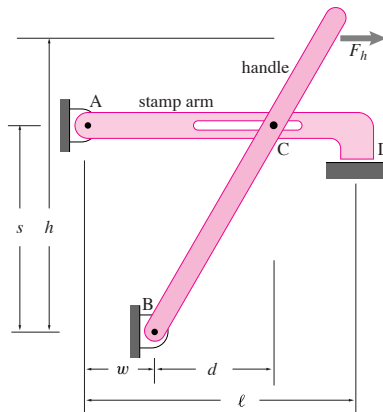


**7.3.2** Pulling on the handle causes the stamp arm to press down at D. Neglect gravity and assume that the hinges at A and B, as well as the roller at C, are frictionless. Find the force  $N$  that the stamp machine causes on the support at D in terms of some or all of  $F_h$ ,  $w$ ,  $d$ ,  $\ell$ ,  $h$ , and  $s$ .



Problem 7.2

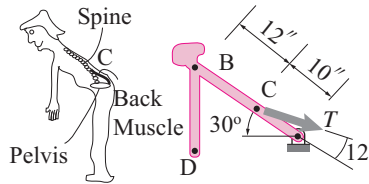
**Answer:**

$$N = (h(w + d)/d\ell) F_h$$

7.3.4 See Problem ?? on page ?. A person with weight  $W = 140 \text{ lbf}$  has an upper body with weight  $0.7W$  with center of mass at C. The back muscles are idealized as a single muscle with one end (the muscle *origin* also at C. Use the idealization and geometry shown.

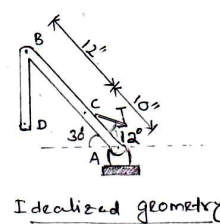
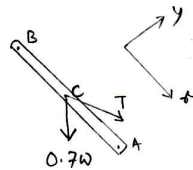
- Find the back-muscle tension and the force of the lower body on the upper body at the hips.
- Repeat the problem but assume

that the person is lifting a 30 lbf load at D.



Problem 7.4

6.3.4



(a)

consider the co-ordinate system with x-axis aligned along the link BC.

Considering moment balance about point A.

$$\Rightarrow \vec{r}_{C/A} \times \vec{T} + \vec{r}_{C/A} \times 0.7\vec{W} = \vec{0} \Rightarrow \vec{T} + 0.7\vec{W} = \vec{0}$$

This will imply that net forces along both axes (x & y) are zero.

considering components along Y-axis

$$T \sin 12^\circ - 0.7W \sin 60^\circ = 0 \Rightarrow (T \times 0.208 - 0.7 \times 140 \times \frac{\sqrt{3}}{2}) \text{ lbf} = 0$$

$$\Rightarrow T = \frac{0.7 \times 140 \times \frac{\sqrt{3}}{2}}{0.208}$$

$$\Rightarrow T = 408.02 \text{ lbf} \quad (\text{Back muscle tension})$$

Force of the lower body on the upper body at the hips will act at point A.

This force will be a reaction to the resultant of forces acting at point C along the link BC.

Net forces at point C, along the link:

$$F_L = T \cos 12^\circ + 0.7W \cos 60^\circ \quad (F_L \text{ is the net force acting along link BC})$$

$$= 399.1 + 49$$

$$= 448.1 \text{ lbf}$$

**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

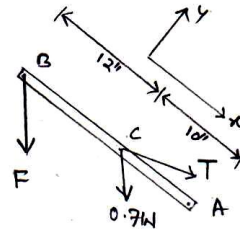
6.3.4 continued

(b) Finding the back muscle tension:

At equilibrium, net moment due to all forces will be zero.

Here we consider moment about point A.

Only the component of the forces along y-axis will contribute for the moment.



