SAMPLE 8.1 Support reactions on a simply supported beam: A uniform beam of length 3 m is simply supported at A and B as shown in the figure. A uniformly distributed vertical load $q = 100\,\mathrm{N/m}$ acts over the entire length of the beam. In addition, a concentrated load $P = 150\,\mathrm{N}$ acts at a distance $d = 1\,\mathrm{m}$ from the left end. Find the support reactions.

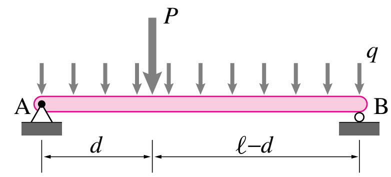

Figure 8.6

Solution Because the beam is supported at A on a pin joint and at B on a roller, the unknown reactions are

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}, \qquad \vec{B} = B_y\hat{\jmath}.$$

The uniformly distributed load q can be replaced by an equivalent concentrated load $W = q\ell$ acting at the center of the beam span. The free-body diagram of the beam, with the concentrated load replaced by the equivalent concentrated load is shown in Fig. 8.7. The moment equilibrium about point A, $\sum \vec{M}_A = \vec{0}$, gives

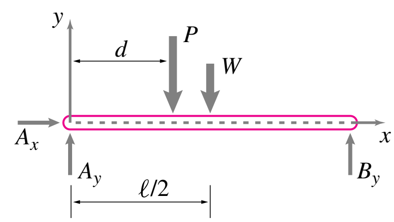

Figure 8.7

$$\left(-Pd - W\frac{\ell}{2} + B_y\ell\right)\hat{k} = \vec{0}$$

$$\begin{aligned}\Rightarrow \quad B_y &= P\frac{d}{\ell} + \frac{1}{2}W \\ &= 150\,\mathrm{N}\cdot\frac{1}{3} + \frac{1}{2}\cdot 300\,\mathrm{N} = 200\,\mathrm{N}.\end{aligned}$$

The force equilibrium, $\sum \vec{F} = \vec{0}$, gives

$$\vec{A} + B_y\hat{\jmath} - P\hat{\jmath} - W\hat{\jmath} = \vec{0}$$

$$\begin{aligned}\Rightarrow \quad \vec{A} &= (-B_y + P + W)\hat{\jmath} \\ &= (-200\,\mathrm{N} + 150\,\mathrm{N} + 300\,\mathrm{N})\hat{\jmath} = 250\,\mathrm{N}\hat{\jmath}.\end{aligned}$$

$$\boxed{\vec{A} = 250\,\mathrm{N}\hat{\jmath}, \quad \vec{B} = 200\,\mathrm{N}\hat{\jmath}}$$

SAMPLE 8.2 Support reactions on a cantilever beam: A 2 kN horizontal force acts at the tip of an 'L'-shaped cantilever beam as shown in the figure. Find the support reactions at A.

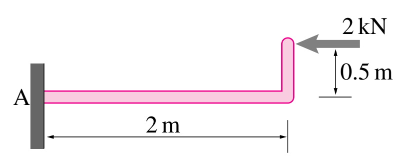

Figure 8.8

Solution The free-body diagram of the beam is shown in Fig. 8.9. The reaction force at A is $\vec{A}$ and the reaction moment is $\vec{M} = M\hat{k}$. Writing the moment-balance equation about point A, $\sum \vec{M}_A = \vec{0}$, we get

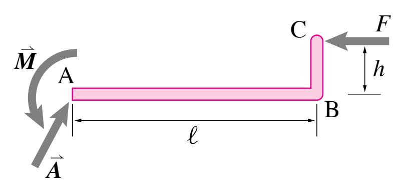

Figure 8.9

$$\begin{aligned}\vec{M} + \vec{r}_{C/A}\times\vec{F} &= \vec{0} \\ \vec{M} + (\ell\hat{\imath} + h\hat{\jmath})\times(-F\hat{\imath}) &= \vec{0} \\ \Rightarrow \quad \vec{M} &= -Fh\,\hat{k} \\ &= -2\,\mathrm{kN}\cdot 0.5\,\mathrm{m}\;\hat{k} \\ &= -1\,\mathrm{kN\cdot m}\;\hat{k}.\end{aligned}$$

The force equilibrium, $\sum \vec{F} = \vec{0}$, gives

$$\begin{aligned}\vec{A} + \vec{F} &= \vec{0} \\ \Rightarrow \quad \vec{A} &= -\vec{F} = -(-2\,\mathrm{kN}\;\hat{\imath}) = 2\,\mathrm{kN}\;\hat{\imath}.\end{aligned}$$

$$\boxed{\vec{A} = 2\,\mathrm{kN}\;\hat{\imath}, \quad \vec{M} = -1\,\mathrm{kN\cdot m}\;\hat{k}}$$

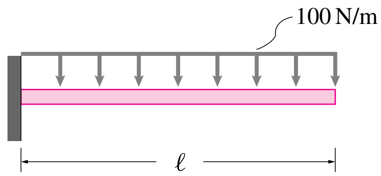

Figure 8.10

SAMPLE 8.3 Net force of a uniformly distributed system: A uniformly distributed vertical load of intensity 100 N/macts on a cantilever beam of length $\ell = 2\,\mathrm{m}$ as shown in the figure.

