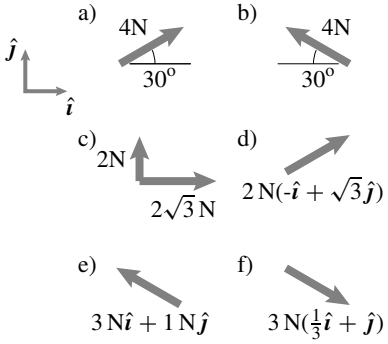
1.1.5 Represent the vector $\vec{r} = 5\text{ m}\hat{\imath} - 2\text{ m}\hat{\jmath}$ in three different ways.

1.1.6 Which one of the following representations of the same vector $\vec{F}$ is wrong and why?



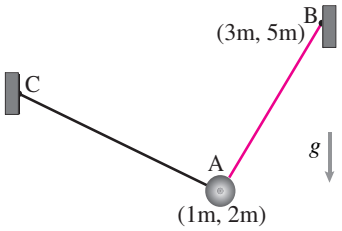
Problem 1.1.6

1.1.7 There are exactly two pairs that describe the same vector in the following pictures. The other two don't match. Match the correct pictures into pairs.



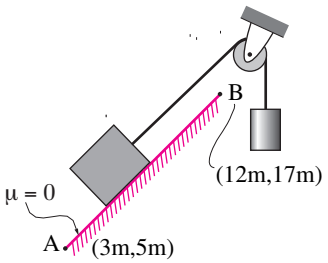
Problem 1.1.7

1.1.8 A string connects a particle A at (1m, 2m) to a support B at (3m, 5m). The tension in the string is 10N. There are other strings also holding the particle in place. What is the force of string AB on the particle?



Problem 1.1.8

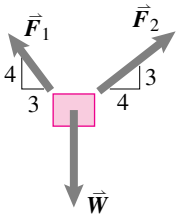
1.1.9 A frictionless ramp connects A at (3m, 5m) to B at (12m, 17m). The ramp pushes a block with a force of 50N normal to the ramp surface. Express the force from the ramp as a vector $\vec{F}$ (ignore the other forces that also act on the block holding it in place).



Problem 1.1.9

1.1.10 Find the sum of forces $\vec{F}_1 = 20\text{ N}\hat{\imath} - 2\text{ N}\hat{\jmath}$, $\vec{F}_2 = 30\text{ N}(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath})$, and $\vec{F}_3 = -20\text{ N}(-\hat{\imath} + \sqrt{3}\hat{\jmath})$.

1.1.11 The forces acting on a block of mass $m = 5$ kg are shown in the figure, where $F_1 = 20$ N, $F_2 = 50$ N, and $W = mg$. Find the sum $\vec{F}$ ($= \vec{F}_1 + \vec{F}_2 + \vec{W}$).



Problem 1.1.11

1.1.12 Given that the sum of four vectors $\vec{F}_i, i = 1$ to 4, is zero, where $\vec{F}_1 = 20\text{ N}\hat{\imath}$, $\vec{F}_2 = -50\text{ N}\hat{\jmath}$, $\vec{F}_3 = 10\text{ N}(-\hat{\imath} + \hat{\jmath})$, find $\vec{F}_4$.

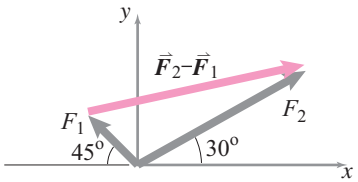
1.1.13 Three forces $\vec{F} = 2\text{ N}\hat{\imath} - 5\text{ N}\hat{\jmath}$, $\vec{R} = 10\text{ N}(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})$ and $\vec{W} = W\text{ N}\hat{\jmath}$ with $W > 0$, sum up to zero. Determine θ and W and draw the force vector $\vec{R}$ clearly showing its direction.

1.1.14 Given that $\vec{R}_1 = 1\text{ N}\hat{\imath} + 1.5\text{ N}\hat{\jmath}$ and $\vec{R}_2 = 3.2\text{ N}\hat{\imath} - 0.4\text{ N}\hat{\jmath}$, find $2\vec{R}_1 + 5\vec{R}_2$.

1.1.15 Find the magnitudes of the forces $\vec{F}_1 = 30\text{ N}\hat{\imath} - 40\text{ N}\hat{\jmath}$ and $\vec{F}_2 = 30\text{ N}\hat{\imath} + 40\text{ N}\hat{\jmath}$. Draw the two forces, representing them with their magnitudes.

