SAMPLE 2.13 **A mass and a pulley.** A block of mass m is held up by applying a force F through a massless pulley as shown in the figure. Assume the string to be massless. Draw free-body diagrams of the mass and the pulley separately and as one system.

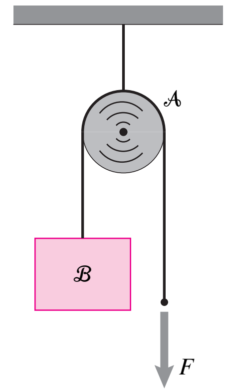


Figure 2.49

Solution The free-body diagrams of the block and the pulley are shown in Fig. 2.50. Since the string is massless and we assume an ideal massless pulley, the tension in the string is the same on both sides of the pulley. Therefore, the force applied by the string on the block is simply F. When the mass and the pulley are considered as one system, the force in the string on the left side of the pulley doesn't show because it is internal to the system.

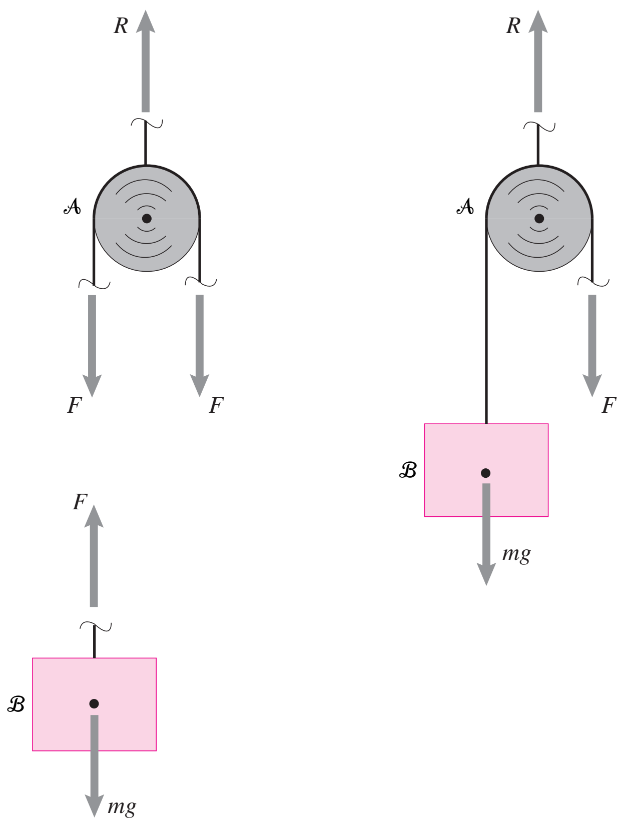


Figure 2.50: The free-body diagrams of the mass, the pulley, and the mass-pulley system. Note that for the purpose of drawing the free-body diagram we need not show that we know that $R = 2F$. Similarly, we could have chosen to show two different rope tensions on the sides of the pulley and reasoned that they are equal as is done in the text.

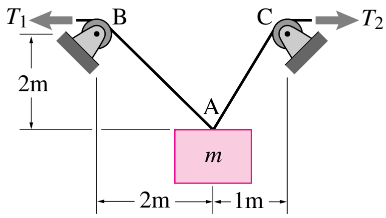


Figure 2.51

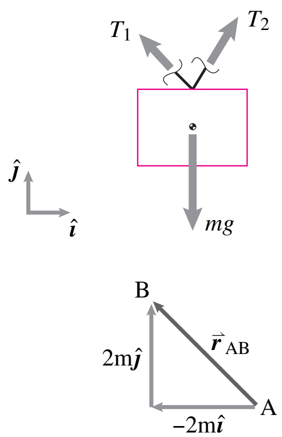


Figure 2.52: Free-body diagram of the block and a diagram of the vector $\vec{r}_{AB}$.

SAMPLE 2.14 Forces in strings. A block of mass m is held in position by strings AB and AC as shown in Fig. 2.51. Draw a free-body diagram of the block and write the vector sum of all the forces shown on the diagram. Use a suitable coordinate system.

Solution To draw a free-body diagram of the block, we first *free* the block. We cut strings AB and AC very close to point A and show the forces applied by the cut strings on the block. We also isolate the block from the earth and show the force due to gravity. (See Fig. 2.52.)