1. Find the net force acting on the beam.
2. Find an equivalent force-couple system at the mid-point of the beam.
3. Find an equivalent force-couple system at the right end of the beam.

Solution

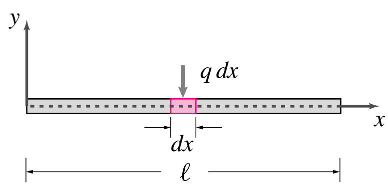

Figure 8.11

1. **The net force:** Because the load is uniformly distributed along the length, we can find the total (or net) load by calculating the load on an infinitesimal segment of length dx of the beam and then integrating over the entire length of the beam. Let the load intensity (load per unit length) be q ($q = 100\,\mathrm{N/m}$, as given). Then the vertical load on segment dx is (see Fig. 8.11),

$$d\vec{F} = q\,dx(-\hat{\jmath}).$$

Therefore, the net force is,

$$\vec{F}_{\text{net}} = \int_0^{\ell} q\,dx(-\hat{\jmath}) = q\,\ell\,\hat{\jmath} = -100\,\mathrm{N/m}\cdot 2\,\mathrm{m}\,\hat{\jmath} = -200\,\mathrm{N}\,\hat{\jmath}.$$

$$\boxed{\vec{F}_{\text{net}} = -200\,\mathrm{N}\,\hat{\jmath}}$$

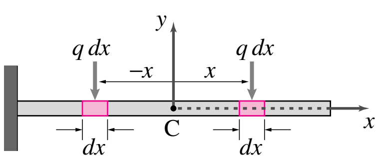

Figure 8.12

2. **The equivalent system at the mid-point:** We have already calculated the net force that can replace the uniformly distributed load. Now we need to calculate the couple at the mid-point of the beam to get the equivalent force-couple system. Again, consider a small segment of the beam of length dx located at distance x from the mid-point C (see Fig. 8.12). The moment about point C due to the load on dx is $(q\,dx)x(-\hat{k})$. But, we can find a similar segment on the other side of C with exactly the same length dx, at exactly the same distance x, that produces a moment of $(q\,dx)x(+\hat{k})$. The two contributions cancel each other and we have a net zero moment about C. Now, you can imagine the whole beam made up of these pairs that contribute equal and opposite moment about C and thus the net moment about the mid-point is zero. You can also find the same result by straight integration:

$$\vec{M}_{\mathrm{C}} = \int_{-\ell/2}^{+\ell/2} qx\,dx(-\hat{k}) = \left.\frac{qx^2}{2}\right|_{-\ell/2}^{+\ell/2}(-\hat{k}) = \vec{0}.$$

$$\boxed{\vec{F}_{\text{net}} = -200\,\mathrm{N}\,\hat{\jmath}, \text{ and } \vec{M}_{\mathrm{C}} = \vec{0}}$$

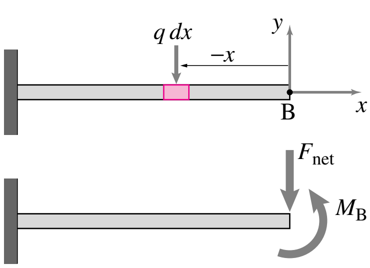

Figure 8.13

3. **The equivalent system at the end:** The net force remains the same as above. We compute the net moment about the end point B, referring to Fig. 8.13, as follows.

$$\begin{aligned}\vec{M}_{\mathrm{B}} &= \int_0^{\ell}(-x\hat{\imath})\times(-q\,dx\,\hat{\jmath}) = -q\int_0^{\ell} x\,dx\hat{k}\\ &= -\frac{q\ell^2}{2}\hat{k} = -\frac{100\,\mathrm{N/m}\cdot 4\,\mathrm{m}^2}{2}\hat{k} = -200\,\mathrm{N{\cdot}m}\hat{k}.\end{aligned}$$

$$\boxed{\vec{F}_{\text{net}} = -200\,\mathrm{N}\,\hat{\jmath} \text{ and } \vec{M}_{\mathrm{B}} = -200\,\mathrm{N{\cdot}m}\hat{k}}$$

SAMPLE 8.4 For the uniformly loaded, simply supported beam shown in the figure, find the shear force and the bending moment at the mid-section c-c of the beam.

