7.3 Mechanisms

We would now like to analyze ways to connect pieces so that a force or moment is amplified, attenuated or redirected.

For completeness, we present the statics recipe for machines, although it is an exact repeat of the recipe used for frames.

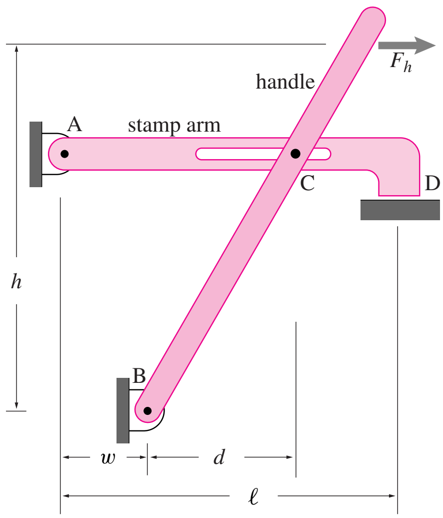
- Draw free-body diagrams of
  - the whole machine; and
  - the separate parts of the machine; and
  - collections of parts of the machine, if such seem likely to be fruitful;
  - Use the principle of action and reaction in the free-body diagrams so that there is only one unknown force at a point where two bodies contact;
- for each free-body diagram write equilibrium conditions. These should yield three independent scalar equations for each non-point part (in 2D)
- solve some or all of the equilibrium equations for the desired unknowns

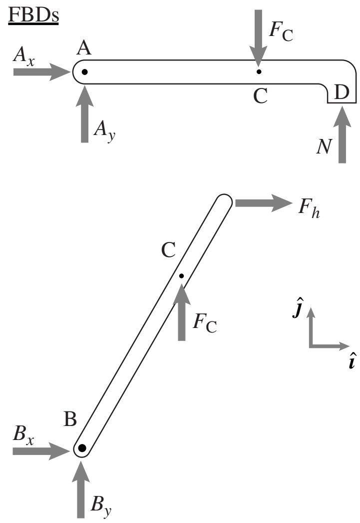
Some useful tricks and shortcuts include:

- for any two-force bodies, assign an equal-valued tension to each end (thus eliminating any need or use for equilibrium equations for that object)
- To minimize calculation, look for a subset of the equilibrium equations that
  - contains your unknowns of interest, and
  - has as many unknowns as scalar equations, and
  - contains as few equations as possible.

Example: Stamp machine

Pulling on the handle (below) causes the stamp arm to press down with a force N at D. We can find N in terms of F_h by drawing free-body diagrams of the handle and stamp arm, writing three equilibrium equations for each piece and then solving these 6 equations for the 6 unknowns (A_x, A_y, F_C, N, B_x, and B_y).





For this problem, the answer can be found more quickly with a judicious choice of equilibrium equations.

$$\begin{aligned}
&\text{For the handle,} \left\{\sum \vec{M}_{/\mathrm{B}} = \vec{0}\right\} \cdot \hat{k} &&\Rightarrow && -hF_h + dF_c &&= 0\\
&\text{For the stamp arm,} \left\{\sum \vec{M}_{/\mathrm{A}} = \vec{0}\right\} \cdot \hat{k} &&\Rightarrow && -(d+w)F_c + \ell N &&= 0.
\end{aligned}$$

$$\text{Eliminating } F_c \quad \Rightarrow \quad N = \frac{h(d+w)}{d\ell} F_h.$$

Note that the stamp force N can be made very large by making d small and thus the handle nearly vertical. Often in structural or machine design, one or another force gets extremely large or small as the design is changed to put pieces in near alignment.

Example: Improved stamp machine
Figure 7.53 shows a stamp machine with all the same components. The method of analysis is identical. However, the design represents an improvement in two ways:

- The lever in the stamp arm amplifies rather than attenuates the stamp force.
- In the previous design, it is harder and harder to generate a given stamp force as the stamped object compresses. In this design, the toggle mechanism associated with the lever arm and sliding pin is in compression. Thus, as the stamping progresses and the handle becomes more vertical, for a constant hand-force, the stamping force increases as the motion progresses.

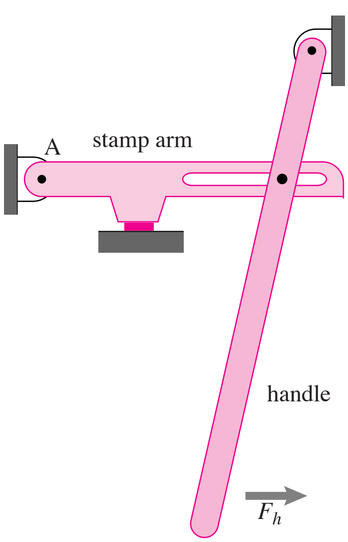


Figure 7.53: An improved stamp machine.