1.1.16 Two forces $\vec{R} = 2\text{ N}(0.16\hat{\imath} + 0.80\hat{\jmath})$ and $\vec{W} = -36\text{ N}\hat{\jmath}$ act on a particle. Find the magnitude of the net force. What is the direction of this force?

1.1.17 In the figure shown, $F_1 = 100$ N and $F_2 = 300$ N. Find the magnitude and direction of $\vec{F}_2 - \vec{F}_1$.



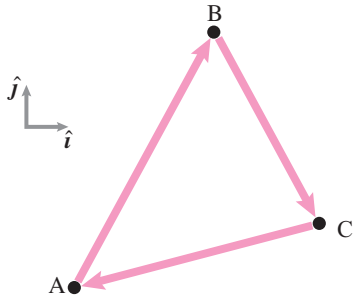
Problem 1.1.17

1.1.18 Two points A and B are located in the xy plane. The coordinates of A and B are (4 mm, 8 mm) and (90 mm, 6 mm), respectively.

1. Draw position vectors $\vec{r}_A$ and $\vec{r}_B$.
2. Find the magnitude of $\vec{r}_A$ and $\vec{r}_B$.
3. How far is A from B?

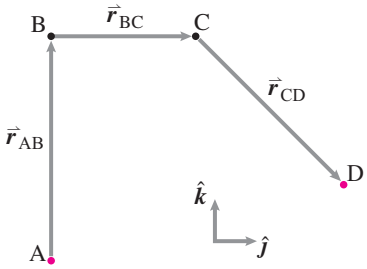
1.1.19 Three position vectors are shown in the figure below. Given that $\vec{r}_{B/A} =$

$3\,\text{m}(\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})$ and $\vec{r}_{C/B} = 1\,\text{m}\hat{\imath} - 2\,\text{m}\hat{\jmath}$, find $\vec{r}_{A/C}$.



Problem 1.1.19

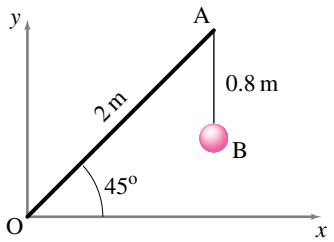
1.1.20 In the figure shown below, the position vectors are $\vec{r}_{AB} = 3\,\text{ft}\hat{k}$, $\vec{r}_{BC} = 2\,\text{ft}\hat{\jmath}$, and $\vec{r}_{CD} = 2(\hat{\jmath} - \hat{k})\,\text{ft}$. Find the position vector $\vec{r}_{AD}$.



Problem 1.1.20

1.1.21 In the figure shown, a ball is suspended with a 0.8 m long cord from a 2 m long hoist OA.

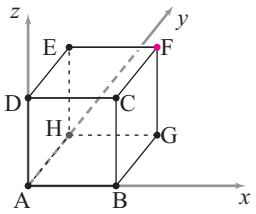
1. Find the position vector $\vec{r}_B$ of the ball.
2. Find the distance of the ball from the origin.



Problem 1.1.21

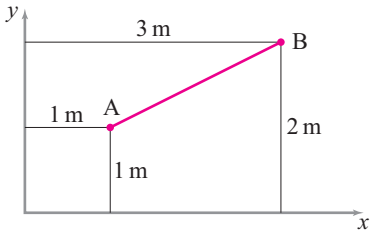
1.1.22 A cube of side 6 ∈ is shown in the figure.

1. Find the position vector of point F, $\vec{r}_F$, from the vector sum $\vec{r}_F = \vec{r}_D + \vec{r}_{C/D} + \vec{r}_{F/c}$.
2. Calculate $|\vec{r}_F|$.
3. Find $\vec{r}_G$ using $\vec{r}_F$.