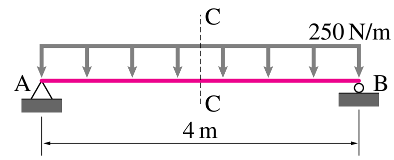

Figure 8.14

Solution To determine the shear force V and the bending moment M at the mid-section c-c, we cut the beam at c-c and draw its free-body diagram as shown in fig. 8.15. For writing force- and moment-balance equations we use the second figure where we have replaced the distributed load with an equivalent single load $F = (q\ell)/2$ acting vertically downward at distance $\ell/4$ from end A.

The force balance, $\sum \vec{F} = \vec{0}$, implies that

$$A_x\hat{\imath} + A_y\hat{\jmath} - V\hat{\jmath} - F\hat{\jmath} = \vec{0}.$$

Dotting with $\hat{\imath}$ and $\hat{\jmath}$, respectively, we get

$$\begin{aligned} A_x &= 0 \\ V &= A_y - F && (8.1) \\ &= A_y - \frac{q\ell}{2}. && (8.2) \end{aligned}$$

From the moment equilibrium about point A, $\sum \vec{M}_{\mathrm{A}} = \vec{0}$, we get

$$M\hat{k} - \left(\frac{q\ell}{2} \cdot \frac{\ell}{4}\right)\hat{k} - V\frac{\ell}{2}\hat{k} = 0$$

$$\Rightarrow \quad M = \frac{q\ell^2 + 4V\ell}{8}. \qquad (8.3)$$

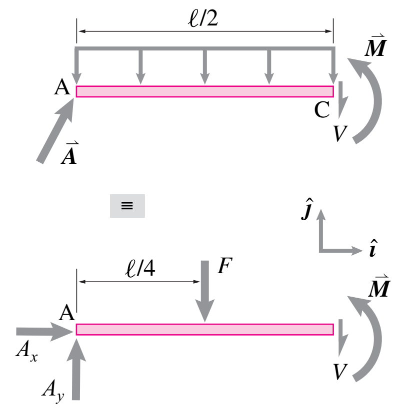

Figure 8.15

Thus, to find V and M we need to know the support reaction $\vec{A}$. From the free-body diagram of the beam in fig. 8.16 and the moment equilibrium equation about point B, $\sum \vec{M}_{\mathrm{B}} = \vec{0}$, we get

$$\begin{aligned} \vec{r}_{\mathrm{A/B}} \times \vec{A} + \vec{r}_{\mathrm{C/B}} \times \vec{Q} &= \vec{0} \\ (-A_y\ell + q\ell\,\frac{\ell}{2})\hat{k} &= \vec{0} \\ \Rightarrow \quad A_y = \frac{q\ell}{2} &= 500\,\mathrm{N}. \end{aligned}$$

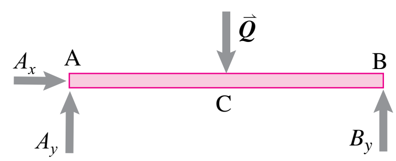

Figure 8.16

Thus $\vec{A} = 500\,\mathrm{N}\,\hat{\jmath}$. Substituting $\vec{A}$ in eqns. (8.2) and (8.3), we get

$$\begin{aligned} V &= 500\,\mathrm{N} - 500\,\mathrm{N} = 0 \\ M &= \frac{(250\,\mathrm{N}\cdot 4\,\mathrm{m})^2 + 0}{8} \\ &= 500\,\mathrm{N{\cdot}m}. \end{aligned}$$

$V = 0, \quad M = 500\,\mathrm{N{\cdot}m}$

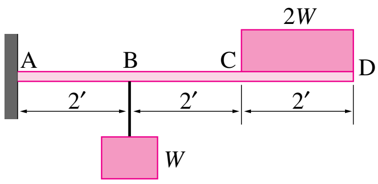

Figure 8.17

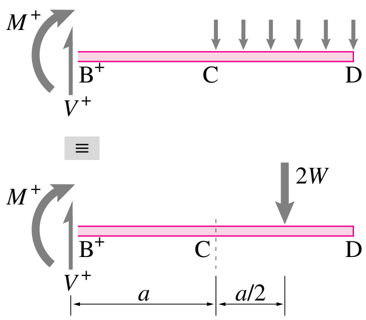

Figure 8.18

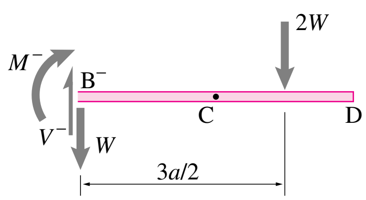

Figure 8.19

SAMPLE 8.5 The cantilever beam AD is loaded as shown in the figure where $W = 200\,\text{lbf}$. Find the shear force and bending moment on a section just left of point B and another section just right of point B.