Non-rigid structures are mechanisms

A non-rigid structure cannot carry all loads and, if not also redundant, has more equilibrium equations than unknown reaction or interaction force components. Such a structure is also called a mechanism. The stamp machine above is a mechanism if we assume there is no contact at D. In particular, the equilibrium equations cannot be satisfied unless $F_h = 0$. Mechanisms have variable configurations. That is, the constraints still allow relative motion.

An attempt to design a rigid structure that turns out to be a mechanism is a design failure. But for machine design, the mechanism aspect of a structure is essential. Even though mechanisms are called 'statically indeterminate' because they cannot carry all possible loads, the desired forces can often be determined using statics. For the stamp machine above, the equilibrium equations are made solvable by treating one of the applied forces, say N, as an unknown, and the other, F_h in this case, as a known. This is a common situation in machine design where you want to determine the loads at one part of a mechanism in terms of loads at another part. For the purposes of analysis, a trick is to make a mechanism determinate by putting a pin on the roller's connection to the ground at the location of any forces with unknown magnitudes but known directions.

Example: Stamp machine with roller
Putting a roller at D, the location of the unknown stamp force, turns the stamp machine into a determinate structure.

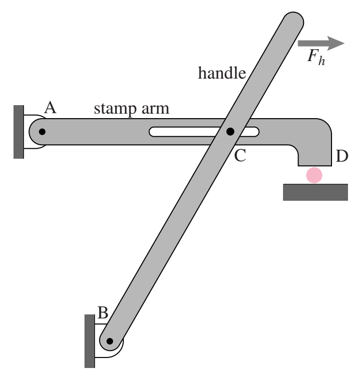


Figure 7.54: Stamp machine with roller.

Pulley and chain drives

Chain and pulley drives may be thought of as spread-out gears (fig. 7.55). The rotation of two shafts is coupled not by the contact of gear teeth but by a belt around a pulley or a chain around a sprocket. For a simple analysis, one draws free-body diagrams for each sprocket or pulley with a little bit of chain as in fig. 7.55b. Note that $T_1 \neq T_2$, unlike the case of an ideal undriven pulley. Applying moment balance we find,

$$\begin{aligned}
&\text{For gear A,} \left\{\sum \vec{\boldsymbol{M}}_{\mathrm{i/A}} = \vec{\mathbf{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad -R_{\mathrm{A}}(T_2 - T_1) + M_{\mathrm{A}} &= 0\\
&\text{For gear B,} \left\{\sum \vec{\boldsymbol{M}}_{\mathrm{i/B}} = \vec{\mathbf{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad R_{\mathrm{B}}(T_1 - T_2) - M_{\mathrm{B}} &= 0.
\end{aligned}$$

$$\text{Eliminating } (T_2 - T_1) \quad \Rightarrow \quad M_B = \frac{R_B}{R_A} M_A \quad \text{or} \quad M_A = \frac{R_A}{R_B} M_B,$$

exactly as for a pair of gears. Note that we cannot find T_2 or T_1 but only their difference. Typically in design if, say, M_A is positive, one would try to keep T_1 as small as possible without the belt slipping or the chain jumping teeth. If T_1 grows, then so must T_2, to preserve their difference. This increase in tension increases the loads on the bearings as well as on the chain or belt itself.

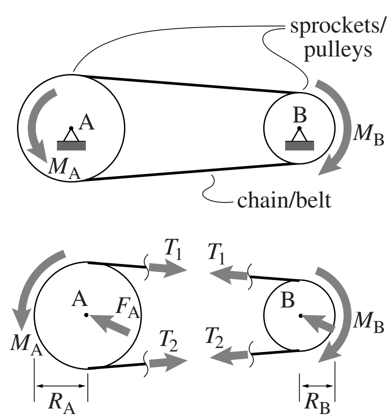


Figure 7.55: a) A chain or pulley drive involving two sprockets or pulleys and one chain or belt, b) free-body diagrams of each of the sprockets/pulleys.

