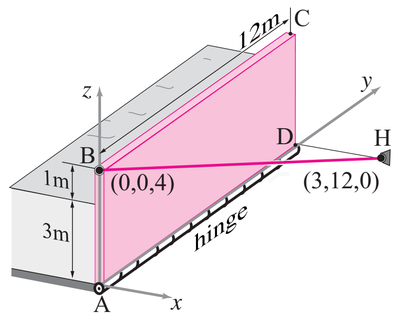
**9.1.3** A 4-meter-high 'door' holds back a stream ($\gamma = 10,000\,\mathrm{N/m^3}$) that is 3m deep and 12m wide. The door is hinged along its bottom and is propped up by a thin rod B that goes from a ball joint at H at (3,12,0) to a ball joint at the upper left corner B of the door at (0,0,4). Neglect the mass of the door. Find the axial-force in the rod BH.

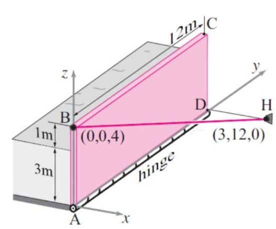



Problem 9.3

**Answer:**
$(117\gamma/2)\,\mathrm{m}^3 = 5.85 * 10^5\,\mathrm{N}$

**8.1.3**

Given, $\gamma = 1000$ N / m$^3$, $l = 12$ m, and $h = 3$ m.

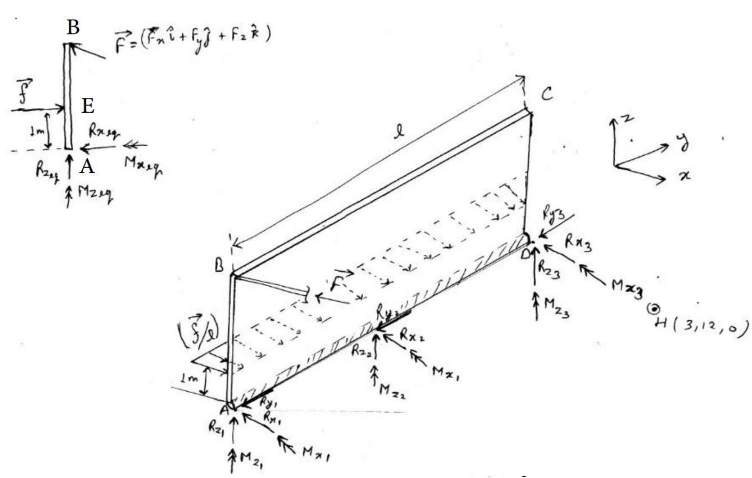



Let the axial force in the rod be $\vec{F} = F\,\hat{\boldsymbol{r}}_{HB}$

$$\Rightarrow \vec{F} = \frac{F}{13}\left(-3\hat{\imath} - 4\hat{\jmath} + 12\hat{k}\right)\ \mathrm{N}$$

Now the pressure force on door is,

$\vec{f} = \frac{1}{2}\gamma h^{2} l\,\hat{\imath}$ , and this equivalent force of the linear force distribution passes 1/3 up from the edge AD. Hence, $\vec{r}_{AE} = \frac{h}{3}\hat{k} = 1\hat{k}$ m




$\sum \vec{M}_A = \vec{0}$ on the door ABCD

$\Rightarrow \hat{\jmath}\,.\left(\vec{r}_{AB} \times \vec{F}\right) + \hat{\jmath}\,.\left(\vec{r}_{AE} \times \vec{f}\right) = 0$ , Let $\vec{r}_{AB} = b\hat{k} = 4\hat{k}$ m

$$\Rightarrow \left(\frac{-3}{13}\right) F\, b + \frac{1}{2}\gamma \frac{h^3}{3} l = 0$$

$$\Rightarrow F = \left(\frac{13}{18}\right) \frac{\gamma\, h^3 l}{b}$$

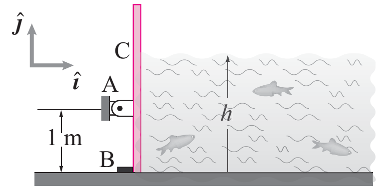
$$\Rightarrow \boxed{F = 5.85 \times 10^4\ \mathrm{N}}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**9.1.4** Water is held in a reservoir by a board with negligible weight that is 5 meters long. It is hinged 1 meter off the bottom at A and kept from leaking by a seal at B. Assume $\rho = 1000\,\text{kg}/\text{m}^3$, $g = 10\,\text{N}/\text{kg}$.

a) What is $h$ when the board starts to pull away from the stop at B?

b) At that $h$ what is the force of the hinge on the board?