Solution To find the desired internal forces, we need to make a cut at a section just to the left of B and one just to the right of B. We first take the one that is to the right of point B. The free-body diagram of the right part of the cut beam is shown in fig. 8.18. Note that if we selected the left part of the beam, we would need to determine support reactions at A. The uniformly distributed load $2W$ of the block sitting on the beam can be replaced by an equivalent concentrated load $2W$ acting at point E, at distance $a/2$ from the end D of the beam.

Let us denote the shear force by V^+ and the bending moment by M^+ at the section of our interest. Now, from the force equilibrium of the part-beam BD we get

$$\begin{aligned} V^+\hat{\boldsymbol{\jmath}} - 2W\hat{\boldsymbol{\jmath}} &= \vec{\mathbf{0}} \\ \Rightarrow \quad V^+ &= 2W \\ &= 400\,\text{lbf}. \end{aligned}$$

The moment equilibrium about point B, $\sum \vec{\boldsymbol{M}}_B = \vec{\mathbf{0}}$, gives

$$\begin{aligned} -M^+\hat{\boldsymbol{k}} - 2W\cdot\frac{3a}{2}\hat{\boldsymbol{k}} &= \vec{\mathbf{0}} \\ \Rightarrow \quad M^+ &= -3Wa \\ &= -1200\,\text{lb}\cdot\text{ft}. \end{aligned}$$

Now, we determine the internal forces at a section just to the left of point B. Let the shear and bending moment at this section be V^- and M^-, respectively, as shown in the free-body diagram (fig. 8.19). Note that load W acting at B is now included in the free-body diagram since the beam is now cut just a teeny bit left of this load.

From the force equilibrium of the part-beam, we have

$$\begin{aligned} V^-\hat{\boldsymbol{\jmath}} - W\hat{\boldsymbol{\jmath}} - 2W\hat{\boldsymbol{\jmath}} &= \vec{\mathbf{0}} \\ \Rightarrow \quad V^- &= 3W \\ &= 600\,\text{lbf} \end{aligned}$$

and, from moment equilibrium about point B, $\sum \vec{\boldsymbol{M}}_B = \vec{\mathbf{0}}$, we get

$$\begin{aligned} -M^-\hat{\boldsymbol{k}} - 2W\cdot\frac{3a}{2}\hat{\boldsymbol{k}} &= \vec{\mathbf{0}} \\ \Rightarrow \quad M^- &= -3Wa \\ &= -1200\,\text{lb}\cdot\text{ft}. \end{aligned}$$

$$M^+ = M^- = -1200\,\text{lb}\cdot\text{ft}, \quad V^+ = 400\,\text{lbf}, \quad V^- = 600\,\text{lbf}$$

Note that the bending moment remains the same on either side of point B but the shear force jumps by $V^+ - V^- = 200\,\text{lbf} = W$ as we go from right to the left. This jump is expected because a concentrated load W acts at B, in between the two sections we consider. Concentrated external forces cause a jump in shear, and concentrated external moments cause a jump in the bending moment.

SAMPLE 8.6 Tension in a bar: A T-shaped bar is fixed in a wall at one end and is acted on by three forces as shown in the figure. Find the tension in the rod at

1. section a-a, and
2. section b-b.

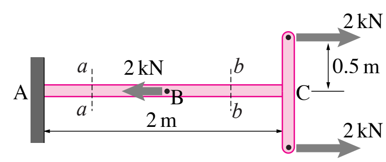

Figure 8.20

Solution

1. Let us cut the bar at section a-a and consider the part of the bar to the right of the cut-section. The free-body diagram of this part of the bar is shown in fig. 8.21. The scalar force balance in the horizontal direction gives

$$\begin{aligned} -T - F + 2F &= 0 \\ \Rightarrow \quad T &= F \\ &= 2\,\text{kN}. \end{aligned}$$

At section a-a:$T = 2\,\text{kN}$

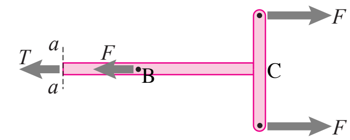

Figure 8.21

2. Now, we cut the bar at section b-b and again consider the section of the bar to the right of the cut-section. The free-body diagram of this part of the bar is shown in fig. 8.22. Again, the force balance in the horizontal direction gives

$$\begin{aligned} -T + 2F &= 0 \\ \Rightarrow \quad T &= 2F \\ &= 4\,\text{kN}. \end{aligned}$$

