

SAMPLE 2.1 Equivalent force on a particle: Four forces $\vec{F}_1 = 2\text{ N}\hat{i} - 1\text{ N}\hat{j}$, $\vec{F}_2 = -5\text{ N}\hat{j}$, $\vec{F}_3 = 3\text{ N}\hat{i} + 12\text{ N}\hat{j}$, and $\vec{F}_4 = 1\text{ N}\hat{i}$ act on a particle. Find the equivalent force on the particle.

Solution The equivalent force on the particle is the net force, *i.e.*, the vector sum of all forces acting on the particle. Thus,

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \\ &= (2\text{ N}\hat{i} - 1\text{ N}\hat{j}) + (-5\text{ N}\hat{j}) + (3\text{ N}\hat{i} + 12\text{ N}\hat{j}) + (1\text{ N}\hat{i}) \\ &= 6\text{ N}\hat{i} + 6\text{ N}\hat{j}.\end{aligned}$$

$$\vec{F}_{\text{net}} = 6\text{ N}(\hat{i} + \hat{j})$$

Note that there is no net couple since all the four forces act at the same point. This is always true for particles. Thus, the equivalent force-couple system for particles consists of only the net force.

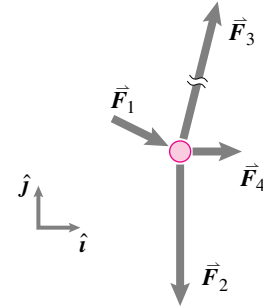


Figure 2.5

SAMPLE 2.2 Equivalent force with no net moment: In the figure shown, $F_1 = 50\text{ N}$, $F_2 = 10\text{ N}$, $F_3 = 30\text{ N}$, and $\theta = 60^\circ$. Find the equivalent force-couple system about point D of the structure.

Solution From the given geometry, we see that the three forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ pass through point D. Thus they are *concurrent* forces. Since point D is on the line of action of these forces, we can simply slide the three forces to point D without altering their mechanical effect on the structure. Then the equivalent force-couple system at point D consists of only the net force, $\vec{F}_{\text{net}}$, with no couple (the three forces passing through point D produce no moment about D). This is true for all concurrent forces. Thus,

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \\ &= F_1(\cos\theta\hat{i} - \sin\theta\hat{j}) - F_2\hat{j} + F_3\hat{i} \\ &= (F_1\cos\theta + F_3)\hat{i} - (F_1\sin\theta + F_2)\hat{j} \\ &= (50\text{ N} \cdot \frac{1}{2} + 30\text{ N})\hat{i} - (50\text{ N} \cdot \frac{\sqrt{3}}{2} + 10\text{ N})\hat{j} \\ &= 55\text{ N}\hat{i} - 53.3\text{ N}\hat{j},\end{aligned}$$

and $\vec{M}_D = \vec{0}.$

$$\vec{F}_{\text{net}} = 55\text{ N}\hat{i} - 53.3\text{ N}\hat{j}, \vec{M}_D = \vec{0}$$

Graphically, the solution is shown in Fig. 2.7

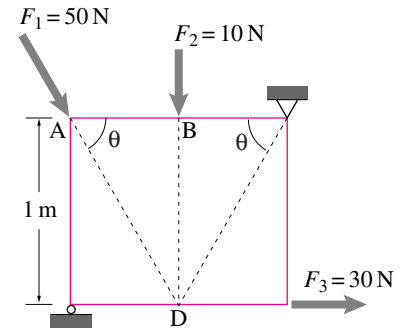


Figure 2.6

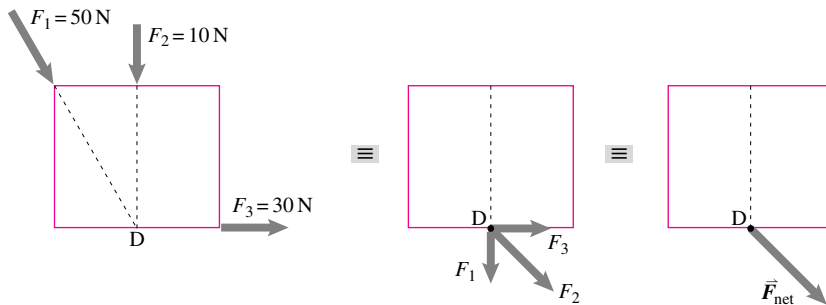


Figure 2.7:

