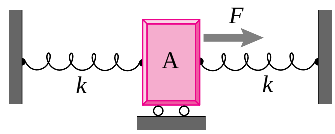
**7.1.1** Find the force $F$ required to push the massless block by 1 cm to the right if $k = 500\,\mathrm{N/m}$.



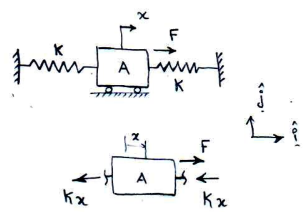

Problem 7.1

**Answer:**
10 N

---

6.1.1

Find the force 'F' required to push the block by 1cm to the right.




Applying force balance to the block A,

$$\sum \underline{F} = \underline{0}$$

Let the block be moved through the distance '$x$'.

All the forces acting on the block are $\sum \underline{F}$

$\Rightarrow \quad \sum \underline{F} = F\hat{\underline{i}} + Kx(-\hat{\underline{i}}) + Kx(-\hat{\underline{i}})$

As the block is in equilibrium, we have $\sum \underline{F} = \underline{0}$

$\Rightarrow \quad \{ F\hat{\underline{i}} - Kx\hat{\underline{i}} - Kx\hat{\underline{i}} = \underline{0} \}$

$\{ \}\cdot\hat{\underline{i}} \quad \Rightarrow \quad F - 2Kx = 0$

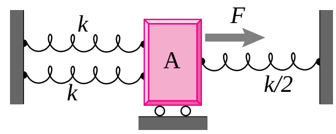
$\Rightarrow \quad F = 2Kx$

Given that $x = 1\,\mathrm{cm} = 0.01\,\mathrm{m}$ and $K = 500\,\mathrm{N/m}$

$\therefore$ Force required to move the block by 1cm to the right is, $F = 2 \times 500\,\frac{N}{m} \cdot 0.01\,\mathrm{m} = 10\,\mathrm{N}$ (or) $\boxed{F = 10\,\mathrm{N}}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**7.1.2** A force $F = 20\,\mathrm{N}$ is applied on the massless block shown in the figure. Find the displacement of the block for equilibrium if $k = 100\,\mathrm{N/cm}$.

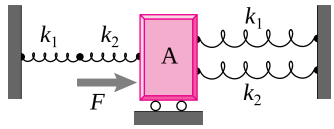



Problem 7.2

**Answer:**
0.08 cm

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**7.1.3** A network of relaxed springs holds a massless block as shown in the figure where $k_1 = 100\,\mathrm{N/cm}$ and $k_2 = 400\,\mathrm{N/cm}$. If the block is pushed to the right by 2 cm, find the force $F$ to hold the block in equilibrium.

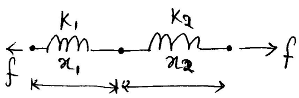



Problem 7.3

**Answer:**
1160 N

6.1.3

In fig. 1, the springs $K_1$ and $K_2$ to the left of A are in series




Let the elongation in spring be $x_1$ & $x_2$

then $f = K_1 x_1 = K_2 x_2$

and total elongation is $(x_1 + x_2) = x$

$$f = K_{eq}(x_1 + x_2)$$

$$\Rightarrow \boxed{\frac{f}{K_{eq}} = \frac{f}{K_1} + \frac{f}{K_2}}$$

$$\Rightarrow \boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}}$$

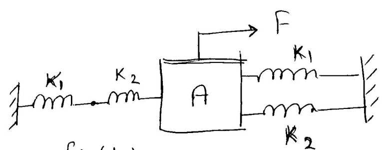
In fig(iii) $\rightarrow$

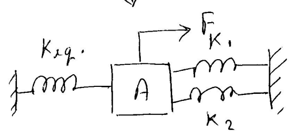
$$\left[\sum \vec{F} = \vec{0}\right] \cdot \hat{i}$$