At section b-b:$T = 4\,\text{kN}$

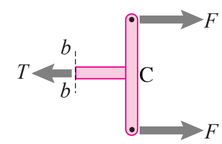

Figure 8.22

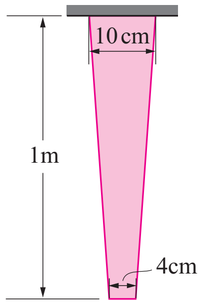

Figure 8.23

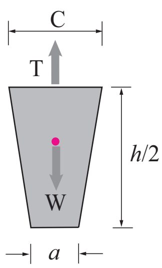

Figure 8.24

SAMPLE 8.7 Tension in a tapered bar due to self weight: A tapered bar of height 1 m, top width 10 cm, bottom width 4 cm and uniform thickness 4 cm hangs upside down from a ceiling. If the density of the material is $7500\,\mathrm{kg/m^3}$, find the tension in the rod halfway from the top. Use $g \approx 10\,\mathrm{m/s^2}$.

Solution Let us cut the bar at a section halfway from the top. The free-body diagram of the bar below the cut is shown in fig. 8.24. From the scalar force balance in the vertical direction, we have

$$T = W$$

where W is the weight of the lower part of bar below the cut section. Now, $W = \rho A t g$ where A is the frontal area, t is the thickness, and $\rho = 7500\,\mathrm{kg/m^3}$ is the density of the rod material. We need to compute W.

The width of the bar at the cut section is $c = (a+b)/2$ where $a = 4\,\mathrm{cm}$ and $b = 10\,\mathrm{cm}$. The frontal area of the bar-part is $A = (a+c)/2 \cdot (h/2)$ where $h = 1\,\mathrm{m}$. Thus,

$$\begin{aligned} W &= \rho\left(\frac{a+c}{2}\cdot\frac{h}{2}\right)tg \\ &= 7500\,\mathrm{kg/m^3}\left(\frac{0.04\,\mathrm{m}+0.07\,\mathrm{m}}{2}\cdot\frac{1\,\mathrm{m}}{2}\cdot(0.04\,\mathrm{m})\right)10\,\mathrm{m/s^2} \\ &= 82.5\,\mathrm{N}. \end{aligned}$$

Thus, $T = 82.5\,\mathrm{N}$.

$T = 82.5\,\mathrm{N}$

SAMPLE 8.8 **A simple frame:** A 2 m high and 1.5 m wide rectangular frame ABCD is loaded with a 1.5 kN horizontal force at B and a 2 kN vertical force at C. Find the internal forces and moments at the mid-section e-e of the vertical leg AB.

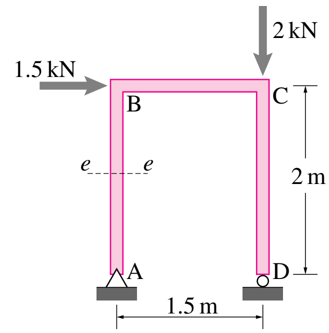

Figure 8.25

Solution To find the internal forces and moments, we need to cut the frame at the specified section e-e and consider the free-body diagram of either AE or EBCD. No matter which of the two we select, we will need the support reactions at A or D to determine the internal forces. Therefore, let us first find the support reactions at A and D by considering the free-body diagram of the whole frame (fig. 8.26). The moment balance about point A, $\sum \vec{M}_A = \vec{0}$, gives

$$
\begin{aligned}
\vec{r}_B \times \vec{F}_1 + \vec{r}_C \times \vec{F}_2 + \vec{r}_D \times \vec{D} &= \vec{0} \\
h\hat{\jmath} \times F_1\hat{\imath} + (h\hat{\jmath} + \ell\hat{\imath}) \times (-F_2\hat{\jmath}) + \ell\hat{\imath} \times D\hat{\jmath} &= \vec{0} \\
-F_1 h\hat{k} - F_2\ell\hat{k} + D\ell\hat{k} &= \vec{0} \\
\Rightarrow \quad D &= F_1\frac{h}{\ell} + F_2 \\
&= 1.5\,\text{kN} \cdot \frac{2}{1.5} + 2\,\text{kN} \\
&= 4\,\text{kN}.
\end{aligned}
$$

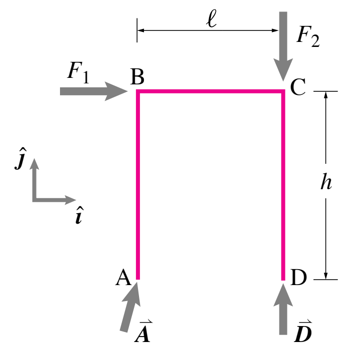

Figure 8.26

From force equilibrium, $\sum \vec{F} = \vec{0}$, we have

$$
\begin{aligned}
\vec{A} &= -\vec{F}_1 - \vec{F}_2 - \vec{D} \\
&= -F_1\hat{\imath} + F_2\hat{\jmath} - D\hat{\jmath} \\
&= -1.5\,\text{kN}\hat{\imath} - 2\,\text{kN}\hat{\jmath}.
\end{aligned}
$$

Now we draw the free-body diagram of AE to find the shear force V, axial (tensile) force T, and the bending moment M at section e-e.