To write the vector sum of all the forces, we need to write the forces as vectors. To write these vectors, we first choose an xy coordinate system with basis vectors $\hat{\imath}$ and $\hat{\jmath}$ as shown in Fig. 2.52. Then, we express each force as a product of its magnitude and a unit vector in the direction of the force. So,

$$\vec{T}_1 = T_1\hat{\lambda}_{AB} = T_1\frac{\vec{r}_{AB}}{|\vec{r}_{AB}|},$$

where $\vec{r}_{AB}$ is a vector from A to B and $|\vec{r}_{AB}|$ is its magnitude. Neglecting the size of pulleys, we get, from the given geometry,

$$\begin{aligned}\vec{r}_{AB} &= -2\,\mathrm{m}\hat{\imath} + 2\,\mathrm{m}\hat{\jmath}\\ \Rightarrow\qquad \hat{\lambda}_{AB} &= \frac{2\,\cancel{\mathrm{m}}(-\hat{\imath}+\hat{\jmath})}{\sqrt{2^2+2^2}\,\cancel{\mathrm{m}}} = \frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}).\end{aligned}$$

Thus,

$$\vec{T}_1 = T_1\frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}).$$

Similarly,

$$\begin{aligned}\vec{T}_2 &= T_2\frac{1}{\sqrt{5}}(\hat{\imath}+2\hat{\jmath})\\ m\vec{g} &= -mg\hat{\jmath}.\end{aligned}$$

Now, we write the sum of all the forces:

$$\begin{aligned}\sum\vec{F} &= \vec{T}_1 + \vec{T}_2 + m\vec{g}\\ &= \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\imath} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\jmath}.\end{aligned}$$

The $\hat{\imath}$ and $\hat{\jmath}$ components of the net force depend on the values of the scalars (magnitudes) T_1, T_2 and mg.

$$\boxed{\sum\vec{F} = \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\imath} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\jmath}}$$

SAMPLE 2.15 Two bodies connected by a massless spring. Two carts A and B are connected by a massless spring. The carts are pulled to the left with a force F and to the right with a force T as shown in Fig. 2.53. Assume the wheels of the carts to be massless and frictionless. Draw free-body diagrams of

- cart A,
- cart B, and
- carts A and B together.

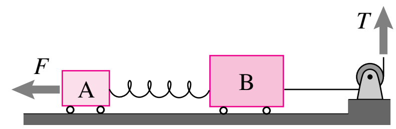


Figure 2.53: Two carts connected by a massless spring

Solution The three free-body diagrams are shown in Fig. 2.54 (a) and (b). In Fig. 2.54 (a) the force F_s is applied by the spring on the two carts. Why is this force the same on both carts? In Fig. 2.54(b) the spring is a part of the system. Therefore, the forces applied by the spring on the carts and the forces applied by the carts on the spring are internal to the system. Therefore these forces do not show on the free body diagram.

Note that the normal reaction of the ground can be shown either as separate forces on the two wheels of each cart or as a resultant reaction.

