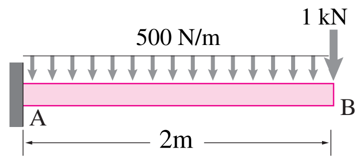
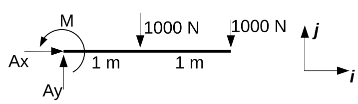
8.1.1 A cantilever beam AB is loaded as shown in the figure. Find the support reactions on the beam at the left end A.



Problem 8.1

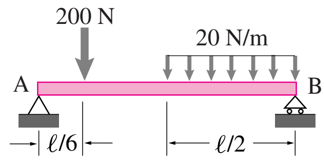


$$
\begin{aligned}
A_x &= 0 \\
A_y - 1000 - 1000 &= 0 \\
A_y &= 2000 \\
M - 1 \cdot 1000 - 2 \cdot 1000 &= 0 \\
M &= 3000
\end{aligned}
$$

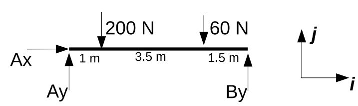
$\vec{F}_A = 2000\hat{\boldsymbol{\jmath}}\,\mathrm{N},\ \vec{M}_A = 3000\hat{\boldsymbol{k}}\,\mathrm{N \cdot m}.$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.2 A simply supported beam AB of length $\ell = 6\,\mathrm{m}$ is partly loaded with a uniformly distributed load as shown in the figure. In addition, there is a concentrated load acting at $\ell/6$ from the left end A. Find the support reactions on the beam.



Problem 8.2

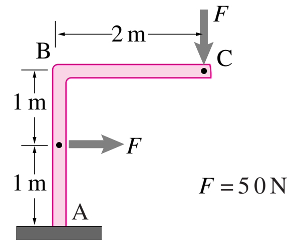


$$A_x = 0$$
$$A_y - 200 - 60 + B_y = 0$$
$$A_y = 260 - B_y$$
$$-1 \cdot 200 - 4.5 \cdot 60 + 6B_y = 0$$
$$6B_y = 470$$
$$B_y = 78.3$$
$$A_y = 260 - 78.3 = 182$$

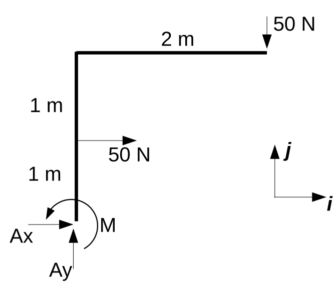
$\vec{F}_A = 182\hat{\jmath}\,\mathrm{N}$, $\vec{F}_B = 78.3\hat{\jmath}\,\mathrm{N}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.3 An (inverted) L-shaped frame is loaded with two equal concentrated forces of magnitude 50 N each as shown in the figure. Find the support reactions at A.



Problem 8.3

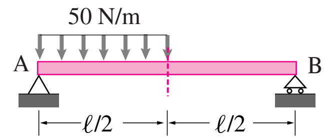


$$A_x + 50 = 0$$
$$A_x = -50$$
$$A_y - 50 = 0$$
$$A_y = 50$$
$$M - 1 \cdot 50 - 2 \cdot 50 = 0$$
$$M = 150$$

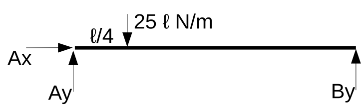
$\vec{F}_A = (-50\hat{\imath} + 50\hat{\jmath})\,\mathrm{N},\ \vec{M}_A = 150\hat{\boldsymbol{k}}\,\mathrm{N\cdot m}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.4 Find the shear force and the bending moment at the mid section of the simply supported beam shown in the figure.



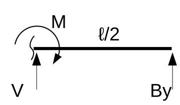
Problem 8.4



First find reaction at B.

$$-\frac{\ell}{4}\cdot 25\ell + \ell B_y = 0$$
$$B_y = \frac{25\ell}{4}$$

Now find shear and bending moment from FBD of right half of beam.

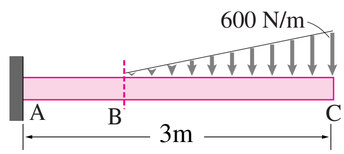


$$V + B_y = 0$$
$$V = -B_y = -\frac{25\ell}{4}$$
$$-M + \frac{\ell}{2}B_y = 0$$
$$M = \frac{\ell}{2}B_y = \frac{25}{8}\ell^2$$

$V = -\frac{25}{4}\ell\ \mathrm{N/m}$, $M = \frac{25}{8}\ell^2\ \mathrm{N/m}$

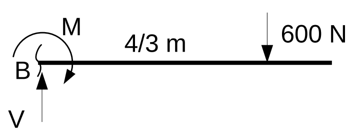
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.5 A cantilever beam ABC is loaded with a linearly variable distributed load along two thirds of its span. The intensity of the load at the right end is 600 N/m. Find the shear force and the bending moment at section B of the beam.



Problem 8.5

Equivalent force is (2 m) · (600 N/m) / 2 = 600 N at 2/3 m from C.



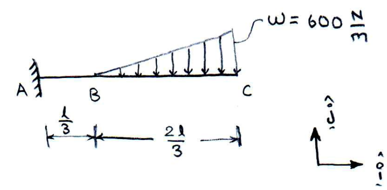
$$V - 600 = 0$$
$$V = 600$$
$$-M - \tfrac{4}{3} \cdot 600 = 0$$
$$M = -800$$

$V = 600\,\mathrm{N},\ M = -800\,\mathrm{N \cdot m}$

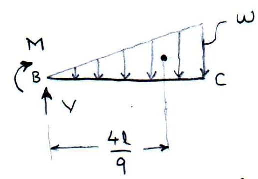
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

7.1.5

Find the shear force and the bending moment at B.