Problem 9.4

**Answer:**
Water starts to spill at $h = 3r_{AB} = 3\,\text{m}$.
Assuming no friction at B, $\vec{F}_A = 2.25 * 10^5 \hat{\imath}\,\text{N}$

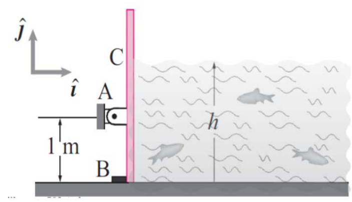
**8.1.4**

(a) Given, $\rho = 10^3$ kg/m$^3$

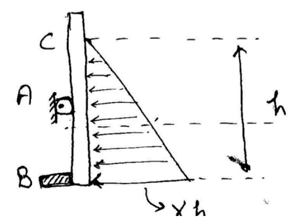
$g = 10$ N/kg

$\Rightarrow \gamma = \rho g = 10^4$ N/m$^3$

Length of the board, $l = 5$ m



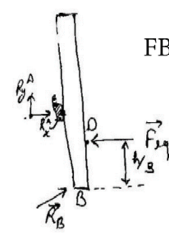

The forces acting on the board are shown in the FBD (I). The equivalent force of the linearly distributed force passes through the point D which is 1/3 up from B. When the board just begins to detach from point B, $R_B = 0$. Hence, from the FBD (II),




$$\sum \vec{M}_A = \vec{0}$$

$$\Rightarrow \vec{r}_{AD} \times \vec{F}_{eq} = \vec{0}$$

$$\Rightarrow \vec{r}_{AD} \times \left(\int_A p\,\hat{n}\,d\vec{A}\right) = \vec{0}$$



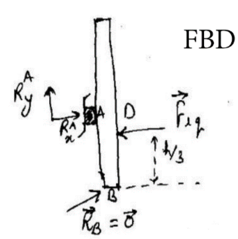

FBD (I)

In order for this equation to hold true, the distance from point A to point D (the point of application of equivalent force due to pressure on the board) has to be zero.

$$\vec{r}_{AD} = \vec{0}$$

$\therefore$ h/3 = AB

so $\boxed{h = 3\text{ AB} = 3\text{m}}$




FBD (II)

(b) $\sum \vec{F} = \vec{0}$

$$\Rightarrow \vec{R}_A + \vec{F}_{eq} = \vec{0}$$

$$\Rightarrow \vec{R}_A - \tfrac{1}{2}\gamma h^2 l\,\hat{\imath} = \vec{0}$$

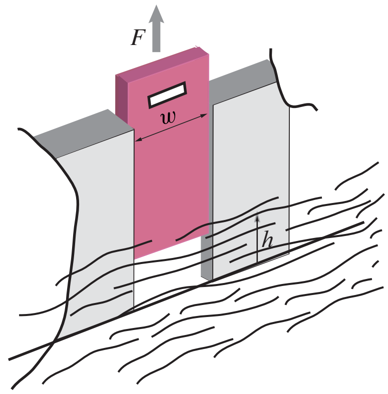
$$\therefore\ \vec{R}_A = \tfrac{1}{2}\gamma h^2 l\,\hat{\imath}$$

$$\boxed{\vec{R}_A = 22.5 \text{ x } 10^4 \hat{\imath}\text{ N}}$$

**9.1.6** A sluice gate is a dam that can be opened. Sometimes it is just a board in a slot that is opened by pulling up the board. For water with density $\rho$ and depth $h$ pressing against a board with width $w$ pressing against one face of the slot (the face away from the water) with coefficient of friction $\mu$.

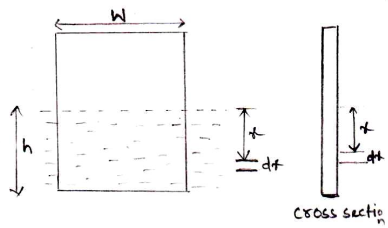
a) find the force $F$ needed to pull up the board in terms of $g, \rho, \mu, h$, and $w$.

b) Find the force assuming $g = 10\,\mathrm{m/s^2}, h = 1\,\mathrm{m}, w = 1\,\mathrm{m}, \mu = 0.5$ and $\rho = 1000\,\mathrm{kg/m^3}$.




Problem 9.6

8·1·6




Magnitude of friction force depends on the reaction force on the object from the support surface.