Four-bar linkages

Four-bar linkages often, confusingly, have 3 bars, the fourth piece is something bigger that the linkage is attached to. A planar mechanism with four pieces connected in a loop by hinges is a four-bar linkage. Four-bar linkages are remarkably common. After a single object connected at a hinge (like a gear or lever), a four-bar linkage is one of the simplest

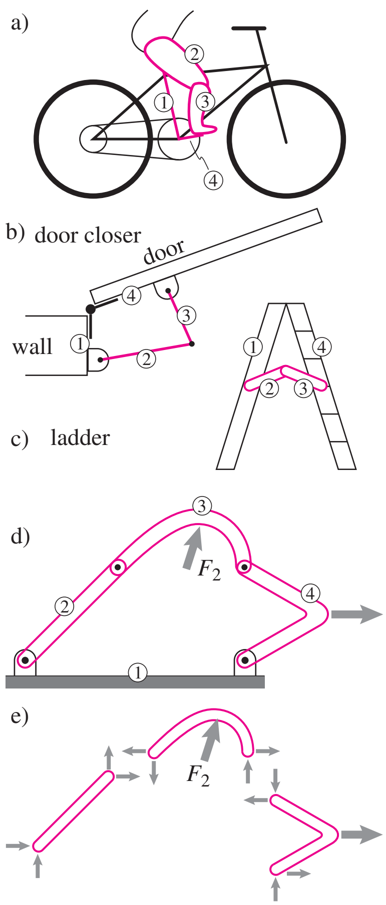


Figure 7.56: Four bar linkages. a) A bicycle, thigh, calf, and crank, b) a door closer, c) a folding ladder, d) a generic mechanism, e) free-body diagrams of the parts of a generic mechanism.

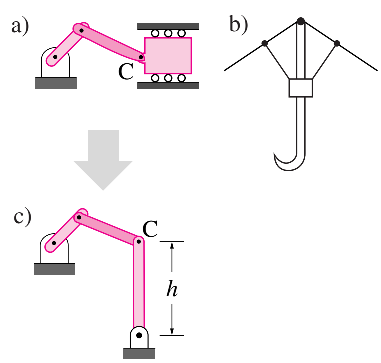


Figure 7.57: a) a slider crank, b) an umbrella has a slider-crank mechanism, c) the equivalent four-bar linkage, at least when $h \to \infty$.

mechanisms that can move in just one way (*i.e.,* have just one degree of freedom).

A reasonable model of a person seated on a bicycle uses a 4-bar linkage (fig. 7.56a). The whole bicycle frame is one bar, the human thigh is the second, the calf is the third, and the bicycle crank is the fourth. The four hinges are the hip joint, the knee joint, the pedal axle, and the bearing at the bicycle crank axle. A more sophisticated model of the system would include the ankle joint and the foot would make up a fifth bar.

A standard door-closing mechanism is part of a 4-bar linkage (fig. 7.56b). The door jamb and door are two bars and the mechanism pieces make up the other two.

A standard folding ladder design is, until locked open, a 4-bar linkage (fig. 7.56c).

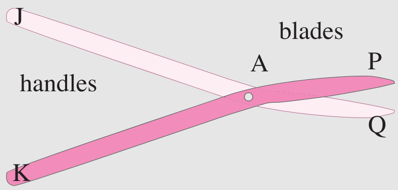
An abstracted 4-bar linkage with two loads is shown in fig. 7.56d with free-body diagrams in fig. 7.56e. If one of the applied loads is given, then the other applied load along with interaction and reaction forces compose nine unknown components (after using the principle of action and reaction). With three equilibrium equations for each of the three bars, all these unknowns can be found.

Slider crank

A mechanism closely related to a four-bar linkage is a slider crank (fig. 7.57a). An umbrella is one example (rotated 90° in fig. 7.57b). If the sliding part is replaced by a bar, as in fig. 7.57c, the point C moves in a circle instead of a straight line. If the height h is very large, then the arc traversed by C is nearly a straight line so the motion of the four-bar linkage is almost the same as the slider crank. For this reason, slider cranks are sometimes regarded as a special case of a four-bar linkage in the limit as one of the bars gets infinitely long.

7.6 Shears with gears

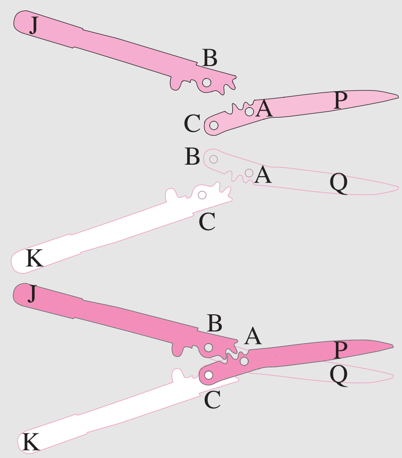
Many cutters, pliers and shears are essentially two levers pivoting against each other. For example these shears



consist of two levers, JAQ and KAP, pivoted at A. The hands squeeze the handles at J and K, causing a cutting force on an object between the blades at P and Q. The force at P, say, is $|KA|/|AP|$ times the force at K (from moment balance about A using a free-body diagram of KAP). Two possible deficiencies of this bi-lever design are that

- One may want more mechanical advantage but not longer handles, and
- For a given hand strength (available force at J and K) the force at the cutting edge gets less and less as the location of the cut force at P and Q moves farther out on the blade, away from A.