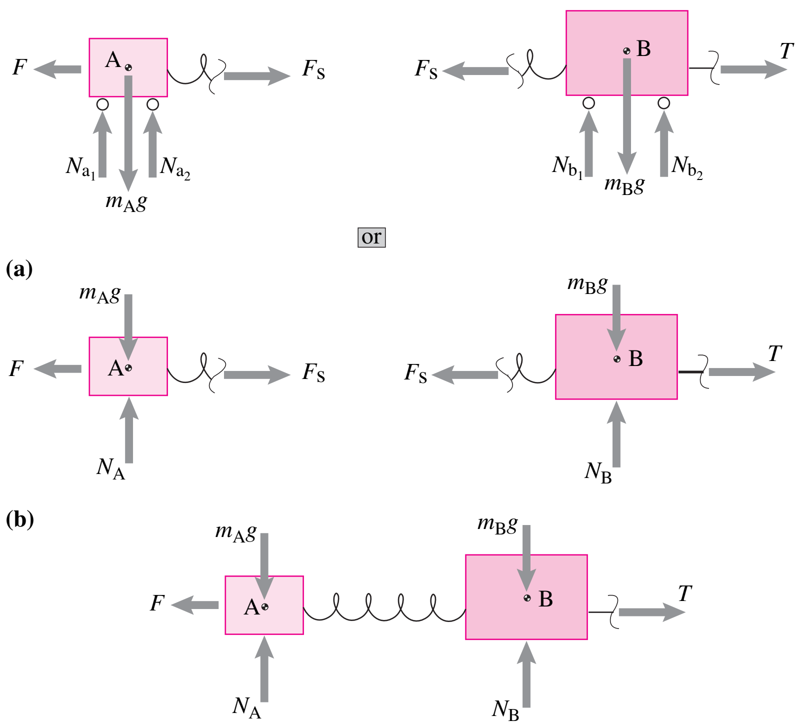


Figure 2.54: Free-body diagrams of (a) cart A and cart B separately and (b) cart A and B together

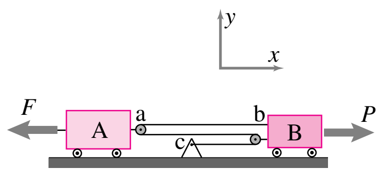


Figure 2.55: Two carts connected by massless pulleys.

SAMPLE 2.16 **Two carts connected by pulleys.** The two masses shown in Fig. 2.55 have frictionless bases and round frictionless pulleys. The inextensible massless cord connecting them is always taut. Mass A is pulled to the left by force F and mass B is pulled to the right by force P as shown in the figure. Draw free-body diagrams of each mass.

Solution Let the tension in the cord be T. Since the pulleys and the cord are massless, the tension is the same in each section of the cord. This equality is clearly shown in the Free-body diagrams of the two masses below.

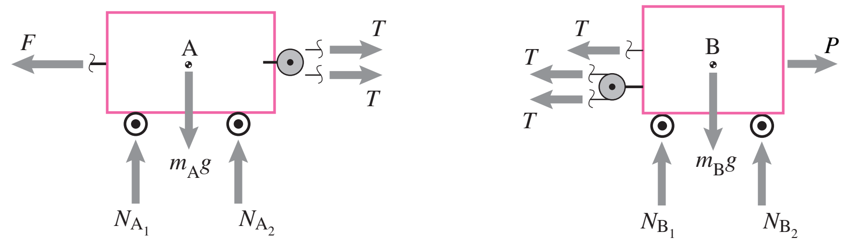


Figure 2.56: Free-body diagrams of the two masses.

Comments: We have shown unequal normal reactions on the wheels of mass B. In fact, the two reactions would be equal only if the forces applied by the cord on mass B satisfy a particular condition. Can you see what condition must be satisfied for, say, $N_{A_1} = N_{A_2}$.

[Hint: think about the moment balance about the center-of-mass A.]

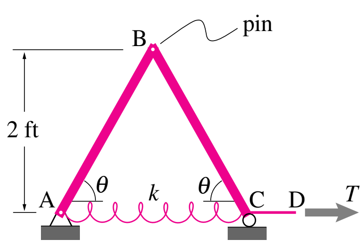
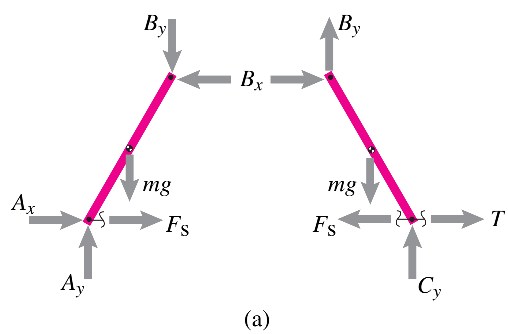


Figure 2.57

SAMPLE 2.17 **Structures with pin connections.** A horizontal force T is applied on the structure shown in the figure. The structure has pin connections at A and B and a roller support at C. Bars AB and BC are rigid. Draw free-body diagrams of each bar and the structure including the spring.

Solution The free-body diagrams are shown in figure 2.58. Note that there are both vertical and horizontal forces at the pin connections because pins restrict translation in any direction. At the roller support at point C there is only vertical force from the support (T is an externally applied force).



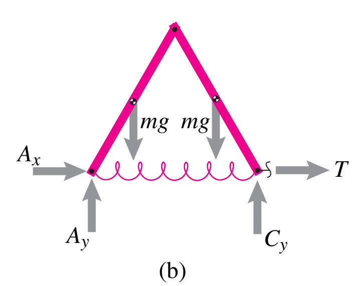


Figure 2.58: Free-body diagrams of (a) the individual bars and (b) the structure as a whole.

SAMPLE 2.18 **The four-bar linkage.** The four-bar linkage① shown in the figure is pushed to the right with a force F. Pins A, C & D have negligible friction but joint B is rusty and resists rotation (has non-negligible friction). Draw free-body diagrams of each of the bars separately and of the whole structure. Use consistent notation for the interaction forces and moments. Clearly mark the action-reaction pairs. Neglect gravity.

Solution An ideal pin resists any relative translation of the pinned parts by exerting forces on them. We could draw three separate free-body diagrams of the pin, the first part, and the second part. Usually, however, we let the pin be a part of one of the objects and just draw two free-body diagrams.

Because rusty joint B resists relative rotation we also show a moment at point B acting on rods AB and BC.

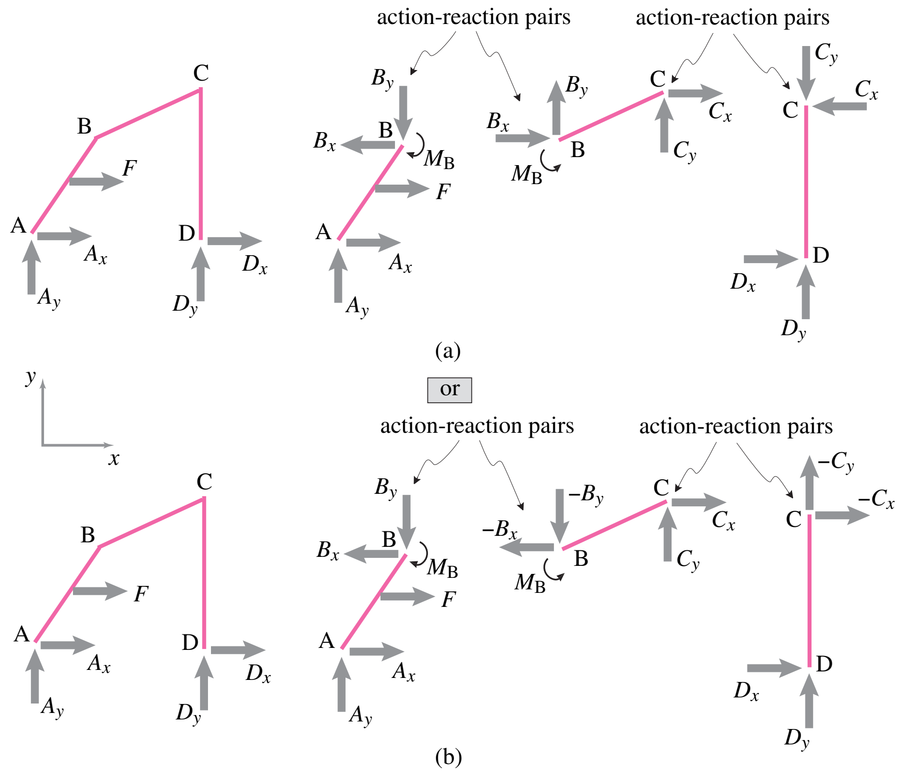


Figure 2.60: Style 1, components: Free-body diagrams of the structure and the individual bars. The forces shown in (a) and (b) are the same.

Figure 2.60 shows the forces in terms of their x- and y-components. The directions of the force components are shown by the arrows and the scalar multipliers are labeled as A_x, A_y, etc. Here the word scalar is sloppily called 'magnitude', even though it can be positive or negative. Therefore, a force, shown as an arrow in the positive x-direction with magnitude A_x, is the same as that shown as an arrow in the negative x-direction with 'magnitude' $-A_x$. Thus, the free-body diagrams in Fig. 2.60(a) show exactly the same forces as in Fig. 2.60(b).

In Fig. 2.61, we show the forces by an arrow in an arbitrary direction. The corresponding labels represent their magnitudes. The angles represent the unknown directions of the forces.

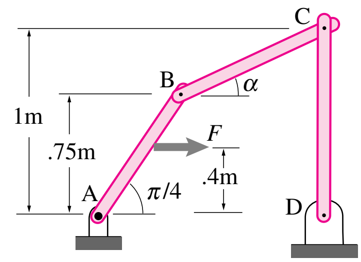


Figure 2.59: A four bar linkage. Force F is the only load. Pin B has non-negligible friction, the other pins have negligible friction.