Sum of moments about point A:

$$(T \sin 12^\circ - 0.7W \sin 60^\circ) \times 10'' - (F \sin 60^\circ) \times 22'' = 0$$

$$\Rightarrow 2.08T - 6.062W - 19.05F = 0$$

$$\Rightarrow 2.08T - 6.062 \times 140 - 19.05 \times 30 = 0$$

$$\Rightarrow 2.08T = 924.3 + 571.5$$

$$\Rightarrow T = 719.13 \text{ lbf}$$

Finding the force of the lower body on the upper body:

Total force along the link BC (along x-axis)

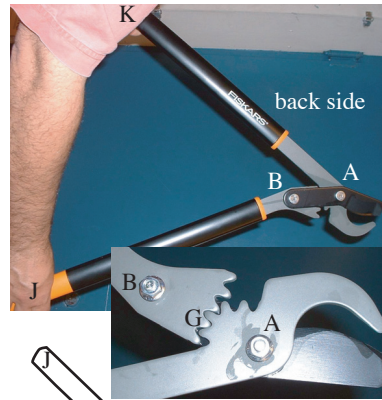
$$= T \cos 12^\circ + 0.7W \cos 60^\circ + F \cos 60^\circ$$

$$= 719.13 \cos 12^\circ + 0.7 \times 140 \times \cos 60^\circ + 30 \cos 60^\circ$$

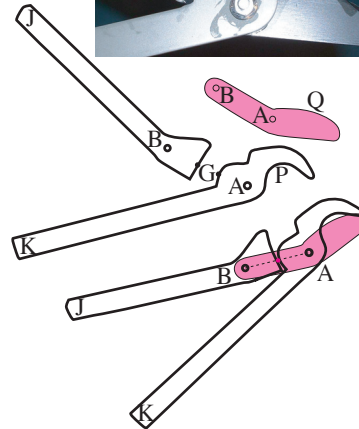
$$= 703.415 + 49 + 15$$

$$= 767.415 \text{ lbf}$$

**7.3.9** Gear teeth on handle JB mesh with teeth on handle-and-blade KAP at point G midway between hinges A and B. Assume that in the configuration of interest J, B and A are co-linear; that K, A and Q are co-linear and that the cutting contact points Q and P are effectively coincident, that angle  $JAK = 20^\circ$ ,  $JB = 40$  cm,  $BA = 6$  cm,  $KA = 46$  cm,  $AP = AQ = 3$  cm, and that the co-linear squeezing forces at J and K are 100 N.



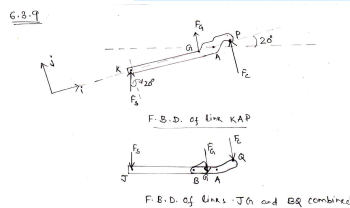
- Find the cutting force at Q and P.
- Replace this design with one that has no tooth engagement at G. But instead handle JB and blade BAQ are welded together as one piece. Assuming the same geometry as before, what then is the cutting force at Q and P?
- Without detailed calculations, explain the ratio of the two answers above.



#### Problem 7.9

##### Answer:

Either by looking at part KAP or at part BAQ, if we think of moment balance about A we see that the cutting force has to fight about twice the torque in the gear mechanism as in the ungeared mechanism. For example KAP is aided in its cutting by the torque from the force at G.



- (a) Consider the F.B.D. of link KAP.
- In the configuration of interest points K, A, P will be on a straight line.
- The forces acting on it are as follows:
- $F_J$ : squeezing force. Let's say its direction is perpendicular to link JB.
  - $F_K$ : Force due to meshing of gears. Direction of this force will depend on the direction of  $F_J$  and the way gear teeth are meshing.
  - $F_G$ : cutting force. Direction of this force will be perpendicular to the contact surface at point of contact. Here we have taken it perpendicular to line KAP.
- Point A is the hinge about which the links are rotating.