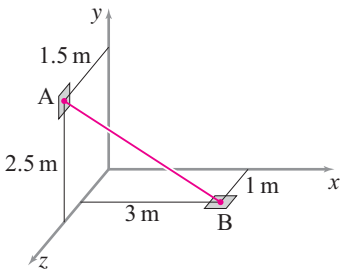
Problem 1.1.22

1.1.23 Find the unit vector $\hat{\lambda}_{AB}$, directed from point A to point B shown in the figure.



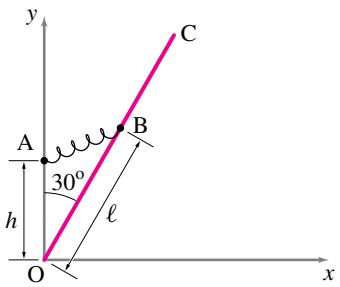
Problem 1.1.23

1.1.24 Find a unit vector along string BA and express the position vector of A with respect to B, $\vec{r}_{A/B}$, in terms of the unit vector.



Problem 1.1.24

1.1.25 In the structure shown in the figure, $\ell = 2\,\text{ft}, h = 1.5\,\text{ft}$. The force on B from the spring is $\vec{F} = k\vec{r}_{AB}$, where $k = 100\,\text{lbf/ft}$. Find a unit vector $\hat{\lambda}_{AB}$ along AB and calculate the spring force $\vec{F} = F\hat{\lambda}_{AB}$.



Problem 1.1.25

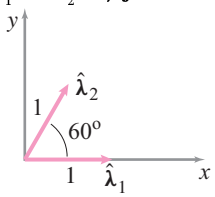
1.1.26 Express the vector $\vec{r}_A = 2\,\text{m}\hat{\imath} - 3\,\text{m}\hat{\jmath} + 5\,\text{m}\hat{k}$ in terms of its magnitude and a unit vector indicating its direction.

1.1.27 Let $\vec{F} = 10\,\text{lbf}\hat{\imath} + 30\,\text{lbf}\hat{\jmath}$ and $\vec{W} = -20\,\text{lbf}\hat{\jmath}$. Find a unit vector in the direction of the net force $\vec{F} + \vec{W}$, and express the net force in terms of the unit vector.

1.1.28 Let $\hat{\lambda}_1 = 0.80\hat{\imath} + 0.60\hat{\jmath}$ and $\hat{\lambda}_2 = 0.5\hat{\imath} + 0.866\hat{\jmath}$.

1. Show that $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are unit vectors.
2. Is the sum of these two unit vectors also a unit vector? If not, find a unit vector along the sum of $\hat{\lambda}_1$ and $\hat{\lambda}_2$.

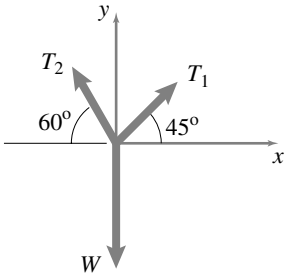
1.1.29 For the unit vectors $\hat{\lambda}_1$ and $\hat{\lambda}_2$ shown below, find the scalars α and β such that $\alpha\hat{\lambda}_1 - 3\hat{\lambda}_2 = \beta\hat{\jmath}$.



Problem 1.1.29

1.1.30 If a mass slides from point A towards point B along a straight path and the coordinates of points A and B are (0 ∈, 5 ∈, 0 ∈) and (10 ∈, 0 ∈, 10 ∈), respectively, find the unit vector $\hat{\lambda}_{AB}$ directed from A to B along the path.

1.1.31 In the figure shown, $T_1 = 20\sqrt{2}\,\text{N}, T_2 = 40\,\text{N}$, and W is such that the sum of the three forces equals zero. If W is doubled, find α and β such that $\alpha\vec{T}_1, \beta\vec{T}_2$, and $2\vec{W}$ still sum up to zero.

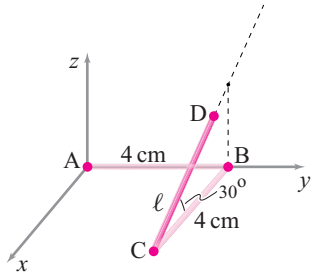


Problem 1.1.31

1.1.32 In the figure shown, rods AB and BC are each 4 cm long and lie along y and x axes, respectively. Rod CD is in the xz plane and makes an angle $\theta = 30^\circ$ with the x-axis.