Let the shear force and the bending moment be V and M respectively as shown in the FBD.



The distributed load shown in the FBD is equivalent to a point force of magnitude $\frac{w}{2}\left(\frac{2l}{3}\right)$ acting at distance $\frac{2}{3}\left(\frac{2l}{3}\right)$ from B.

From the equilibrium of the beam segment BC.

$\sum \underline{F} = \underline{0}$

$\Rightarrow \left\{ V\hat{j} + \frac{1}{2}\frac{2l}{3}w(-\hat{j}) = \underline{0} \right\}$

$\{\}\cdot\hat{j} \quad \Rightarrow \quad V - \frac{l}{3}w = 0 \quad \Rightarrow \quad V = \frac{wl}{3}$

$\sum \underline{M}_{/B} = \underline{0}$

$\Rightarrow \quad -M\hat{k} + \frac{4l}{9}\hat{i} \times \left[\frac{1}{2}\frac{2l}{3}w(-\hat{j})\right] = \underline{0}$

$\Rightarrow \quad -M - \frac{4}{27}wl^2 \quad \Rightarrow \quad M = \frac{-4}{27}wl^2$

Given that $w = 600\ N/m$ and $l = 3m$

$\therefore$ Shear force at B is $V = \frac{wl}{3} = \frac{1}{3}\,600\,\frac{N}{m}\,3m \Rightarrow \boxed{V = 600\,N}$

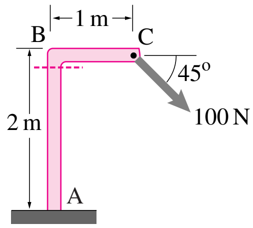
and Bending moment at B is $M = \frac{-4}{27}wl^2 = \frac{-4}{27}\,600\,\frac{N}{m}\,(3m)^2$

$\Rightarrow \boxed{M = -800\ Nm}$

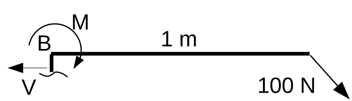
As the bending moment obtained at B is negative, it is in the opposite direction to the assumed direction.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.6 Analyze the frame shown in the figure and find the shear force and the bending moment at the end of the vertical section of the frame.



Problem 8.6



"Smile" sign convention for bending moment is ambiguous for vertical members—use positive direction as shown.

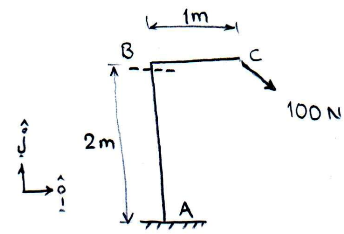
$$-V + 100\frac{\sqrt{2}}{2} = 0$$
$$V = 50\sqrt{2}$$
$$-M - 100\frac{\sqrt{2}}{2} = 0$$
$$M = -50\sqrt{2}$$

$V = 50\sqrt{2}\,\mathrm{N},\ M = -50\sqrt{2}\,\mathrm{N \cdot m}$

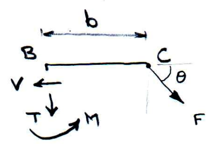
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

7.1.6

Find the shear force and bending moment at the end of the vertical section.



Consider the FBD of the frame at BC



From the equilibrium of BC,

$\sum \underline{F} = \underline{0}$

$\Rightarrow \left\{ V(-\hat{\imath}) + T(-\hat{\jmath}) + F\cos\theta\,\hat{\imath} + F\sin\theta(-\hat{\jmath}) = \underline{0} \right\}$

$\{\,\}\cdot\hat{\imath} \quad \Rightarrow \quad -V + F\cos\theta = 0 \quad \Rightarrow \quad V = F\cos\theta$

Also $\{\,\}\cdot\hat{\jmath} \quad \Rightarrow \quad -T - F\sin\theta = 0 \quad \Rightarrow \quad T = -F\sin\theta$

$\sum \underline{M}_{/B} = \underline{0}$

$\Rightarrow M\hat{k} + \underline{r}_{C/B} \times \underline{F} = \underline{0}$

$\Rightarrow M\hat{k} + b\hat{\imath} \times (F\cos\theta\,\hat{\imath} - F\sin\theta\,\hat{\jmath}) = \underline{0}$

$\Rightarrow \left\{ M\hat{k} - Fb\sin\theta\,\hat{k} = \underline{0} \right\}$

$\{\,\}\cdot\hat{k} \quad \Rightarrow \quad M = Fb\sin\theta$

Given that $F = 100\,N$, $b = 1m$ and $\theta = 45°$

The shear force at B, $V = 100\,(N)\cos 45° \Rightarrow \boxed{V = 70.71\,N}$

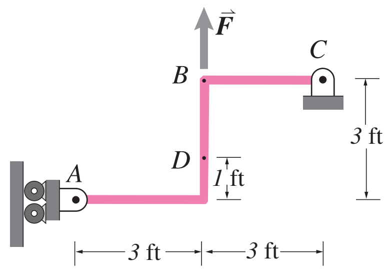
The bending moment at B, $M = 100\,(N)\;1\,(m)\;\sin 45°$

$\Rightarrow \boxed{M = 70.71\,Nm}$

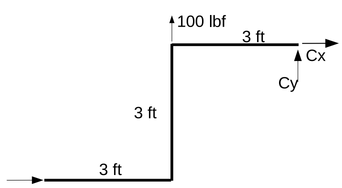
8.1.7 A force $F = 100\,\text{lbf}$ is applied to the bent rod shown. Before doing any calculations, try to figure out the tension at D in your head.

a) Find the reactions at A and C.

b) Find the tension, shear and bending moment at the section D. Check your answer against what you figured out in your head.