**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

6.3.9 continued

At equilibrium moments about point A will be balanced.

$$\sum \vec{M}_A = \vec{0}$$

$$\Rightarrow \vec{M}_{F_S/A} + \vec{M}_{F_G/A} + \vec{M}_{F_C/A} = \vec{0}$$

$$\Rightarrow \vec{r}_{K/A} \times \vec{F}_S + \vec{r}_{G/A} \times \vec{F}_G + \vec{r}_{P/A} \times \vec{F}_C = \vec{0}$$

$$\Rightarrow (KA)(-\hat{i}) \times [F_S \sin 20^\circ \hat{i} + (F_S \cos 20^\circ) \hat{j}] + (GA)(-\hat{i}) \times (F_G) \hat{j} + (AP) \hat{i} \times (F_C) \hat{j} = \vec{0}$$

$$\Rightarrow KA \cdot F_S \cos 20^\circ (-\hat{k}) + GA \cdot F_G (-\hat{k}) + AP \cdot F_C (\hat{k}) = \vec{0}$$

$$\Rightarrow (-46 \text{ cm} \cdot 100 \cos 20^\circ \text{ N}) \hat{k} - (3 \text{ cm} F_G) \hat{k} + (3 \text{ cm} F_C) \hat{k} = \vec{0}$$

$$\Rightarrow 46 \times (100 \cos 20^\circ) + 3 F_G - 3 F_C = 0 \quad \text{--- (1)}$$

(c) From the "F.B.D. of links JG and BQ combined" let us consider the equilibrium of link JG alone.

Moments about point B will be balanced at equilibrium

$$\sum \vec{M}_B = \vec{0}$$

$$\Rightarrow \vec{M}_{F_S/B} + \vec{M}_{F_G/B} = \vec{0}$$

$$\Rightarrow \vec{r}_{J/B} \times \vec{F}_S + \vec{r}_{G/B} \times \vec{F}_G = \vec{0}$$

$$\Rightarrow JB(-\hat{i}') \times F_S(-\hat{j}') + BG(\hat{i}') \times F_G(-\hat{j}') = \vec{0} \quad \text{F.B.D. of JG alone}$$

$$\Rightarrow (JB \cdot F_S) (-\hat{k}') + (BG \cdot F_G) (-\hat{k}') = \vec{0}$$

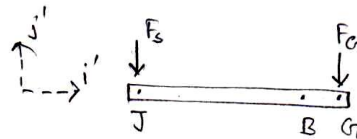
$$\Rightarrow JB \cdot F_S = BG \cdot F_G$$

$$\Rightarrow 40 \text{ cm} \times 100 \text{ N} = 3 \text{ cm} \times F_G \Rightarrow 4000 = 3 F_G \quad \text{--- (2)}$$

Substituting (2) in (1), we get

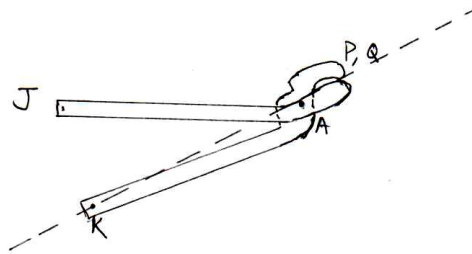
$$46 \times (100 \cos 20^\circ) + 4000 - 3 F_C = 0$$

$$\Rightarrow F_C = 2774.2 \text{ N} \quad \text{[cutting force at Q and P]}$$

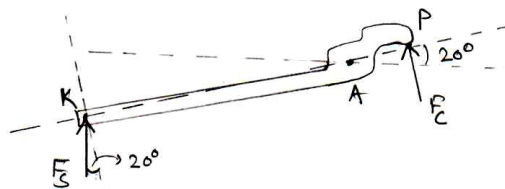


6.3.9 continued

(b) The new geometry is as follows



Points K, A and P are collinear.