From the force equilibrium of part AE, we get

$$
\begin{aligned}
\vec{A} - V\hat{\imath} + T\hat{\jmath} &= \vec{0} \\
(A_x - V)\hat{\imath} + (A_y + T)\hat{\jmath} &= \vec{0} \\
\Rightarrow \quad V = A_x &= -1.5\,\text{kN} \\
T = -A_y &= 2\,\text{kN}.
\end{aligned}
$$

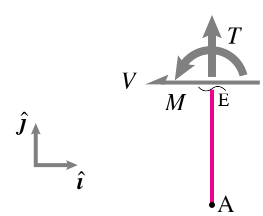

Figure 8.27

From the moment equilibrium about point A, $\sum \vec{M}_A = \vec{0}$, we have

$$
\begin{aligned}
M\hat{k} + \frac{h}{2}\hat{\jmath} \times (-V\hat{\imath}) &= \vec{0} \\
M\hat{k} + V\frac{h}{2}\hat{k} &= \vec{0} \\
\Rightarrow \quad M &= -V\frac{h}{2} \\
&= -(-1.5\,\text{kN}) \cdot \frac{2\,\text{m}}{2} \\
&= 1.5\,\text{k N·m}.
\end{aligned}
$$

$$V = 1.5\,\text{kN}, \quad T = 2\,\text{kN}, \quad M = 1.5\,\text{k N·m}$$

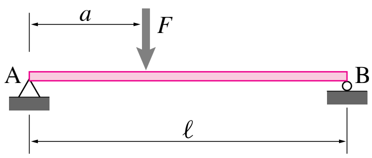

Figure 8.28

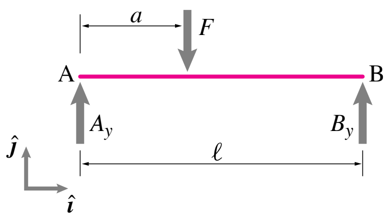

Figure 8.29

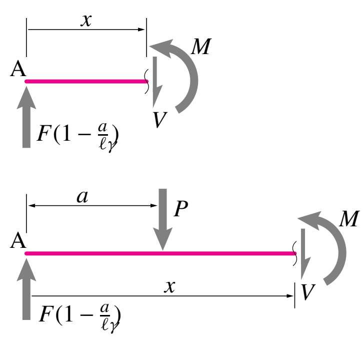

Figure 8.30

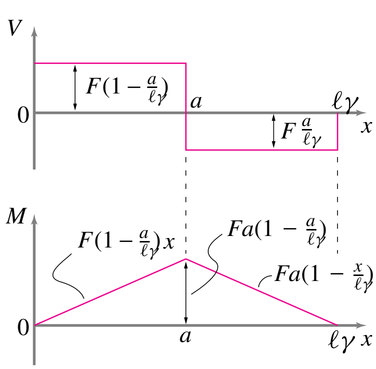

Figure 8.31

SAMPLE 8.9 Shear force and bending moment diagrams: A simply supported beam of length $\ell = 2\,\mathrm{m}$ carries a concentrated vertical load $F = 100\,\mathrm{N}$ at a distance a from its left end. Find and plot the shear force and the bending moment along the length of the beam for $a = \ell/4$.

Solution We first find the support reactions by considering the free-body diagram of the whole beam shown in fig. 8.29. By now, we have developed enough intuition to know that the reaction at A will have no horizontal component since there is no external force in the horizontal direction. Therefore, we take the reactions at A and B to be only vertical. Now, from the moment equilibrium about point B, $\sum \vec{\boldsymbol{M}}_{\mathrm{B}} = \vec{\mathbf{0}}$, we get

$$\begin{aligned} F(\ell-a)\hat{\boldsymbol{k}} - A_y\ell\hat{\boldsymbol{k}} &= \vec{\mathbf{0}} \\ \Rightarrow \quad A_y &= \frac{F(\ell-a)}{\ell} \\ &= F\left(1-\frac{a}{\ell}\right) \end{aligned}$$

and from the force equilibrium in the vertical direction, $(\sum \vec{\boldsymbol{F}} = \vec{\mathbf{0}}) \cdot \hat{\boldsymbol{\jmath}}$, we get

$$B_y = F - A_y = F\frac{a}{\ell}.$$

Now we make a cut at an arbitrary (variable) distance x from A where $x < a$ (see fig. 8.30). Carrying out the force balance and the moment balance about point A, we get, for $0 \le x < a$,

$$V = A_y = F\left(1-\frac{a}{\ell}\right) \tag{8.4}$$

$$M = Vx = F\left(1-\frac{a}{\ell}\right)x \tag{8.5}$$

Thus V is constant for all $x < a$ but M varies linearly with x.