Problem 8.7



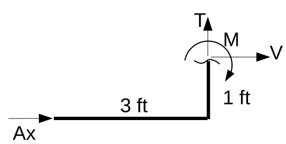
Find reaction at A by balancing moments at C.

$$-3 \cdot 100 + 3A_x = 0$$
$$A_x = 100$$

3

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Use FBD of lower section to find tension, shear and bending moment. ”Smile” sign convention for bending moment is ambiguous for vertical members—use positive direction as shown.



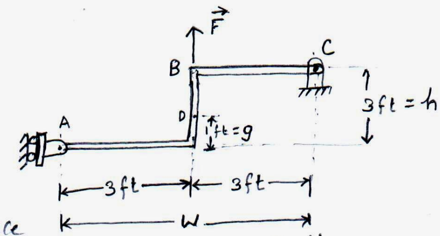
$$100 + V = 0$$
$$V = -100$$
$$T = 0$$
$$1 \cdot 100 - M = 0$$
$$M = 100$$

$T = 0$, $V = -100\,\text{lbf}$, $M = 100\,\text{ft}\cdot\text{lbf}$

7.1.7 :

a) Find the reactions at A and C.

b) Find the tension, shear and bending moment at D.



Solution: Let's call the horizontal distance between A and C as "w" and vertical distance as "h".

Now, let's consider the FBD of the Bent rod "ABC":

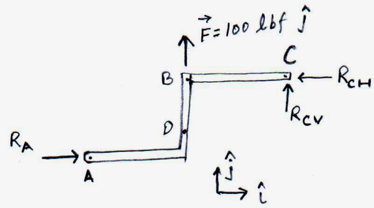


fig: FBD of the bent rod "ABC"

Since point A is supported by a roller, and the rod is free to rotate at this point, the vertical reaction and moment reactions at point A are zero. Similarly, no reaction bending moment is offered by joint at C.

By force balance of this rod (based on the FBD shown above), we get:

$$\sum \vec{F} = \vec{0} \Rightarrow R_A \hat{i} + F\hat{j} - R_{CH}\hat{i} + R_{cv}\hat{j} = \vec{0} \quad \text{——— (i)}$$

$$\{\text{equ.(i)}\} \cdot \hat{i} \Rightarrow R_A - R_{CH} = 0 \Rightarrow R_A = R_{CH} \quad \text{——— (ii)}$$

$$\{\text{equ.(i)}\} \cdot \hat{j} \Rightarrow F + R_{cv} = 0 \Rightarrow R_{cv} = -F \quad \text{——— (iii)}$$

Moment balance of the rod about point C gives:

$$\sum \vec{M}_{/C} = \vec{0} \Rightarrow \vec{r}_{A/C} \times \vec{R}_A + \vec{r}_{B/C} \times \vec{F} = \vec{0}$$

$$\Rightarrow (-w\hat{i} - h\hat{j}) \times R_A \hat{i} + \left(-\frac{w}{2}\hat{i}\right) \times F\hat{j} = \vec{0}$$

$$\Rightarrow \left\{\left(R_A h - \frac{Fw}{2}\right)\hat{k} = \vec{0}\right\}$$

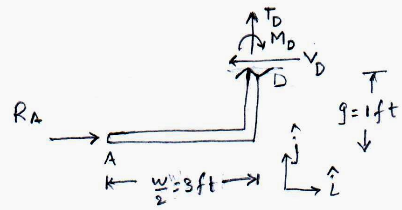
$$\{\ \}\cdot\hat{k} \Rightarrow R_A h = \frac{Fw}{2} \Rightarrow R_A = \frac{Fw}{2h} \quad \text{——— (iv)}$$

By substituting the given values, $F = 100\ lbf$, $w = 6ft$, $h = 3ft$; we get,

$$\boxed{R_A = 100\ lbf\ ;\ R_{CH} = 100\ lbf\ ;\ \text{and } R_{cv} = -100\ lbf}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

7.1.7 (b) To determine the section forces at point 'D', let us consider the FBD of the rod from point 'A' to point 'D'



By force balance of this section we get:

$$\underbrace{\left\{ R_A \hat{i} - V_D \hat{i} + T_D \hat{j} = \vec{0} \right\}}$$

$$\{ \quad \} \cdot \hat{j} \Rightarrow T_D = 0$$

$$\{ \quad \} \cdot \hat{i} \Rightarrow V_D = R_A$$

By moment balance of this portion about point D we get

$$\Sigma \vec{M}_{/D} = 0$$

$$\Rightarrow \vec{r}_{A/D} \times (R_A \hat{i}) - M_D \hat{k} = \vec{0}$$

$$\Rightarrow \left[-\left(\frac{w}{2} \hat{i} + g \hat{j} \right) \right] \times R_A \hat{i} - M_D \hat{k} = \vec{0}$$

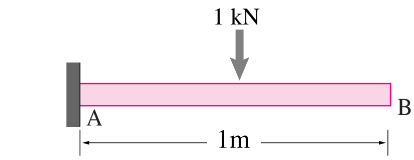
$$\Rightarrow (R_A g - M_D) \hat{k} = \vec{0}$$

$$\{ \} \cdot \hat{k} \Rightarrow M_D = R_A g$$

Substituting the given values we get

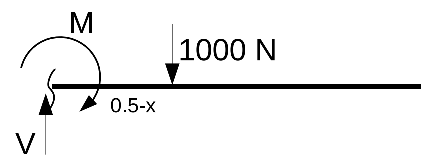
$$\boxed{T_D = 0 \; ; \; V_D = 100 \text{ lbf} \; ; \; M_D = 100 \text{ lbf} \cdot \text{ft}}$$

8.1.8 Draw the shear force and the bending moment diagram for the cantilever beam shown in the figure.



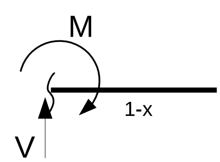
Problem 8.8

Cut left of force.