Here one face of the slot will apply that reaction force on the board.

The reaction force will be equal to the force exerted by water on the board.

Since force at a particular point due to water depends on depth of water at that point, it will vary with distance from surface.

From the cross sectional figure shown above, pressure due to water at a depth $x$ from the surface is given by

$$P_x = \rho g x$$

Due to this pressure, horizontal force acting on a small element of height $dx$ is given by

$$dF_h = \rho g x\,(w \cdot dx)$$

$w \cdot dx$ :- it is the area of the small element under consideration.

Now total force can be found by integrating above

$$\int_0^{F_h} dF_h = \int_0^h \rho g x\, w \cdot dx$$

$$\Rightarrow F_h = \rho g w \int_0^h x\, dx$$

$$\Rightarrow F_h = \frac{\rho g w h^2}{2}$$

8.1.6. continued

(a) Force required to pull up the board

$$F = 4F_h$$

$$= \frac{4\rho g w h^2}{2}$$

(b) Substituting the values

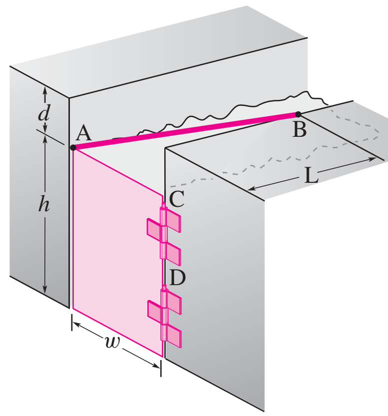
$$F = \frac{0.5 \times 1000\,kg/m^3 \times 10\,m/s^2 \times 1\,m \times (1\,m)^2}{2}$$

$$= 2500\ kg\frac{m}{s^2} = 2500\,N$$

**9.1.8** A door holds back the water at a lock on a canal. The water surface is at the top of the door. The rope AB keeps it from swinging open. The door has hinges at C and D. The height of the door is $h$, the width $w$. The point B is a distance $d$ above the top of the door and is set back a distance $L$. The weight density of the water is $\gamma$.

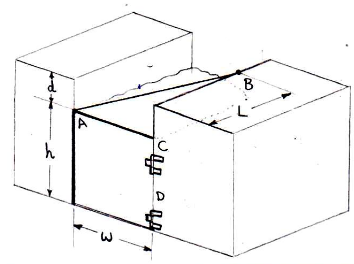
a) What is the total force of the water on the door?

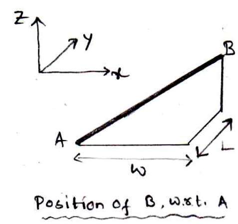
b) What is the tension in the rope AB?




Problem 9.8

8.1.8




(a) Total force of water on the door :-

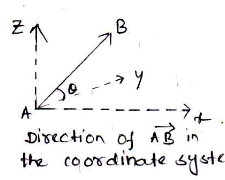
Note: Calculation of this is similar to that as followed in problem 8.1.8

$$F_h = \frac{\rho g w h^2}{2}$$

Weight density water is given as $\gamma = \rho g$

$$\therefore F_h = \frac{\gamma w h^2}{2}$$

(b) Moments due to forces exerted by water and that by rope about hinge will be balanced at equillibrium.




Direction of AB is shown in the figure.
Angle made by AB with y axis is θ.

Then we can say that

$$\cos(\theta) = \frac{L}{AB}$$

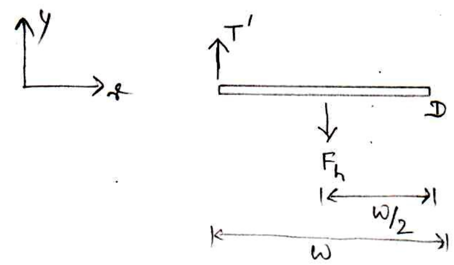
$AB = \sqrt{w^2+L^2+d^2}$ [from the figure ' position of B wrt A ]

$$\therefore \cos(\theta) = \frac{L}{\sqrt{w^2+L^2+d^2}}.$$

We can see that, Only the component of tension $\vec{T_{AB}}$ in rope AB which is along Y-axis [T'] will exert a moment about CD.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.8 continued




$T' \rightarrow$ component of tension $\vec{T}_{AB}$ along Y-axis.