Now we make a cut at an arbitrary x to the right of load F, *i.e.*, $a < x \le \ell$. Again, from the force balance in the vertical direction, we get

$$V = -F + F\left(1-\frac{a}{\ell}\right) = -F\frac{a}{\ell} \tag{8.6}$$

and from the moment balance about point A,

$$\begin{aligned} M &= Fa + Vx \\ &= Fa - F\frac{a}{\ell}x \\ &= Fa\left(1-\frac{x}{\ell}\right). \end{aligned} \tag{8.7}$$

Although eqn. (8.5) is strictly valid for $x < a$ and eqn. (8.7) is strictly valid for $x > a$, substituting $x = a$ in these two equations gives the same value for $M(= Fa(1-a/\ell))$ as it must because there is no reason to have a jump in the bending moment at any point along the length of the beam. The shear force V, however, does jump because of the concentrated load F at $x = a$.

Now, we plug in $a = \ell/4 = 0.5\,\mathrm{m}$, and $F = 100\,\mathrm{N}$, in eqns. (8.4)–(8.7) and plot V and M along the length of the beam by varying x. The plots of $V(x)$ and $M(x)$ are shown in fig. 8.31.

SAMPLE 8.10 Shear force and bending moment diagrams by superposition: For the cantilever beam and the loading shown in the figure, draw the shear force and the bending moment diagrams by

1. considering all the loads together, and
2. considering each load (of one type) at a time and using superposition.

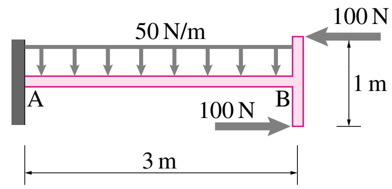

Figure 8.32

Solution

1. *$V(x)$ and $M(x)$ with all forces considered together:* The horizontal forces acting at the end of the cantilever are equal and opposite and, therefore, produce a couple. So, we first replace these forces by an equivalent couple $M_{\text{applied}} = 100\,\text{N} \cdot 1\,\text{m} = 100\,\text{N·m}$. Since we have a cantilever beam, we can consider the right hand side of the beam after making a cut anywhere for finding V and M without first finding the support reactions.

   Let us cut the beam at an arbitrary distance x from the right hand side. The free-body diagram of the right segment of the beam is shown in fig. 8.33. From the force balance, $\sum \vec{F} = \vec{0}$, we find that

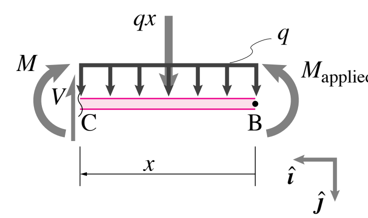

Figure 8.33

$$
\begin{aligned}
-V\hat{\jmath} + qx\hat{\jmath} &= \vec{0} \\
\Rightarrow \quad V &= qx \qquad (8.8)\\
&= (50\,\text{N/m})x.
\end{aligned}
$$

Thus the shear force varies linearly along the length of the beam with

$$
\begin{aligned}
V(x=0) &= 0,\\
\text{and} \qquad V(x=3\,\text{m}) &= 150\,\text{N}.
\end{aligned}
$$

The moment balance about point C, $\sum \vec{M}_C = \vec{0}$, gives

$$-M\hat{k} - qx \cdot \frac{x}{2}\hat{k} + M_{\text{applied}}\hat{k} = \vec{0}$$

where the moment due to the distributed load is most easily computed by considering an equivalent concentrated load qx acting at $x/2$ from the end B. Thus,

$$
\begin{aligned}
\Rightarrow \quad M &= M_{\text{applied}} - q\frac{x^2}{2} \qquad (8.9)\\
&= 100\,\text{N·m} - 50\,\text{N/m} \cdot \frac{x^2}{2}. \qquad (8.10)
\end{aligned}
$$

Thus, the bending moment varies quadratically with x along the length of the beam. In particular, the values at the ends are

$$
\begin{aligned}
M(x=0) &= 100\,\text{N·m}\\
\text{and} \qquad M(x=3\,\text{m}) &= -125\,\text{N·m}.
\end{aligned}
$$

The shear force and the bending moment diagrams obtained from eqns. (8.8) and (8.9) are shown in fig. 8.34. Note that $M = 0$ at $x = 2\,\text{m}$ as given by eqn. (8.9).