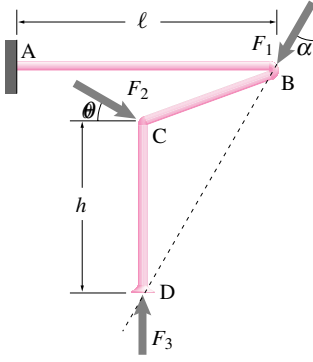


Figure 2.8

SAMPLE 2.3 An equivalent force-couple system: Three forces $F_1 = 100 \text{ N}$, $F_2 = 50 \text{ N}$, and $F_3 = 30 \text{ N}$ act on a structure as shown in the figure where $\alpha = 30^\circ$, $\theta = 60^\circ$, $\ell = 1 \text{ m}$ and $h = 0.5 \text{ m}$. Find the equivalent force-couple system about point D.

Solution The net force is the sum of all applied forces, i.e.,

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \\ &= F_1(-\sin \alpha \hat{i} - \cos \alpha \hat{j}) + F_2(\cos \theta \hat{i} - \sin \theta \hat{j}) + F_3 \hat{j} \\ &= (-F_1 \sin \alpha + F_2 \cos \theta) \hat{i} + (-F_1 \cos \alpha - F_2 \sin \theta + F_3) \hat{j} \\ &= (-100 \text{ N} \cdot \frac{1}{2} + 50 \text{ N} \cdot \frac{1}{2}) \hat{i} + (-100 \text{ N} \cdot \frac{\sqrt{3}}{2} - 50 \text{ N} \cdot \frac{\sqrt{3}}{2} + 30 \text{ N}) \hat{j} \\ &= -25 \text{ N} \hat{i} - 99.9 \text{ N} \hat{j}.\end{aligned}$$

Forces $\vec{F}_1$ and $\vec{F}_3$ pass through point D. Therefore, they do not produce any moment about D. So, the net moment about D is the moment caused by force $\vec{F}_2$:

$$\begin{aligned}\vec{M}_D &= \vec{r}_{C/D} \times \vec{F}_2 \\ &= h \hat{j} \times F_2(\cos \theta \hat{i} - \sin \theta \hat{j}) \\ &= -F_2 h \cos \theta \hat{k} \\ &= -50 \text{ N} \cdot 0.5 \text{ m} \frac{1}{2} \hat{k} = -12.5 \text{ N} \cdot \text{m} \hat{k}.\end{aligned}$$

The equivalent force-couple system is shown in Fig. 2.9

$$\vec{F}_{\text{net}} = -25 \text{ N} \hat{i} - 99.9 \text{ N} \hat{j} \quad \text{and} \quad \vec{M}_D = -12.5 \text{ N} \cdot \text{m} \hat{k}$$

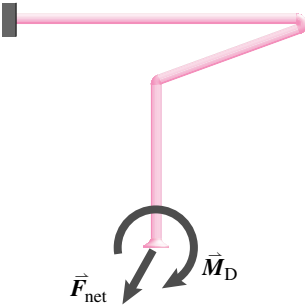


Figure 2.9

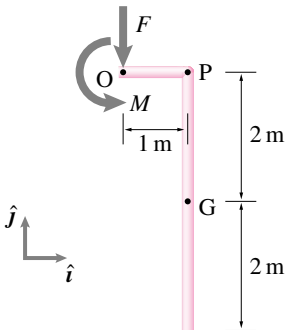


Figure 2.10

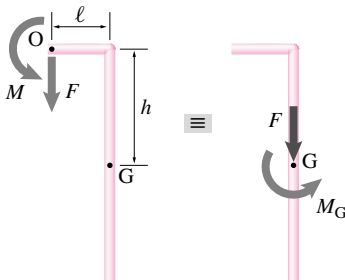


Figure 2.11

SAMPLE 2.4 Translating a force-couple system: The net force and couple acting about point O on the 'L' shaped bar shown in the figure are 100 N and $20 \text{ N} \cdot \text{m}$, respectively. Find the net force and moment about point G.