The Fiskars company, known mostly for scissors using the basic design above, has some designs that address these deficiencies. The loppers in problem 7.3.9 use a 3-piece mechanism to address these issues. Here, even more elaborately, are Fiskars shears using 4 moving parts.



The two identical blades AP and AQ are hinged at A. The two identical handles JB and KC are hinged to the blades at B and C. Each handle also has gear teeth at the end that engage gear teeth on the opposite blade. Let's take P and Q to be the point of contact of the object being cut.

Let's try to understand the mechanism without detailed analysis (see homework problem 7.3.10). To start, forget handle KC and assume that blade BAQ is held firmly by something outside.

(continued...)

7.6 Shears with gears (continued)

Blade CAP is attached at A about which it is free to spin. Handle JB is attached at B about which it is free to spin. But JB and CAP roll against each other with engaged gear teeth. So if handle JB rotates counter-clockwise about B, then CAP rotates clockwise about A.

Although the gear teeth are complex-looking, there is always an *effective* contact point G between handle JB and blade CAP on the line segment AB. G is effectively a hinge between JB and CAP. You can think of the handle as a lever with force points at J, B and G. Thus blade CAP is closed by the force on the gear teeth at G. The shorter BG, the bigger the forces at B and G.

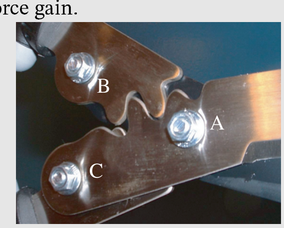
Simultaneously you could think of blade CAP as fixed with blade BAQ and handle KC hinged to it and geared to each other at G' (not shown). Thus blade CAP is also closed by an upwards force at C from handle KC. Similarly blade BAQ is closed by a downwards force at B from handle JB and a downwards force at G' from handle KC.

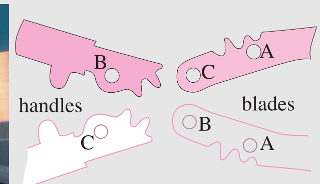
The effective hinges G and G' have locations which change as the blades close. When the blades are wide open, G and G' are near A. When the blades are closed, G and G' have moved to about the midpoint between B and A and C and A, respectively.

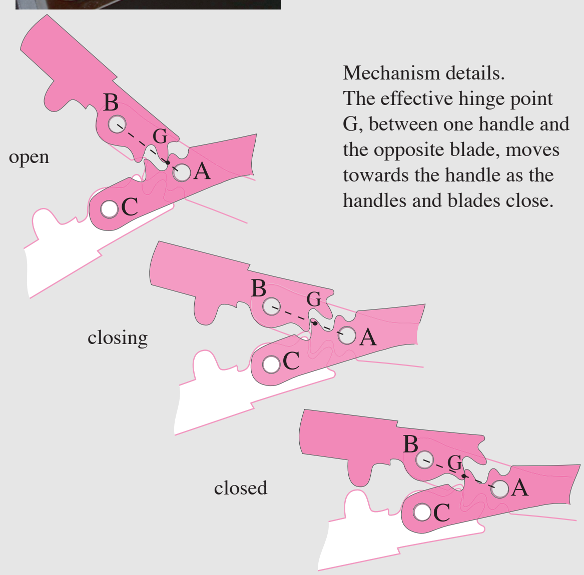
If G was *at* A, then this 4-piece design would be equivalent to a standard 2-piece cutter. Because BG is shorter than BA, this design gives a bigger downwards force at B.

The shape of the geared curves makes the distance BG decrease, and the distance AG increase, as the blades close. Thus for given forces acting at J and Q, as the blades close the force at B increases, the force at G increases, and the lever-arm AG increases. These three effects partially compensate for the standard scissors problem, the decreasing mechanical advantage from the distance AP increasing as the blades close.

Another way to see the mechanical advantage of this design compared to the 2-piece design is to see that during a cut the handle angle decrease is greater than the blade angle decrease. Following the general rule for mechanisms, a motion attenuation is a force gain.







Mechanism details.
The effective hinge point G, between one handle and the opposite blade, moves towards the handle as the handles and blades close.