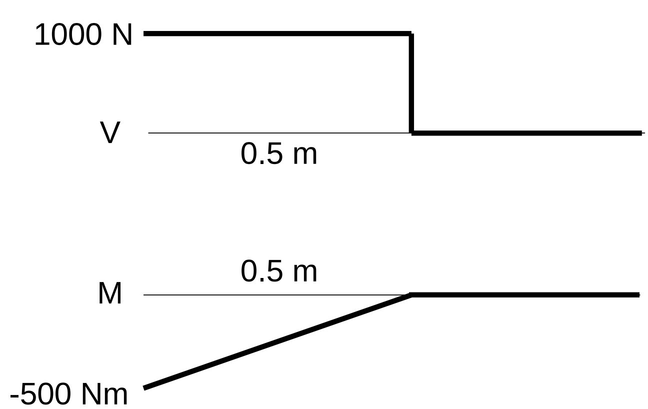
$$V - 1000 = 0$$
$$V = 1000$$
$$-M - (0.5 - x)1000 = 0$$
$$-M - 500 + 1000x = 0$$
$$M = 1000x - 500$$

Cut right of force.



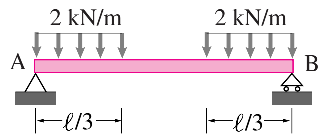
$$V = 0$$
$$M = 0$$

Draw shear and bending moment diagrams.



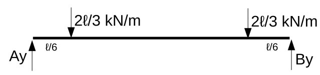
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.9 A simply supported beam AB is loaded along one thirds of its span from both ends by a uniformly distributed load of intensity 2 kN/m. Draw the shear force and the bending moment diagram of the beam.



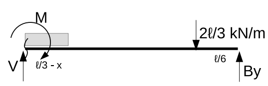
Problem 8.9

Get reaction at B.



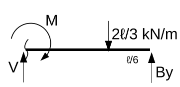
$$-\tfrac{\ell}{6}\tfrac{2\ell}{3} - \tfrac{5\ell}{6}\tfrac{2\ell}{3} + \ell B_y = 0$$
$$\ell B_y = \tfrac{2\ell^2}{3}$$
$$B_y = \tfrac{2\ell}{3}$$

Cut in first third. Equivalent force is $2(\frac{\ell}{3} - x)$ at $\frac{\ell}{6} - \frac{x}{2}$ from cut.



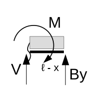
$$V - 2(\tfrac{\ell}{3} - x) - \tfrac{2\ell}{3} + \tfrac{2\ell}{3} = 0$$
$$V = \tfrac{2\ell}{3} - 2x$$
$$-M - \left(\tfrac{\ell}{6} - \tfrac{x}{2}\right) \cdot 2(\tfrac{\ell}{3} - x) - \left(\tfrac{5\ell}{6} - x\right)\tfrac{2\ell}{3} + (\ell - x)\tfrac{2\ell}{3} = 0$$
$$-M - \left(\tfrac{\ell^2}{9} - \tfrac{2\ell x}{3} + x^2\right) - \left(\tfrac{5\ell^2}{9} - \tfrac{2\ell x}{3}\right) + \tfrac{2\ell^2}{3} - \tfrac{2\ell x}{3} = 0$$
$$\tfrac{2\ell x}{3} - x^2 = M$$

Cut in middle third.



$$V - \tfrac{2\ell}{3} + \tfrac{2\ell}{3} = 0$$
$$V = 0$$
$$-M - (\tfrac{5\ell}{6} - x)\tfrac{2\ell}{3} + (\ell - x)\tfrac{2\ell}{3} = 0$$
$$-M + \tfrac{\ell^2}{9} + \tfrac{2\ell x}{3} - \tfrac{2\ell x}{3} = 0$$
$$M = \tfrac{\ell^2}{9}$$

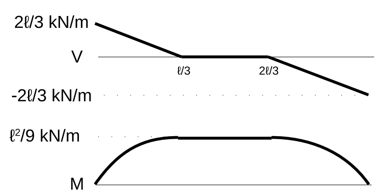
Cut in last third. Equivalent force is $2(\ell - x)$ at $\frac{\ell - x}{2}$ from cut.



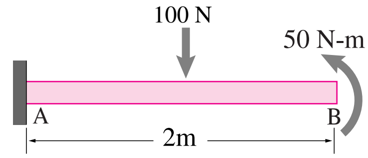
$$V - 2(\ell - x) + \tfrac{2\ell}{3} = 0$$
$$V = \tfrac{4\ell}{3} - 2x$$
$$-M - \tfrac{\ell - x}{2} \cdot 2(\ell - x) + \tfrac{2\ell}{3}(\ell - x) = 0$$
$$-M - (\ell^2 - 2\ell x + x^2) + \tfrac{2\ell^2}{3} - \tfrac{2\ell x}{3} = 0$$
$$-\tfrac{\ell^2}{3} + \tfrac{4\ell x}{3} - x^2 = M$$

Graph result.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

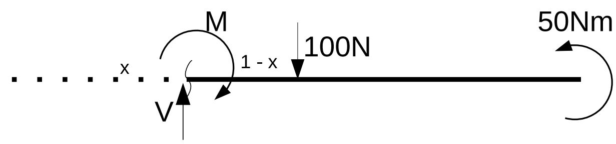

2ℓ/3 kN/m
V
ℓ/3
2ℓ/3
-2ℓ/3 kN/m
ℓ²/9 kN/m
M

8.1.10 The cantilever beam shown in the figure is loaded with a concentrated load and a concentrated moment as shown in the figure. Draw the shear force and the bending moment diagram of the beam.



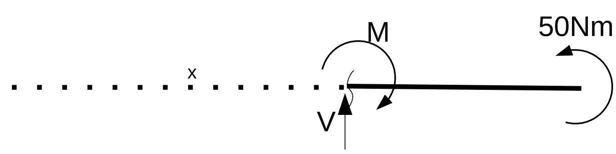
Problem 8.10

Cut in first half.