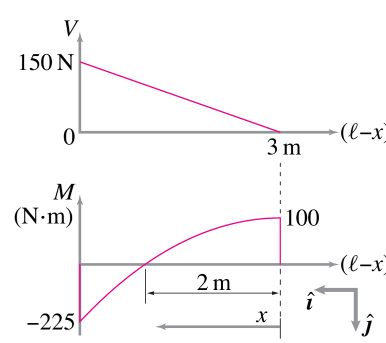

Figure 8.34

2. *$V(x)$ and $M(x)$ by superposition:* Now we consider the cantilever beam with only one type of load at a time. That is, we first consider the beam only with the uniformly distributed load and then only with the end couple. We draw the shear force and the bending moment diagrams for each case separately and then just *add them up*. That is superposition.

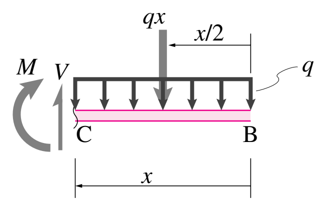

Figure 8.35

So, first let us consider the beam with the uniformly distributed load. The free-body diagram of a segment CB, obtained by cutting the beam at a distance x from the end B, is shown in fig. 8.35. Once again, from force balance, we get

$$V = qx \qquad \text{for } 0 \le x \le \ell \tag{8.11}$$

and from the moment balance about point C, $\sum \vec{\boldsymbol{M}}_{\mathrm{C}} = \vec{\mathbf{0}}$, we get

$$M = -qx \cdot \frac{x}{2} = -q\frac{x^2}{2} \qquad \text{for } 0 \le x \le \ell. \tag{8.12}$$

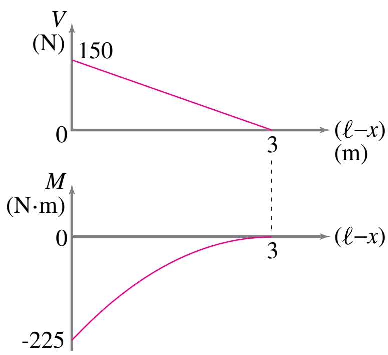

Figure 8.36

Figure 8.36 shows the plots of V and M obtained from eqns. (8.11) and (8.12), respectively, with the values computed from $x = 0$ to $x = 3\,\mathrm{m}$ with $q = 50\,\mathrm{N/m}$ as given.

Now we take the beam with only the end couple and repeat our analysis. A cut section of the beam is shown in fig. 8.37. In this case, it should be obvious that from force balance and moment balance about any point, we get

$$\begin{aligned} V &= 0 \\ \text{and} \qquad M &= M_{\text{applied}}. \end{aligned}$$

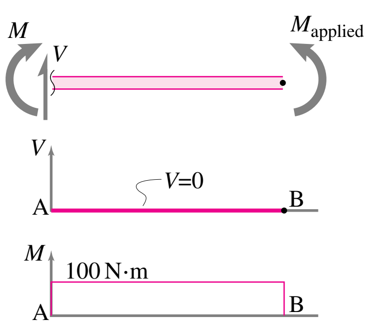

Figure 8.37

Thus, both the shear force and the bending moment are constant along the length of the beam as shown in fig. 8.37.

Now superimposing (adding) the shear force diagrams from Figs. 8.36 and 8.37, and similarly, the bending moment diagrams from Figs. 8.36 and 8.37, we get the same diagrams as in fig. 8.38.

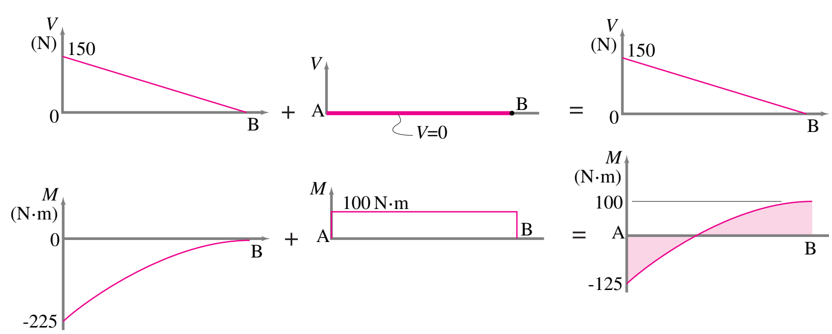

Figure 8.38: