2.2 Center of mass and center of gravity

2.2.1 An otherwise massless structure is made of four point masses, m, $2m$, $3m$ and $4m$, located at coordinates (0, 1 m), (1 m, 1 m), (1 m, −1 m), and (0, −1 m), respectively. Locate the center of mass of the structure. *

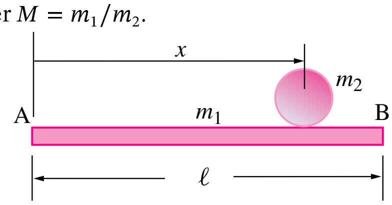
2.2.2 3-D: The following data is given for a structural system modeled with five point masses in 3-D-space:

mass	**coordinates (in m)**
0.4 kg	(1,0,0)
0.4 kg	(1,1,0)
0.4 kg	(2,1,0)
0.4 kg	(2,0,0)
1.0 kg	(1.5,1.5,3)

Locate the center of mass of the system.

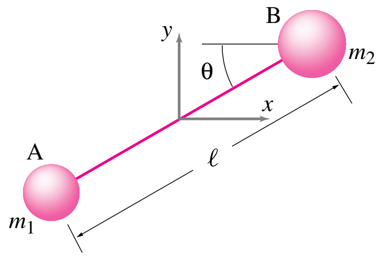
2.2.3 Write a computer program to find the center of mass of a point-mass-system. The input to the program should be a table (or matrix) containing individual masses and their coordinates. (It is possible to write a single program for both 2-D and 3-D cases, write separate programs for the two cases if that is easier for you.) Check your program on Problems 2.2.1 and 2.2.2.

2.2.4 A cylinder of mass m_2 and radius R rolls on a flat circular plate of mass m_1 and length ℓ. Let the position of the cylinder from the left edge of the plate be x. Find the horizontal position of the center of mass of the system as a function of x and a non-dimensional mass parameter $M = m_1/m_2$.



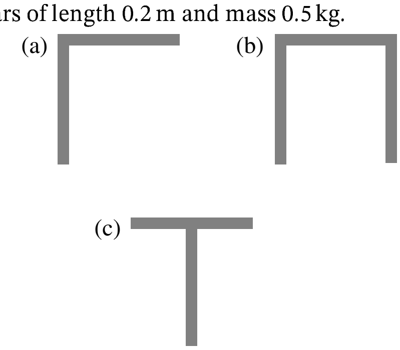
Problem 2.2.4

2.2.5 Two masses m_1 and m_2 are connected by a massless rod AB of length ℓ. In the position shown, the rod is inclined to the horizontal axis at an angle θ. Find the position of the center of mass of the system as a function of angle θ and the other given variables. Check if your answer makes sense by setting appropriate values for m_1 and m_2.



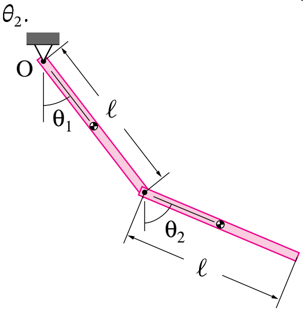
Problem 2.2.5

2.2.6 Find the center of mass of the following composite bars. Each composite shape is made of two or more uniform bars of length 0.2 m and mass 0.5 kg.



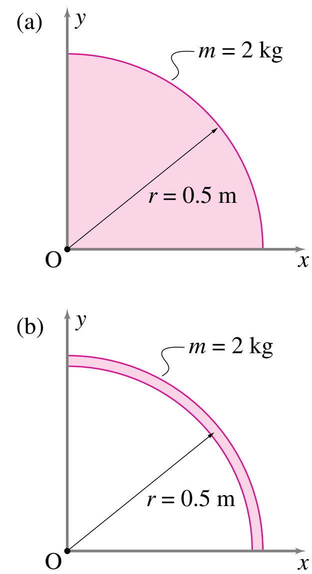
Problem 2.2.6

2.2.7 A double pendulum consists of two uniform bars of length ℓ and mass m each. The pendulum hangs in the vertical plane from a hinge at point O. Taking O as the origin of a xy coordinate system, find the location of the center of gravity of the pendulum as a function of angles θ_1 and θ_2.



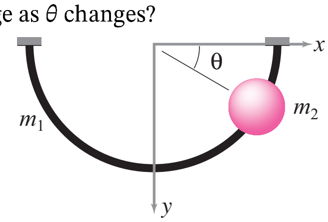
Problem 2.2.7

2.2.8 Find the center of mass of the following two objects [Hint: set up and evaluate the needed integrals.]



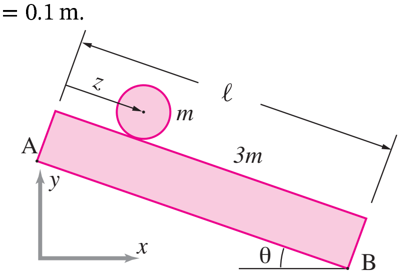
Problem 2.2.8

2.2.9 A semicircular ring of radius R = 1 m and mass m_1 = 0.1 kg rests in the vertical plane. A bead of mass m_2 = 0.25 kg slides on the ring. Find the position of the center of mass of the ring-bead-system at an instant when $\theta = 30^\circ$. How does the center of mass position change as θ changes?



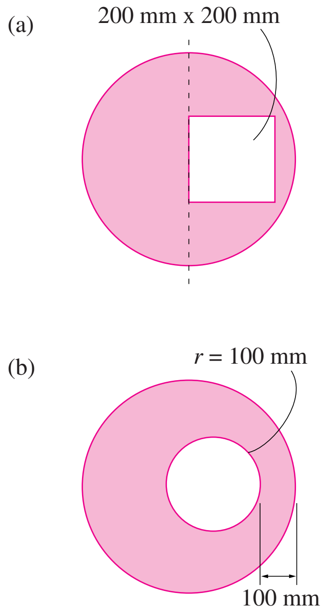
Problem 2.2.9

2.2.10 A uniform circular disk of mass m and radius R rolls on an inclined rectangular plate of mass $3m$ and dimensions $2R \times \ell$. Point A is on the y axis and point B is on the x axis. Find the coordinates of the center of mass of the system for m = 1 kg, ℓ = 1 m, z = 0.2 m, and R = 0.1 m.



Problem 2.2.10

2.2.11 Find the center of mass of the following plates obtained from cutting out a section from a uniform circular plate of mass 1 kg (prior to removing the cutout) and radius 1/4 m.



Problem 2.2.11