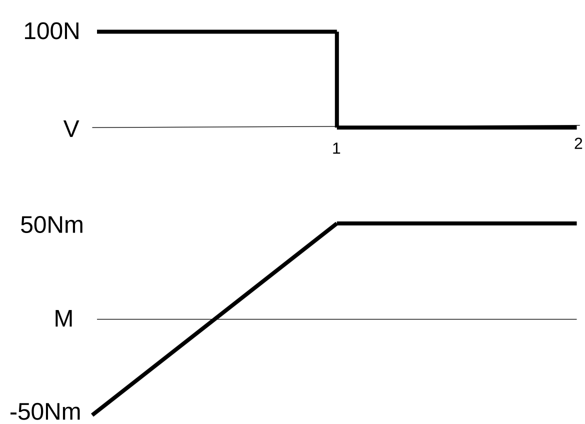
$$V - 100 = 0$$
$$V = 100$$
$$-M - (1-x)100 + 50 = 0$$
$$-100 + 100x + 50 = M$$
$$-50 + 100x = M$$

Cut in second half.



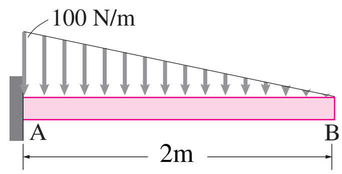
$$V = 0$$
$$-M + 50 = 0$$
$$M = 50$$

Graph the result.

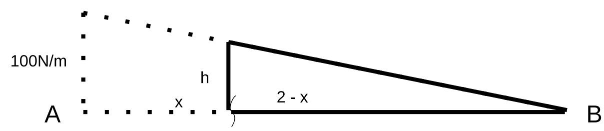


Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.11 A cantilever beam AB is loaded with a triangular shaped distributed load as shown in the figure. Draw the shear force and the bending moment diagrams for the entire beam.



Problem 8.11

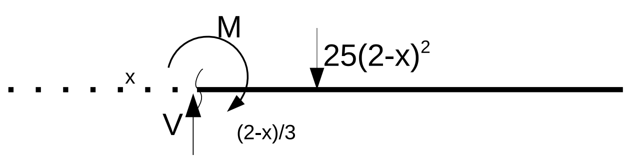


$$\frac{h}{2-x} = \frac{100}{2}$$
$$h = 50(2-x)$$

Equivalent load is

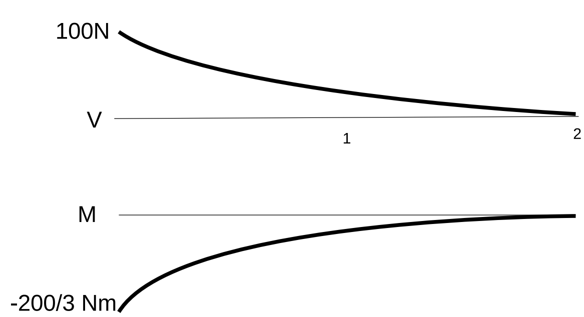
$$\frac{50(2-x)(2-x)}{2} = 25(2-x)^2$$

at $\frac{2-x}{3}$ right of the cut.



$$V - 25(2-x)^2 = 0$$
$$V = 25(2-x)^2$$
$$-M - \frac{2-x}{3}25(2-x)^2 = 0$$
$$-\frac{25}{3}(2-x)^3 = M$$

Graph the result.

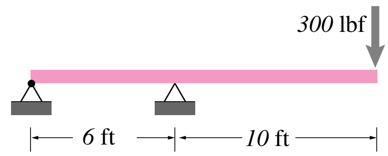


Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

8.1.12 A regulation 16 ft diving board is supported as shown.

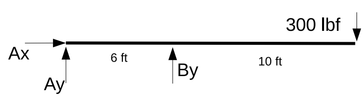
a) Where is the bending moment the greatest and how big is it there?

b) Draw a bending moment diagram for this board.



Problem 8.12

Let A be at the left support and B be at the right support. Find load at B by balancing moments at A.

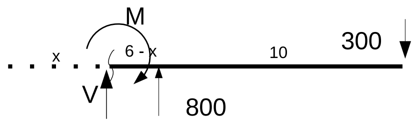


$$
\begin{aligned}
6B_y - 16 \cdot 300 &= 0 \\
6B_y &= 4800 \\
B_y &= 800
\end{aligned}
$$

Cut to left of B.

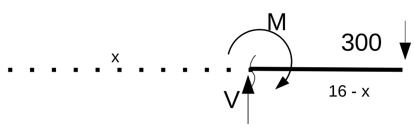
7

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*



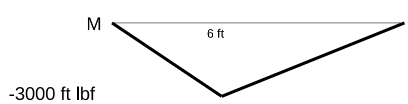
$$
\begin{aligned}
-M + (6-x)800 - (16-x)300 &= 0\\
4800 - 800x - 4800 + 300x &= M\\
-500x &= M
\end{aligned}
$$

Cut to right of B.



$$
\begin{aligned}
-M - (16-x)300 &= 0\\
-4800 + 300x &= M
\end{aligned}
$$

b) Graph the bending moment.



a) From the graph, the maximum magnitude bending moment is -3000 ft lbf and it occurs at point B.

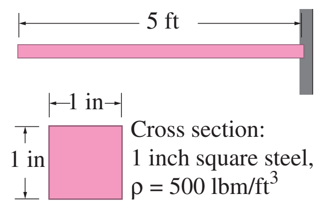
8.1.13 The cantilever steel beam is loaded by its own weight.

a) Find the bending moment and shear force at the free and at the clamped end.

b) Draw a shear force diagram

c) Draw a bending moment diagram

d) The tension stress σ in the beam at the top edge where it is biggest is given by $\sigma = 12M/h^3$ where $h = 1 \in$ for this beam. The strength (the maximum tension stress the material can bear) of soft steel is about $\sigma_{\text{max}} = 30,000\,\text{lbf}/\ \in^2$. What is the longest a beam with this cross section can be made and still not fail?