①Although there are only three bars here, it is called a *four-bar linkage* because the rigid ground connection AD is also counted as another "bar" in the linkage.

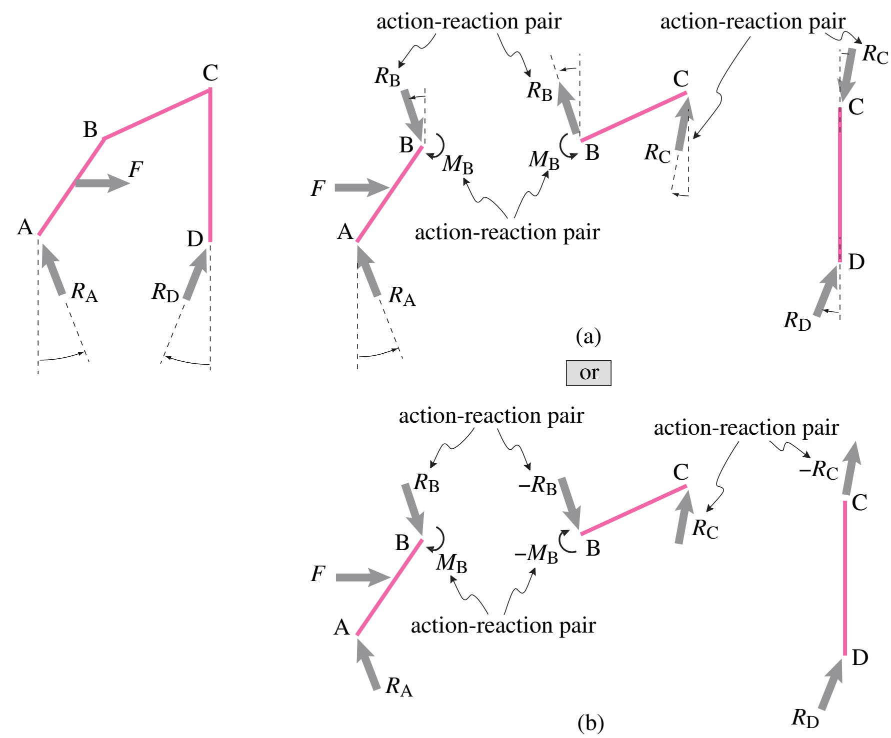


Figure 2.61: Style 2, magnitude times drawn unit vector: The forces in (a) have arcs indicating angles which would show the directions of the forces. The FBDs in (b) are more informal and not recommended. Why not? There is no visible notation showing the directions of the forces.

In Fig. 2.62, we show yet another way of drawing and labeling the free-body diagrams, where the forces are labeled as vectors.

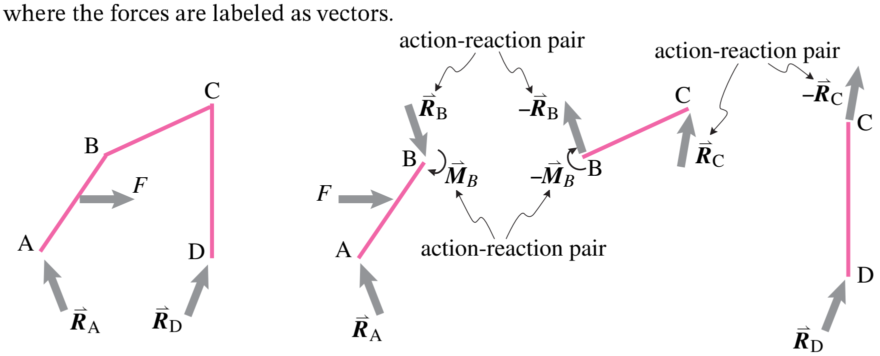


Figure 2.62: Style 3, vector notation: The labels of forces as vectors indicate both their magnitude and direction. The arrows are arbitrary and merely indicate that a force or a moment acts at those locations.

Note: Bar CD is a two-force member. We could have used that to simplify all of the free-body diagrams above by showing equal and opposite forces at C and D that were parallel to the line CD.