F.B.D. of KAP [direction of  $F_S$  and  $F_C$  are same as before]

At equilibrium moments about point A will be balanced,

$$\sum \vec{M}_A = \vec{0}$$

$$\Rightarrow \vec{M}_{F_S/A} + \vec{M}_{F_C/A} = \vec{0}$$

$$\Rightarrow \vec{r}_{K/A} \times \vec{F}_S + \vec{r}_{P/A} \times \vec{F}_C = \vec{0}$$

$$\Rightarrow (KA)(-\hat{i}'') \times [F_S \sin 20^\circ \hat{i}'' + (F_C \cos 20^\circ) \hat{j}''] \\ + (PA) \hat{i}'' \times (F_C \hat{j}'')$$

$$\Rightarrow (KA \cdot F_S \cos 20^\circ)(-\hat{k}'') + (PA \cdot F_C)(\hat{k}'') = \vec{0}$$

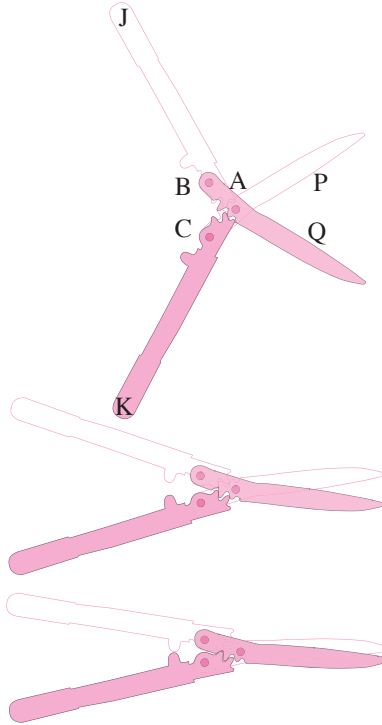
$$\Rightarrow (46 \text{ cm} \cdot 100 \cos 20^\circ \text{ N})(-\hat{k}'') + (3 \text{ cm} \cdot F_C)(\hat{k}'') = \vec{0}$$

$$\Rightarrow F_C = 1440.86 \text{ N} \quad [\text{cutting force at Q and P}]$$

**7.3.10** These 4-piece shears use the mechanism in problem 7.3.9 twice over. Co-linear hand forces  $F_{JK}$  are applied to handles JB and KC at J and K. Handle JB is hinged to blade BAQ at B. Handle KC is hinged to blade CAP at C. The blades are hinged to each other at A. Handle JB is effectively hinged to CAP, by means of gear teeth, at G, a point on the line segment AB. Similarly KC is effectively hinged to BAQ at point G' on the segment AC. The cut object presses with colinear forces  $F_{PQ}$  on the blades at P and Q. See box ?? on page ?? for more pictures of these shears.

Assume  $F_{JK} = 100\text{ N}$ ,  $JB = KC = 30\text{ cm}$ ,  $AB = AC = 6\text{ cm}$ ,  $AG = AG' = 3\text{ cm}$ , and  $AP = AQ = 20\text{ cm}$ . Assume AB, BJ, AC, and CK all make angles of  $\pm 20^\circ$  with a horizontal line. Assume P and Q are coincident and on a horizontal line extending from A.

- Find  $F_{PQ}$ .
- Replace this design with one where JB is welded to BAQ at B, KC is welded to CAP at C, and there are no contacting gears. In this same geometry what is  $F_{PQ}$ ?
- Give a quantitative estimate, but not a detailed calculation that tells you the ratio of the forces in the above two problems?

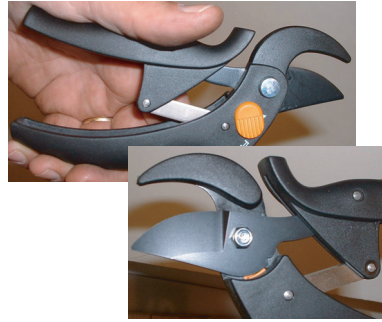


Problem 7.10

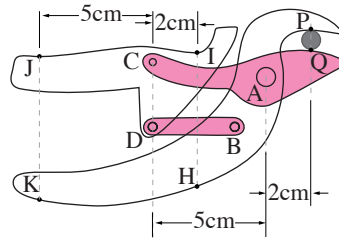
**Answer:**

The mechanism multiplies the force at B and C by a factor of 2 compared to having the handle hinged at A. The force at G also gets (a shade less than) this force but with half the lever arm. Together they give a force multiplication of (a shade less than)  $2+1=3$ .