$F_h \rightarrow$ Force exerted by water

F.B.D. of door [Top view]

At equilibrium net moment about point D will be zero

$$\Sigma \vec{M}_{/D} = \vec{0}$$

$$\Rightarrow \vec{M}_{T'/D} + \vec{M}_{F_h/D} = \vec{0}$$

$$\Rightarrow \vec{r}_{T'/D} \times \vec{T'} + \vec{r}_{F_h/D} \times \vec{F_h} = \vec{0}$$

$$\Rightarrow (-w\,\hat{i}) \times (T'\hat{j}) + (-\tfrac{w}{2}\,\hat{i}) \times [F_h(-\hat{j})] = \vec{0}$$

$$\Rightarrow -wT'(\hat{k}) + (\tfrac{w}{2} \cdot F_h)\,\hat{k} = \vec{0}$$

$$\Rightarrow \{-w \cdot T\cos\theta\,(\hat{k}) + (\tfrac{w}{2} \cdot F_h)\,\hat{k} = \vec{0}\} \cdot \hat{k} \quad [\because T' = T \cdot \cos\theta]$$

$$\Rightarrow T = \frac{F_h}{2 \cdot \cos\theta}$$

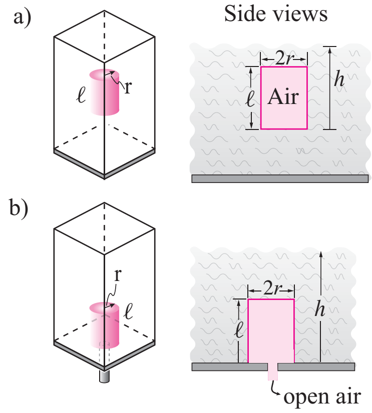
$$\Rightarrow T = \frac{1}{2} \cdot \frac{\gamma w h^2}{2} \times \frac{\sqrt{w^2+L^2+d^2}}{L}$$

$$\Rightarrow T = \frac{\gamma w h^2}{4L} \sqrt{w^2+L^2+d^2}$$

**9.1.9** This problem somewhat explains the workings of some toilet valves. Open the tank of a toilet and look at the rubber piece at the bottom that sits on the bottom but then floats after initially lifted by the turning of the flush lever. The puzzle this problem solves is this: Why does the valve stick to the bottom, but then float when lifted.

a) A hollow cylinder with an open bottom (like an upside down but open can) is filled with air but is under water. What force is required to hold it under water (in terms of $\rho, r, h, \ell$ and $g$?

b) The same can is on the bottom of a tank of water and its edges are sealed. The bottom is open to atmospheric air. How much force is needed to hold the can down now (so there is no force from the bottom of the tank onto the edges of the cylinder)?




Problem 9.9

**Answer:**
(a) $\rho g \pi r^2 \ell$
(b) $-\rho g \pi r^2 (h-\ell)$, note the minus sign, it now takes force to lift the can.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

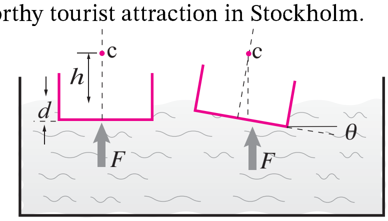
**9.1.14** *Challenge:* This challenge problem is closely related to the challenge problem above, but is much more famous. It seems to have been first solved by Leonard Euler and Pierre Bouguer in about 1735. This solution seems to be the first mechanics problem in which the significance of the area moment of inertia was appreciated [this is a hint].

For simplicity assume that a boat is shaped like a box with width $h$ and and length into the paper of $b$. Assume that the boat floats with its bottom a depth $d$ under water. Now rotate the boat about an axis at the surface of the water and along its length (into the paper). Imagine that giant hands hold the boat in this position. In this rotated position the effect of the water pressure on the boat is a buoyant force and moment. This is equivalent to a force that is displaced slightly sideways.

Your goal is to find the height of the point that the line of action of this force intersects a mast of the boat. For small angles of boat tip the location is independent of the amount of tip.

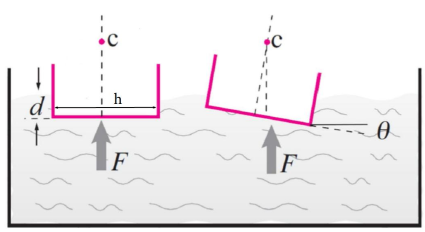
This point is called the *metacenter* of the hull, and its distance up from the centroid of the boat's submerged volume is the hull's *metacentric height*. The condition of boat stability is that the metacenter be above the center of mass of the boat (thus the moment of the buoyant forces about the center of mass will tend to restore the boat to level).