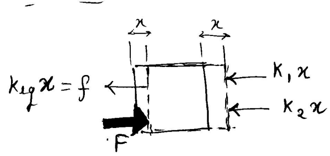
$$F = K_{eq}x + K_1 x + K_2 x$$

$$= \frac{K_1 K_2 x}{K_1 + K_2} + K_1 x + K_2 x$$

$$= 2\left[\frac{400}{5} + 100 + 400\right] \mathrm{N}$$

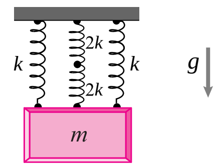



$$\boxed{F = 1160\ \mathrm{N}}$$

**7.1.4** A block of mass $m = 300\,\text{kg}$ hangs from the ceiling with the help of a network of springs in series and parallel as shown. Taking $k = 20\,\text{kN/m}$ and $g = 10\,\text{m/s}^2$, find the stretch in the two side (the left and right) springs.



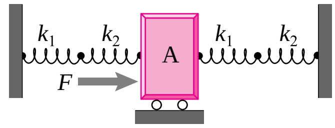

Problem 7.4

**Answer:**
5 cm

**7.1.5** For the arrangement of springs shown in the figure, $k_1 = 50\,\mathrm{N/cm}$ and $k_2 = 100\,\mathrm{N/cm}$. Find

a) the equivalent spring stiffness of the arrangement,

b) the displacement of the block if a force $F = 50\,\mathrm{N}$ acts on the block.

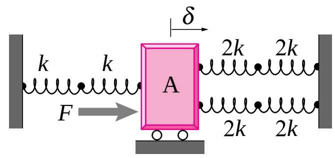



Problem 7.5

**Answer:**
$k_e = 66.7\,\mathrm{N/cm}$, $\delta = 0.75\,\mathrm{cm}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**7.1.7** A massless block is held in position by a network of springs shown in the figure. If the block is displaced to the right by 1 cm from the relaxed position of the springs, a force of $F = 50\,\text{N}$ is required to keep the block in equilibrium. Find the value of $k$.



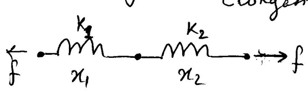

Problem 7.7

**Answer:**
$k = 20\,\text{N/cm}$

6.1.7

Springs on the left of A in fig(1) are in series and two similar sets of springs on the right of A are in parallel.

$\Rightarrow$ Left of A, Let $x_1$ & $x_2$ are elongations in springs



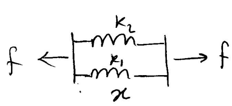

$f = k_1 x_1 = k x_2$

if $f = k_{eq}(x_1 + x_2)$

$\therefore \boxed{k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}}$

Here springs are in series

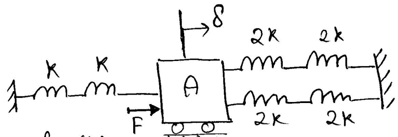
$\Rightarrow$ Right of A (parallel)



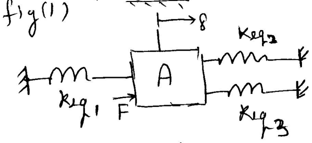

$f = k_1 x + k_2 x$

$\therefore f = k_{eq}(x)$

$\boxed{k_{eq} = k_1 + k_2}$ → parallel

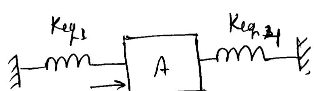



fig(1)

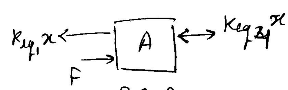



fig(11)

⇓




fig(111)




F.B.D

$\therefore k_{eq1} = \dfrac{k}{2}$ (springs are in series)

$k_{eq2} = k = k_{eq3}$

and $k_{eq4} = 2k$ (springs are in parallel)

so $F = \left(\dfrac{k}{2} + 2k\right)\delta$

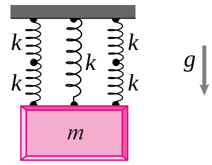
$F = 50\,\text{N}$

$\delta = 1\,\text{cm}$

$\therefore \boxed{k = 20\,\text{N/cm}}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

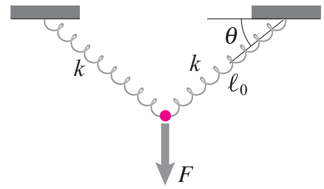
**7.1.8** A box weighing 1000 N is hung from the ceiling using a network of springs, each with stiffness $k = 500\,\text{N/cm}$. Find the stretch in each spring.