**7.3.11** The garden cutters shown are a 4-bar linkage. Estimate the locations of points, as needed, using the given dimensions as a scale (the drawn clippers are shrunk slightly from reality to simplify the numbers).



- If the handle is squeezed with a pair of 50 N forces at J and K what is the cutting force at P and Q?
- If the handle is squeezed with a pair of 50 N forces at I and H what is the cutting force at P and Q?
- If this design was changed by eliminating link DB and welding handle JC DI to the blade CAQ, what would be the answers to the two questions above.
- Describe in words, the reasons for the similarities and differences between the answers above.



Problem 7.11

**Answer:**

$$F_P = 125 \text{ N}$$

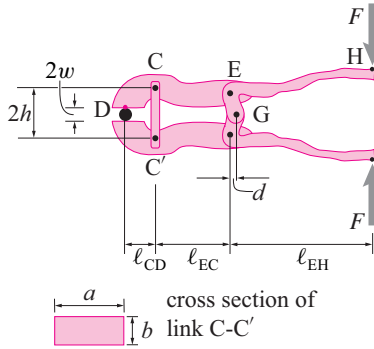
$$F_Q = 125 \text{ N}$$

For the load at I,  $F_P = 75 \text{ N}$ . For the load at J,  $F_P = 250 \text{ N}$ .

With the welded handle there is just a simple lever and the mechanical advantage comes from the horizontal distance between the load and hinge A. For the 4 bar mechanism the force at C is the applied vertical load, no matter where it is applied. So the lever arm is the horizontal distance from A to C.

**7.3.12** The pliers shown are made of five pieces modeled as rigid: HEG and its mirror image, DCE and its mirror image, and link CC'. You may assume that the geometry is symmetric about a horizontal line (the top is a mirror image of the bottom). The load  $F$  and dimensions shown are given. (See also similar problem ??)

- Find the force squeezing the piece at D;
- Find the tension in CC';
- What happens to the squeezing force if  $d$  is made smaller, approaching zero? Why can't this work in practice?



Problem 7.12

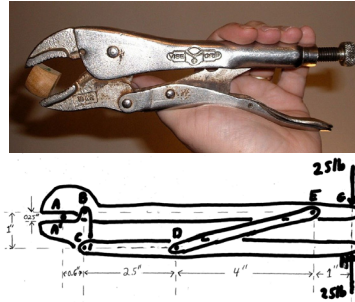
**Answer:**

$$F_D = \ell_{EC}(\ell_{EH} - d)F/d\ell_{CD}$$

$$T_{CC'} = (\ell_{EH}/d - 1)(\ell_{EC}/\ell_{CD} + 1)F$$

As  $d \rightarrow 0$ ,  $F_D \rightarrow \infty$ . Two problems: the amount of motion goes to zero and the assumption of rigidity becomes non-negligibly inaccurate.

**7.3.13** For simplicity the vice grips shown in the photo are approximated as in the drawing. Round piece AA' is gripped between the upper handle/jaw ABEG and the lower jaw A'BC. The upper handle ABEG is pinned to the lower jaw A'BC at B. Handle CDH is pinned to the lower jaw at C and to the bar DE at D. Bar DE is pinned to the upper handle ABEG at E. The 25 lbf forces act at G and H as shown. Dimensions are as shown. What is the magnitude of the force at A?