1. Find $\vec{r}_{AD}$ in terms of the variable length ℓ.
2. Find ℓ and α such that

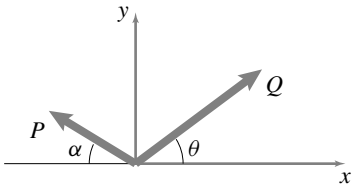
$$\vec{r}_{AD} = \vec{r}_{AB} - \vec{r}_{BC} + \alpha\hat{k}.$$



Problem 1.1.32

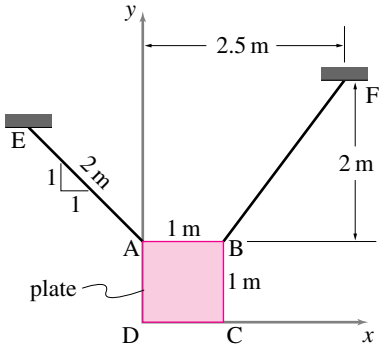
1.1.33 In Problem 1.1.32, find ℓ such that the length of the position vector $\vec{r}_{AD}$ is 6 cm.

1.1.34 Let two forces $\vec{P}$ and $\vec{Q}$ act in the directions shown in the figure. You are allowed to change the direction of the forces by changing the angles α and θ while keeping the magnitudes fixed. What are the values of α and θ that maximize the magnitude of $\vec{P} + \vec{Q}$?



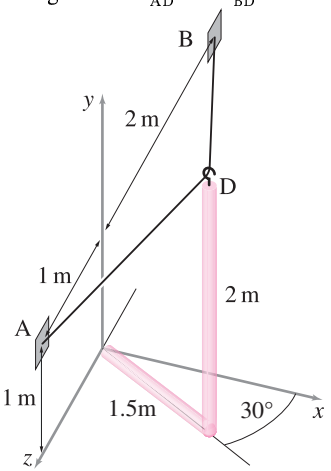
Problem 1.1.34

1.1.35 A 1 m × 1 m square board is supported by two strings AE and BF. The tension in the string BF is 20 N. Express this tension as a vector.



Problem 1.1.35

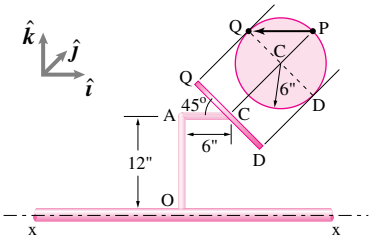
1.1.36 The top of an L-shaped bar, shown in the figure, is to be tied by strings AD and BD to the points A and B in the yz plane. Find the length of the strings AD and BD using vectors $\vec{r}_{AD}$ and $\vec{r}_{BD}$.



Problem 1.1.36

1.1.37 A circular disk of radius 6∈ is mounted on axle x-x at the end of an L-shaped bar as shown in the figure. The disk is tipped 45° with respect to the horizontal bar AC. Two points, D and Q, are marked on the rim of the disk; with CP directly into the page, and Q at the highest point above the center C. Taking the base vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ as shown in the figure ($\hat{\jmath}$ into the page), find

1. the relative position vector $\vec{r}_{Q/P}$,
2. the magnitude $|\vec{r}_{Q/P}|$.



Problem 1.1.37

1.1.38 Write the vectors $\vec{F}_1 = 30\,\text{N}\hat{\imath} + 40\,\text{N}\hat{\jmath} - 10\,\text{N}\hat{k}$, $\vec{F}_2 = -20\,\text{N}\hat{\jmath} + 2\,\text{N}\hat{k}$, and $\vec{F}_3 = -10\,\text{N}\hat{\imath} - 100\,\text{N}\hat{k}$ as a list of numbers (rows or columns). Find the sum of the forces using a computer.

1.1.39 Let $\alpha\vec{F}_1 + \beta\vec{F}_2 + \gamma\vec{F}_3 = \vec{0}$, where $\vec{F}_1, \vec{F}_2,$ and $\vec{F}_3$ are as given in Problem 1.1.38. Solve for $\alpha, \beta,$ and γ using a computer.

1.1.40 Let $\vec{r}_n = 1\,\text{m}(\cos\theta_n\hat{\imath} + \sin\theta_n\hat{\jmath})$, where $\theta_n = \theta_0 - n\Delta\theta$. Using a computer generate the required vectors and find the sum

$$\sum_{n=0}^{44} \vec{r}_i, \quad \text{with } \Delta\theta = 1^\circ \quad \text{and} \quad \theta_0 = 45^\circ.$$

1.1.41 Find two non-zero and non-parallel vectors $\vec{A}$ and $\vec{B}$ so that $|\vec{A} + 2\vec{B}| = 2|\vec{A} + \vec{B}|$. *