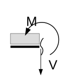


Problem 8.13

Let $\rho = 500\,\text{lbm}/\text{ft}^3$ be the mass density of the bar. [Note: Not 500 lbm/ in^3.] Let $A = 1\,\text{in}^2$ be the cross-sectional area, and $\ell = 5\,\text{ft}$ be the length. Then the mass per unit length is ρA and the weight per unit length is $\rho A g$, where g is the gravitational acceleration.

a)

Free end. Cut infinitesimally right of end.



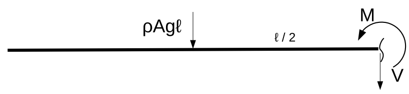
$$V = 0$$
$$M = 0$$

Clamped end. Cut infinitesimally left of clamp. Find equivalent force.

8

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Total force is $\rho Ag\ell$ equivalent at $\ell/2$.

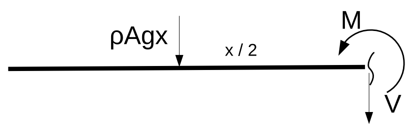


$$-\rho Ag\ell - V = 0$$
$$-\rho Ag\ell = V$$
$$\rho Ag\ell\frac{\ell}{2} + M = 0$$
$$M = -\frac{1}{2}\rho Ag\ell^2$$

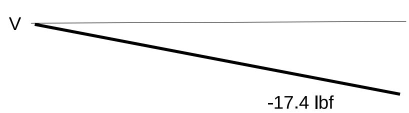
Plugging in the numbers, and using conversion factors 1 lbf = g · 1 lbm and 1 ft = 12 in.

$$\begin{aligned} V &= -1\,\text{in}^2\,500\frac{\text{lbm}}{\text{ft}^3}\,g\,60\,\text{in} \\ &= -30000\,g\,\frac{\text{lbm}}{\text{ft}^3}\,\text{in}^3\,\frac{\text{lbf}}{g\,\text{lbm}}\left(\frac{\text{ft}}{12\,\text{in}}\right)^3 \\ &= -\frac{30000}{12^3}\,\text{lbf} = -17.4\,\text{lbf} \\ M &= -\frac{1}{2}\,500\frac{\text{lbm}}{\text{ft}^3}\,1\,\text{in}^2 g\,(60\,\text{in})^2 \\ &= -900000\,g\,\text{in}^4\,\frac{\text{lbm}}{\text{ft}^3}\left(\frac{\text{ft}}{12\,\text{in}}\right)^4\frac{\text{lbf}}{g\,\text{lbm}} \\ &= -\frac{900000}{12^4}\,\text{ft}\cdot\text{lbf} = -43.4\,\text{ft}\cdot\text{lbf} \end{aligned}$$

b)

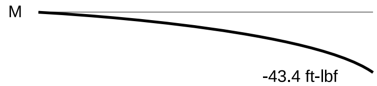


$$-\rho Agx - V = 0$$
$$-\rho Agx = V$$



c)

$$\frac{x}{2}\rho Agx + M = 0$$
$$M = -\frac{1}{2}\rho Agx^2$$



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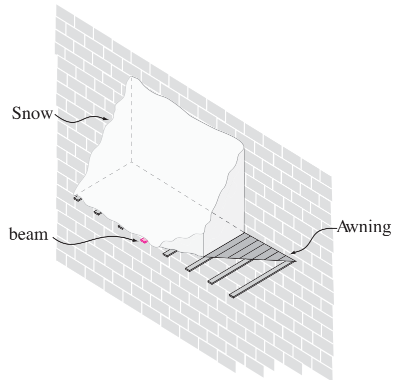
d) The maximum (absolute value) of the moment is when $x = \ell$, so it is $\frac{1}{2}\rho A g \ell^2$ from above. Thus the maximum tension stress is $12/h^3$ times that value, and we want

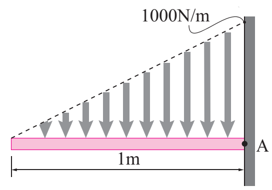
$$\begin{aligned}
\frac{12\frac{1}{2}\rho A g \ell^2}{h^3} &< \sigma \\
\frac{6(1\text{ in})^2}{1\text{ in}^3}\, 500\frac{\text{lbm}}{\text{ft}^3}\, g\, \ell^2 &< \sigma \\
3000\, g \frac{1}{\text{in}}\frac{\text{lbm}}{\text{ft}^3}\, \ell^2 \left(\frac{\text{ft}}{12\text{ in}}\right)^3 \frac{\text{lbf}}{g\,\text{lbm}} &< \sigma \\
\frac{3000}{12^3}\frac{\text{lbf}}{\text{in}^4}\ell^2 &< 30000\frac{\text{lbf}}{\text{in}^2} \\
\ell^2 &< 12^3 \cdot 10\,\text{in}^2 \\
\ell &< 131\text{ in}
\end{aligned}$$

8.1.14 A snow loaded bus-stop awning (shown partially cut away) on the side of a building is supported by horizontal, cantilevered, beams. The loading that is carried by one beam is as shown below.

a) Find the reaction force and couple at the wall at A (the force and moment acting on one beam from the wall).

b) Draw shear force and bending moment diagrams for the beam.

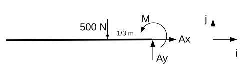




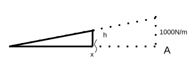
Problem 8.14

Answer:
$F_{Ay} = -500\,\mathrm{N}, M_A = -500/3\,\mathrm{N{\cdot}m}$

The total load acting on one beam is 500 N and it acts equivalently at 1/3 m from A.