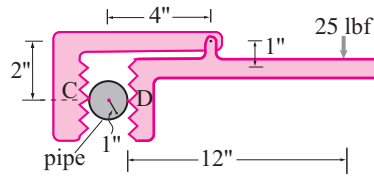
Problem 7.13

**Answer:**

$$F_A = 500 \text{ lbf}$$

**7.3.14 Pipe wrench.** A wrench is used to turn a pipe as shown in the figure. Neglecting the weight of the pipe, find

- the torque of the pipe wrench forces about the center of the pipe
- the forces on the pipe at C and D
- the needed friction coefficient between the wrench and pipe for the wrench not to slip.
- what design change would reduce this needed coefficient of friction (what change of dimensions)?
- given that the design change above is possible, why isn't it used? [hint: implement the design change and calculate the forces on the pipe.]



Problem 7.14

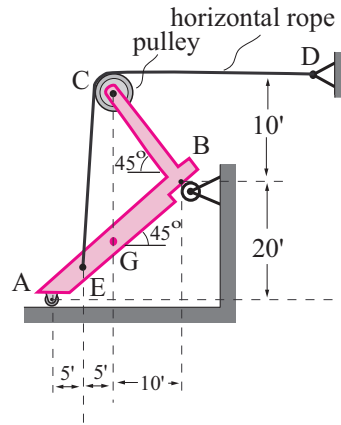
**Answer:**

(d) reduce the dimension marked “2 inches”. The smaller the less the friction needed.

(e) As the “2 inch” dimension is reduced to zero, the needed coefficient of friction goes to zero and the forces squeezing the pipe go to infinity. This is bad because it can damage the pipe. It is also bad because a small pipe deformation will cause the hinge on the wrench to snap through, like a so called “toggle mechanism” and thus not grab at all.

**7.3.15** The center of mass of 200 pound structure AEGBC is at G. It is held by rollers at A and B as well as with the rope which starts at E, wraps around the pulley at C, and ends at D. You can assume the pulley at C has negligible size.

- Find the force of the ground on the structure at A.
- Find the tension in the rope.



Problem 7.15

**Answer:**

$$\vec{R}_A = \vec{0}$$

$$T = 200 \text{ lbf}$$

**7.3.17** The proposed nutcracker design consists of two moving parts: a lever hinged to the fixed base at B and a punch hinged to the fixed base at A. All joints and slots are assumed to have negligible friction.

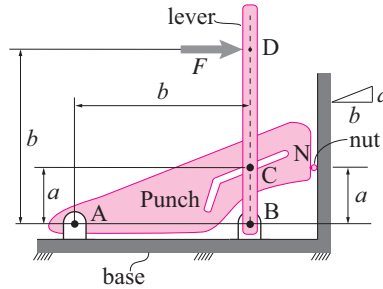
**Mechanism and geometry clarifications:** The vertical lever has a pin at C and a horizontal force  $F$  applied at D. The punch has a slot in which the lever pin slides at C. The slot is parallel to the line AC. The spherical nut is cracked by being squeezed between the vertical surface of the punch at N and the vertical surface attached to the base. Point N at the left edge of the nut is level with the sliding pin at C. The horizontal distance from C to N does not enter the solution, but assume it is  $c$  if you need it for an intermediate calculation.

**Quantities:**  $F = 10 \text{ lbf}$ ,  $a = 2 \text{ in}$ ,  $b = 10 \text{ in}$ .

- a) Find the force acting on the nut at N. A number is desired (i.e., so many lbf force). [Hint: Only substitute in numbers when you have a formula for your answer in terms

of  $a$ ,  $b$  and  $F$ .]

- b) The answer to (a) is conspicuous in its being either much smaller than  $F$ , very similar to  $F$ , or much bigger than  $F$ . Which is it? Explain, in words, why. The best possible answer will generate an approximate formula for the force at N using next-to-no equations.



Problem 7.17

**Answer:**

$$F_N (b(a^2 + b^2)/a^2)) F = 130F = 1300 \text{ lbf}$$

The mechanism uses three tricks to multiply the force: a lever, a wedge, and a toggle. Each of these multiplies by about 5. Thus the nut-force  $F_N$  is on the order of  $5^3 = 125$  times as big as  $F$ .