CHAPTER **1**

Vectors for mechanics: position, force and moment

Mechanical analysis uses vectors. We develop vector skills here using the key vectors for statics, namely relative position, force, and moment. Notational clarity is emphasized because good vector calculation demands distinguishing vectors from scalars. Vector addition is motivated by the need to add forces and relative positions. Dot products are motivated as the tool which reduces vector equations to scalar equations. And cross products are motivated as the formula which correctly calculates the heuristically-motivated quantities of moment and moment about an axis.

In these books you will learn to use the laws of mechanics which were informally introduced in Chapter 1. This means calculating with mechanics-relevant quantities. The most fundamental quantities in mechanics are the two scalars,

- mass m and

- time t,

and the two vectors,

- relative position $\vec{\boldsymbol{r}}_{i/\mathrm{O}}$ (position of point i relative to point O), and
- force $\vec{\boldsymbol{F}}$ (acting on a system of interest) ①

①Actually, one can argue that $\vec{\boldsymbol{F}}$ isn't fundamental because it can be defined in terms of m and $\vec{\boldsymbol{a}}$ which, in turn, are defined in terms of $\vec{\boldsymbol{r}}$ and t. But, both historically and in most engineering practice, $\vec{\boldsymbol{F}}$ is treated as a fundamental quantity.

Scalars are typed with an italic font (t and m) and vectors are typed in bold with a harpoon on top ($\vec{\boldsymbol{r}}_{i/\mathrm{O}}, \vec{\boldsymbol{F}}$). All the other scalars (see box 1.1 on page 47) and vectors (see box 1.2 on page 48) that we use in mechanics are defined in terms of $m, t, \vec{\boldsymbol{r}}_{i/\mathrm{O}}$ and $\vec{\boldsymbol{F}}$. If you are a legitimate reader of this book, you are already good at scalar arithmetic and algebra (adding, subtracting, multiplying and dividing ordinary numbers and symbols representing numbers). For mechanics you also need facility with the vector arithmetic and algebra, as explained in this chapter.

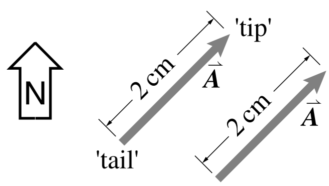

Figure 1.1: Vector $\vec{\boldsymbol{A}}$ is 2 cm long and points Northeast. Two copies of $\vec{\boldsymbol{A}}$ are drawn to show that where the drawing is placed doesn't affect the vector. It's the same vector drawn in two different places.

What is a vector?

Whereas a scalar is just a (possibly dimensional②), single number, a thing with magnitude and a sign,

a vector is a (possibly dimensional) quantity that is fully described by both magnitude and direction.

② By 'dimensional' we mean 'with units' like meters, Newtons, or kg. (We *don't* mean: having an abstract vector-space dimension, as in one, two or three dimensional.

Example: NorthEast 2 cm
As a first vector example, consider a line segment with a length (magnitude) of 2 cm. The segment has a tail end and a head end. It is pointed Northeast. Let's call this vector $\vec{\boldsymbol{A}}$ (see fig. 1.1).

$$\vec{\boldsymbol{A}} \stackrel{def}{=} \text{2 cm long line segment pointed Northeast}$$

In terms of our basic list at the top of the page, $\vec{\boldsymbol{A}}$ is the relative position $\vec{\boldsymbol{r}}_{h/t}$ of its head h relative to its tail t.

Every vector in mechanics is well visualized as an arrow. The direction of the arrow is the direction of the vector. The length of the arrow is proportional to the magnitude of the vector. The magnitude of $\vec{\boldsymbol{A}}$ is a positive scalar indicated by $|\vec{\boldsymbol{A}}|$. A vector does not lose its identity if it is picked up and moved around in space (so long as it is not rotated nor stretched). Thus, both vectors drawn in fig. 1.1 are the same vector $\vec{\boldsymbol{A}}$.

Vector arithmetic makes sense

We have oversimplified. We said that a vector is something with magnitude and direction. In fact, by common modern convention, that's not enough. A one way street sign, for example, is not considered a vector even though it has a magnitude (its mass is, say, half a kilogram) and a direction (the direction that most of the traffic goes). We need more to call something a vector. A thing is only called a vector if, additionally, elementary vector arithmetic, vector addition in particular, has a sensible meaning[3].

The following sentence summarizes centuries of thought and also motivates this chapter:

> The vectors in mechanics have magnitude and direction *and* elementary vector arithmetic operations have sensible physical meanings.

This chapter is about vector arithmetic. In this chapter you will learn how to add and subtract vectors, how to stretch them, how to find their components, and how to multiply them with each other two different ways. Each of these operations has use in mechanics.

[3] In abstract mathematics they don't bother to talk about magnitudes and directions. All they care about is vector arithmetic. So, to the mathematicians, anything which obeys simple vector arithmetic is a vector, arrow-like or not. In math talk, lots of strange things are vectors, like arrays of numbers and functions. As special cases of the mathematicians' 'abstract vectors', the vectors in this book always have magnitude and direction.