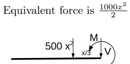


$$A_x = 0$$
$$-500 + A_y = 0$$
$$A_y = 500$$
$$\vec{A} = 500\,\mathrm{N}\hat{\boldsymbol{\jmath}}$$
$$\frac{500}{3} + M = 0$$
$$M = -\frac{500}{3}$$
$$\vec{M} = -500/3\,\mathrm{N}\cdot\mathrm{m}\hat{\boldsymbol{k}}$$



$$\frac{x}{h} = \frac{1}{1000}$$
$$1000x = h$$

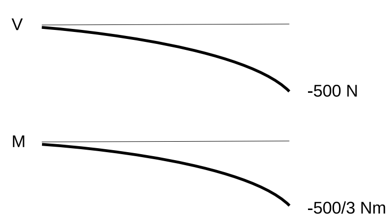
Equivalent force is $\frac{1000x^2}{2}$ at $\frac{2x}{3}$.



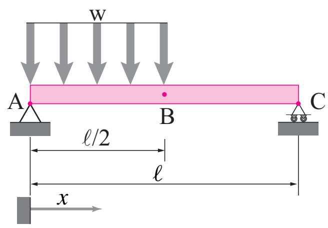
$$-500x^2 - V = 0$$
$$-500x^2 = V$$
$$\frac{x}{3}\,500x^2 + M = 0$$
$$M = -\frac{500}{3}\,x^3$$

10

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*


V
-500 N
M
-500/3 Nm

8.1.15 Draw shear and bending moment diagrams of the beam shown. Clearly label the values of the heights of the curves at jumps, kinks and local maxima (if and where they exist).



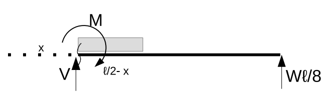
Problem 8.15

Answer:
$V(\ell/2) = -w\ell/8, M(\ell/2) = w\ell^2/16, M_{max} = M(3\ell/8) = 9wl^2/128$

Find the force at C. The equivalent load is $W\ell/2$ at $\ell/4$.

$$-\frac{\ell}{4}\frac{W\ell}{2} + \ell C_y = 0$$
$$C_y = \frac{W\ell}{8}$$

Cut in left half.

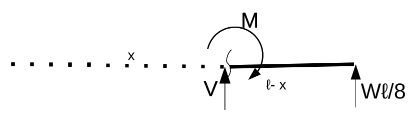


Equivalent force is $W(\ell/2 - x)$ at $\ell/4 - x/2$ from cut.

$$V - W\left(\frac{\ell}{2} - x\right) + \frac{W\ell}{8} = 0$$
$$V - \frac{W\ell}{2} + Wx + \frac{W\ell}{8} = 0$$
$$V - \frac{3W\ell}{8} + Wx = 0$$
$$V = \frac{3W\ell}{8} - Wx$$
$$-M - \left(\frac{\ell}{4} - \frac{x}{2}\right)W\left(\frac{\ell}{2} - x\right) + (\ell - x)\frac{W\ell}{8} = 0$$
$$-W\left(\frac{\ell^2}{8} - \frac{\ell x}{4} - \frac{\ell x}{4} + \frac{x^2}{2}\right) + W\left(\frac{\ell^2}{8} - \frac{\ell x}{8}\right) = M$$
$$\frac{3\ell x W}{8} - \frac{W x^2}{2} = M$$

Cut in right half.

11



$$V + \frac{W\ell}{8} = 0$$
$$V = -\frac{W\ell}{8}$$
$$-M + (\ell - x)\frac{W\ell}{8} = 0$$
$$\frac{W\ell^2}{8} - \frac{W\ell x}{8} = M$$

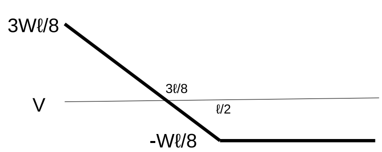
Get V and M at critical points for graphing.

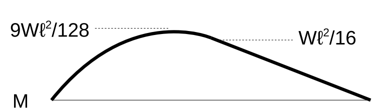
$x = 0$: $V = 3W\ell/8$; $M = 0$.

$x = 3\ell/8$: $V = 0$; $M = 9\ell^2W/64 - 9\ell^2W/128 = 9\ell^2W/128$.

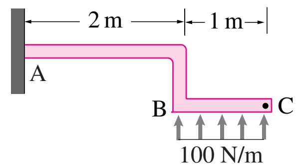
$x = \ell/2$: $V = -W\ell/8$; $M = 3\ell^2W/16 - \ell^2W/8 = \ell^2W/16$.

$x = \ell$: $V = -W\ell/8$. $M = 0$.





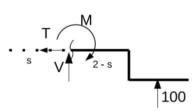
8.1.16 A frame ABC is much like a cantilever beam with a short bent section of length 0.5 m. The frame is loaded as shown in the figure. Draw the shear force and the bending moment diagrams of the entire frame indicating how it differs from an ordinary cantilever beam.



Problem 8.16

Distributed force is equivalent to 100 N at 0.5 m right of B. Measure s along the bent beam from A.

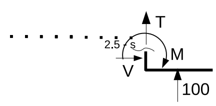
Cut between A and bend.



$$
\begin{aligned}
T &= 0\\
V + 100 &= 0\\
V &= -100\\
-M + (2.5 - s)100 &= 0\\
250 - 100s &= M
\end{aligned}
$$

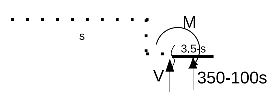
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Cut between bend and B. Use convention shown for positive shear and bending moment.