Solution The net force on a structure is the same about any point since it is just the vector sum of all the forces acting on the structure and is independent of their point of application. Therefore,

$$\vec{F}_{\text{net}} = \vec{F} = -100 \text{ N} \hat{j}.$$

The net moment about a point, however, depends on the location of points of application of the forces with respect to that point. Thus,

$$\begin{aligned}\vec{M}_G &= \vec{M}_O + \vec{r}_{O/G} \times \vec{F} \\ &= M \hat{k} + (-\ell \hat{i} + h \hat{j}) \times (-F \hat{j}) \\ &= (M + F \ell) \hat{k} \\ &= (20 \text{ N} \cdot \text{m} + 100 \text{ N} \cdot 1 \text{ m}) \hat{k} = 120 \text{ N} \cdot \text{m} \hat{k}.\end{aligned}$$

$$\vec{F}_{\text{net}} = -100 \text{ N} \hat{j}, \quad \text{and} \quad \vec{M}_G = 120 \text{ N} \cdot \text{m} \hat{k}$$

SAMPLE 2.5 Checking equivalence of force-couple systems: In the figure shown below, which of the force-couple systems shown in (b), (c), and (d) are equivalent to the force system shown in (a)?

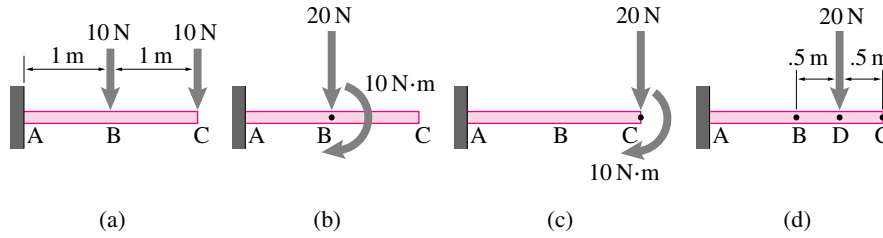


Figure 2.12:

Solution The equivalence of force-couple systems require that (i) the net force be the same, and (ii) the net moment about any reference point be the same. For the given systems, let us choose point B as our reference point for comparing their equivalence. For the force system shown in Fig. 2.12(a), we have,

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 = -10 \text{ N} \hat{j} - 10 \text{ N} \hat{j} = -20 \text{ N} \hat{j} \\ \vec{M}_{B_{\text{net}}} &= \vec{r}_{C/B} \times \vec{F}_2 = 1 \text{ m} \hat{i} \times (-10 \text{ N} \hat{j}) = -10 \text{ N} \cdot \text{m} \hat{k}.\end{aligned}$$

Now, we can compare the systems shown in (b), (c), and (d) against the computed equivalent force-couple system, $\vec{F}_{\text{net}}$ and $\vec{M}_{B_{\text{net}}}$.

- Figure (b) shows exactly the system we calculated. Therefore, it represents an equivalent force-couple system.
- Figure (c): Let us calculate the net force and moment about point B for this system.

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_C = -20 \text{ N} \hat{j} \\ \vec{M}_B &= \vec{M}_C + \vec{r}_{C/B} \times \vec{F}_C \\ &= -10 \text{ N} \cdot \text{m} \hat{k} + 1 \text{ m} \hat{i} \times (-20 \text{ N} \hat{j}) = -30 \text{ N} \cdot \text{m} \hat{k} \neq \vec{M}_{B_{\text{net}}}.\end{aligned}$$

Thus, the given force-couple system in this case is not equivalent to the force system in (a).

- Figure (d): Again, we compute the net force and the net couple about point B:

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_D = -20 \text{ N} \hat{j} \\ \vec{M}_B &= \vec{r}_{D/B} \times \vec{F}_D \\ &= 0.5 \text{ m} \hat{i} \times (-20 \text{ N} \hat{j}) = -10 \text{ N} \cdot \text{m} \hat{k} = \vec{M}_{B_{\text{net}}}.\end{aligned}$$

Thus, the given force-couple system (with zero couple) at D is equivalent to the force system in (a).

(b) and (d) are equivalent to (a); (c) is not.