Euler and Bouguer did the calculation you are asked to do here *after*, *e.g.*, , the launching of the 'great' Swedish ship Vasa, which capsized in the harbor on day 1. This was unfortunate for Sweden at the time, but fortunate now, because the brand-new 375 year old ship is a seeworthy tourist attraction in Stockholm.




Problem 9.14: Ship stability.

**8.1.14**



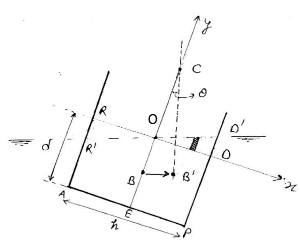

Center of buoyant force is the centroid of the submerged volume of the boat. When the boat tilts, this point shifts from its initial position (say B) to the new position of centroid of the submerged volume (say B'). We need to find this shift in position of the centroid (denoted by $\bar{x}$ and $\bar{y}$), and the metacentric height.

Since the depth (b) of the boat is constant, we consider the area moments in order to find the shift in center of buoyancy. Let the coordinate system be centered at 'O'. The shift in $x$ direction is given as:

$$\bar{x} = \frac{\int_{ARDP} xdA + \int_{D'OD} xdA - \int_{R'OR} xdA}{A_{AR'D'P}}$$

$$\Rightarrow \bar{x} = \frac{0 + \int_0^{h/2} x^2 tan\theta dx - \int_{-h/2}^{0} -x^2 tan\theta dx}{hd}$$

$$\Rightarrow \bar{x} = \frac{tan\theta \int_{-h/2}^{h/2} x^2 dx}{hd} \Rightarrow \bar{x} = \frac{h^2 tan\theta}{12d} \quad (1)$$




The shift in $y$ direction is given as:

$$\bar{y} = \frac{\int_{ARDP} ydA + \int_{D'OD} ydA - \int_{R'OR} ydA}{A_{AR'D'P}}$$

$$\Rightarrow \bar{y} = \frac{-\frac{d}{2}hd + \int_0^{\frac{h}{2}tan\theta} y(\frac{h}{2} - y)cot\theta dy - \int_{-\frac{h}{2}tan\theta}^{0} -y(\frac{h}{2} + y)cot\theta dy}{hd}$$

**8.1.14**

$$\Rightarrow \bar{y} = \frac{-\frac{d}{2}hd + cot\theta\left[\frac{y^2}{2}\frac{h}{2} - \frac{y^3}{3}\right]_0^{\frac{h}{2}tan\theta} + cot\theta\left[\frac{y^2}{2}\frac{h}{2} + \frac{y^3}{3}\right]_{-\frac{h}{2}tan\theta}^{0}}{hd}$$

$$\Rightarrow \bar{y} = -\frac{d}{2} + \frac{h^2tan^2\theta}{24d} \quad (2)$$

For small angle (θ), assuming $tan\ \theta \approx \theta$ and $tan^2\ \theta \approx 0$.

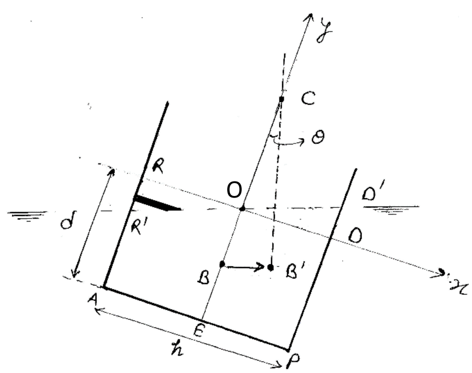
$\therefore\ \bar{x} = \frac{h^2\theta}{12d}\ \ and\ \ \bar{y} = -\frac{d}{2}$

Here $\bar{x}$ and $\bar{y}$ give the approximate position of point B'.

As the center of buoyancy shift in $x$ direction is significant, from ΔBB'C

$$tan\theta \approx \frac{BB'}{BC}$$

$$\Rightarrow BC \approx \frac{BB'}{tan\theta} \approx \frac{\bar{x}}{tan\theta} = \frac{h^2\tan\theta}{12d\tan\theta} = \frac{h^2}{12d}$$




When point C (also called the metacenter) lies above the center of gravity of the ship, then the couple applied by gravity and the buoyant force is of restoring nature and the boat stays in stable equilibrium. This metacentric height (BC in triangle BB'C) is independent of tilt for small angle tip and is a feature of the geometric configuration of the ship.