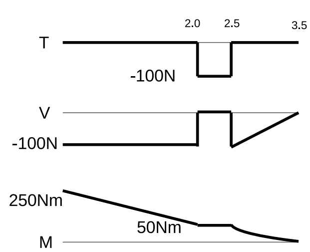


$$T + 100 = 0$$
$$T = -100$$
$$V = 0$$
$$-M + .5(100) = 0$$
$$50 = M$$

Cut between B and C. Equivalent force is $100(3.5 - s)$ at $(3.5 - s)/2$.



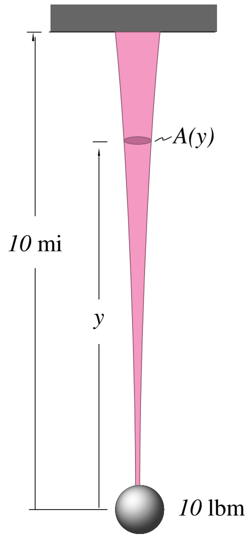
$$T = 0$$
$$V + (350 - s) = 0$$
$$V = 100s - 350$$
$$-M + \frac{3.5 - s}{2}(350 - 100s) = 0$$
$$M = 50(3.5 - s)^2$$



8.1.17 A 10 pound ball is suspended by a long steel wire. The wire has a density of about $500\,\mathrm{lbm}/\,\mathrm{ft}^3$. The strength of the wire (the maximum force per unit area it can carry) is about $\sigma_{\mathrm{max}} = 60,000\,\mathrm{lbf}/\in^2$.

a) First, neglecting the weight of the wire in the calculation of stress, what is the weight of wire needed to hold the weight?

b) Taking into account the weight of the wire in the load calculation, what is the weight of wire needed to hold the weight?



Problem 8.17

Answer:
(b) [Hint: at every height y the cross sectional area must be big enough to hold the weight plus the wire below that point. From this you can set up a differential equation for the cross sectional area A as a function of y. Find appropriate initial conditions and solve the equation. Once solved, the volume of wire can be calculated as $V = \int_0^{10\,\mathrm{mi}} A(y)dy$ and the mass as ρV.]

a) Neglecting the weight of the wire, the tension will be a constant $T = 10$ lbf throughout the wire. We want $\sigma A = T$ for the minimum cross-section A, so $A = T/\sigma$ throughout. Multiplying by $\ell =$ 10 miles to get the volume of the wire, by $\rho = 500\,\mathrm{lbm}/\,\mathrm{ft}^3$ to get the (mass) density and by g to get the weight, we have the total weight w of the wire:

$$\begin{aligned} w &= \frac{T}{\sigma}\ell\rho g \\ &= \frac{10\,\mathrm{lbf}}{60000\,\mathrm{lbf\,in}^{-2}} 10\,\mathrm{mi}\,500\frac{\mathrm{lbm}}{\mathrm{ft}^3} g \\ &= \frac{5}{6}\frac{\mathrm{in}^2\,\mathrm{mi}}{\mathrm{ft}^3}\,\mathrm{lbm}\,g \left(\frac{\mathrm{ft}}{12\,\mathrm{in}}\right)^2 \frac{5280\,\mathrm{ft}}{\mathrm{mi}}\frac{\mathrm{lbf}}{\mathrm{lbm}\,g} \\ &= \frac{5\cdot 5280}{6\cdot 12^2}\,\mathrm{lbf} = 30.6\,\mathrm{lbf} \end{aligned}$$

Where we used the fact that 5280 ft = 1 mi, 12 in = 1 ft and 1 lbf = 1 lbm $\cdot g$ as conversion factors.

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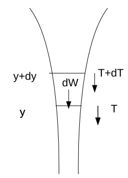
b) If we don't neglect the weight of the wire, the tension will not be a constant.



Let dW be the weight of the differential slice of the wire between y and $y + dy$. $dW = A\,dy\,\rho\,g$.

$$\begin{aligned}
T + dT &= T + A\rho g\,dy \\
\frac{dT}{dy} &= A\rho g \\
\frac{d(\sigma A)}{dy} &= A\rho g \\
\frac{dA}{dy} &= \frac{A\rho g}{\sigma}
\end{aligned}$$

since we want $\sigma A = T$, just as in part a.

Thus

$$A = A_0 e^{\frac{\rho g}{\sigma} y}$$

where A_0 is the cross-section at $y = 0$. $A_0 = T_0/\sigma$.

To get the volume of the wire, we integrate the cross-section along the length.

$$\begin{aligned}
V &= \int_0^\ell \frac{T_0}{\sigma} e^{\frac{\rho g}{\sigma} y} dy \\
&= \frac{T_0}{\sigma} \frac{\sigma}{\rho g} e^{\frac{\rho g}{\sigma} y} \Big|_0^\ell \\
&= \frac{T_0}{\rho g} \left(e^{\frac{\rho g \ell}{\sigma}} - 1 \right)
\end{aligned}$$

And to get the weight we multiply the volume by the density and the gravitational acceleration:

$$\begin{aligned}
w &= \rho g \frac{T_0}{\rho g} \left(e^{\frac{\rho g \ell}{\sigma}} - 1 \right) \\
&= T_0 \left(e^{\frac{\rho g \ell}{\sigma}} - 1 \right)
\end{aligned}$$

Computing

$$\begin{aligned}
\frac{\rho g \ell}{\sigma} &= \frac{500\,\text{lbm ft}^{-3} g\ 10\,\text{mi}}{60000\,\text{lbf in}^{-2}} \\
&= \frac{1}{12} \frac{\text{lbm ft}^{-3} g\ \text{mi}}{\text{lbf in}^{-2}} \left(\frac{\text{ft}}{12\,\text{in}} \right)^2 \frac{5280\,\text{ft}}{\text{mi}} \frac{\text{lbf}}{\text{lbm}\,g} \\
&= \frac{5280}{12^3} = 3.056
\end{aligned}$$

Substituting

$$w = 10\,\text{lbf}\left(e^{3.056} - 1\right) = 202\,\text{lbf}$$