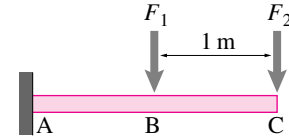


Figure 2.13

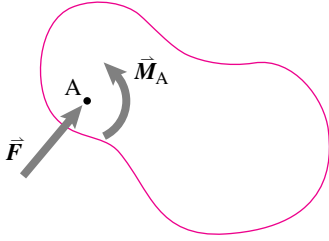


Figure 2.14

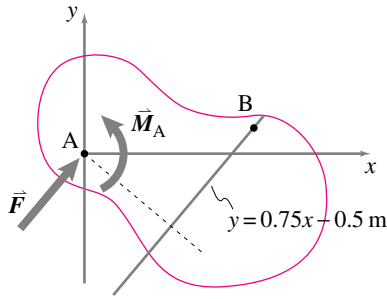


Figure 2.15

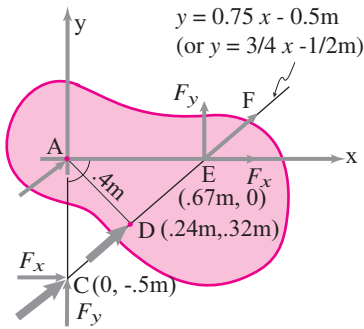


Figure 2.16

SAMPLE 2.6 Equivalent force with no couple: For a body, an equivalent force-couple system at point A consists of a force $\vec{F} = 20\text{ N}\hat{i} + 15\text{ N}\hat{j}$ and a couple $\vec{M}_A = 10\text{ N}\cdot\text{m}\hat{k}$. Find a point on the body such that the equivalent force-couple system at that point consists of only a force (zero couple).

Solution The net force in the two equivalent force-couple systems has to be the same. Therefore, for the new system, $\vec{F}_{\text{net}} = \vec{F} = 20\text{ N}\hat{i} + 15\text{ N}\hat{j}$. Let B be the point at which the equivalent force-couple system consists of only the net force, with zero couple. We need to find the location of point B. Let A be the origin of a xy coordinate system in which the coordinates of B are (x, y) . Then, the moment about point B is,

$$\begin{aligned}\vec{M}_B &= \vec{M}_A + \vec{r}_{A/B} \times \vec{F} \\ &= M_A \hat{k} + (-x\hat{i} - y\hat{j}) \times (F_x \hat{i} + F_y \hat{j}) \\ &= M_A \hat{k} + (-F_y x + F_x y) \hat{k}.\end{aligned}$$

Since we require that $\vec{M}_B$ be zero, we must have

$$\begin{aligned}F_y x - F_x y &= M_A \\ \Rightarrow y &= \frac{F_y}{F_x} x - \frac{M_A}{F_x} \\ &= \frac{15\text{ N}}{20\text{ N}} x - \frac{10\text{ N}\cdot\text{m}}{20\text{ N}} \\ &= 0.75x - 0.5\text{ m}.\end{aligned}$$

This is the equation of a line. Thus, we can select any point on this line and apply the force $\vec{F} = 20\text{ N}\hat{i} + 15\text{ N}\hat{j}$ with zero couple as an equivalent force-couple system.

Any point on the line $y = 0.75x - 0.5\text{ m}$.

So, how or why does it work? The line we obtained is shown in gray in Fig. 2.15. Note that this line has the same slope as that of the given force vector (slope $= 0.75 = F_y/F_x$) and the offset is such that shifting the force $\vec{F}$ to this line counter balances the given couple at A. To see this clearly, let us select three points C, D, and E on the line as shown in Fig. 2.16. From the equation of the line, we find the coordinates of C(0, -0.5m), D(0.24m, 0.32m) and E(0.67m, 0). Now imagine moving the force $\vec{F}$ to C, D, or E. In each case, it must produce the same moment $\vec{M}_A$ about point A. Let us do a quick check.

$\vec{F}$

- at point C: The moment about point A is due to the horizontal component $F_x = 20\text{ N}$, since F_y passes through point A. The moment is $F_x \cdot AC = 20\text{ N} \cdot 0.5\text{ m} = 10\text{ N}\cdot\text{m}$, same as M_A . The direction is counterclockwise as required.

$\vec{F}$

- at point D: The moment about point A is $|\vec{F}| \cdot AD = 25\text{ N} \cdot 0.4\text{ m} = 10\text{ N}\cdot\text{m}$, same as M_A . The direction is counterclockwise as required.

$\vec{F}$

- at point E: The moment about point A is due to the vertical component F_y , since F_x passes through point A. The moment is $F_y \cdot AE = 15\text{ N} \cdot 0.67\text{ m} = 10\text{ N}\cdot\text{m}$, same as M_A . The direction here too is counterclockwise as required.

Once we check the calculation for one point on the line, we should not have to do any more checks since we know that sliding the force along its line of action (line CB) produces no couple and thus preserves the equivalence.