Problem 7.8

**Answer:**
Middle spring: $\delta = 1\,\text{cm}$; side-springs $\delta = 0.5\,\text{cm}$

**7.1.10** Find the stretch in each spring to hold the pin in equilibrium for $F = 10\,\text{kN}$ if the relaxed length (in the horizontal position) of each spring is $\ell_0 = 15\,\text{cm}$ and $k = 10\,\text{kN/m}$.



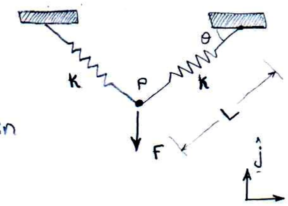

Problem 7.10

6.1.10

Find the stretch in each spring in equilibrium.

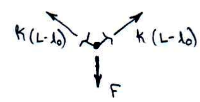
Given that $F = 10\,\text{kN}$, $K = 10\,\text{kN/m}$.

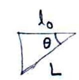
Relaxed length of each spring in horizontal position, $l_0 = 15$ cm.




Let the length of the stretched spring in equilibrium be 'L'.

Consider the FBD of the pin 'P'.




Applying the force balance, $\Sigma \underline{F} = \underline{0}$

$$\Rightarrow \left\{ F(-\hat{j}) + K(L-l_0)(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + K(L-l_0)[-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}] = \underline{0} \right\}$$


$\hat{\lambda} = \cos\theta\,\hat{i} + \sin$

$$\{\ \}\cdot\hat{j} \Rightarrow -F + K(L-l_0)\sin\theta + K(L-l_0)\sin\theta = 0$$

$\cos\theta = \dfrac{l_0}{L}$

$\Rightarrow L = \dfrac{l_0}{\cos\theta}$

$$\Rightarrow 2K\left(\frac{l_0}{\cos\theta} - l_0\right)\sin\theta = F$$

$$\Rightarrow \tan\theta - \sin\theta - \frac{F}{2Kl_0} = 0$$

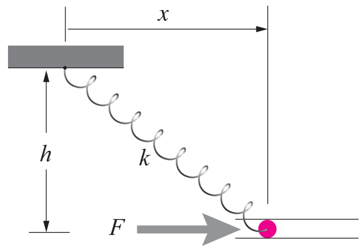
$$\Rightarrow \tan\theta - \sin\theta - \frac{10}{3} = 0$$

Upon solving the above nonlinear equation numerically, we get $\theta = 1.3427$ radians.

$\therefore$ Stretch in the spring, $\delta = \dfrac{l_0}{\cos\theta} - l_0 = 0.5133\,\text{m}$ (or) $\boxed{\delta = 51.33\,\text{cm}}$

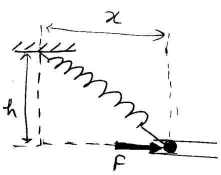
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**7.1.11** A mass slides with negligible friction in a rigid horizontal track. It is also pulled by a zero-rest-length spring ($\ell_0 = 0$) of stiffness 50 kN/m. Find the horizontal position $x$ of the pin if it is in equilibrium with an applied force $F = 1000$ N.




Problem 7.11

6.1.11

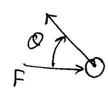



fig(1)

$k \to$ spring stiffness

$k = 50$ kN/m

From the F.B.D [fig(2)]




fig(2)

$$\left[\sum \vec{F} = \vec{0}\right] \cdot \hat{i}$$

$$\Rightarrow \quad F - K\delta \cos\theta = 0$$

$$F = K\delta\cos\theta$$

$$\delta = \sqrt{x^2 + h^2} \qquad \text{and } F = 1000\,\text{N}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+h^2}}$$

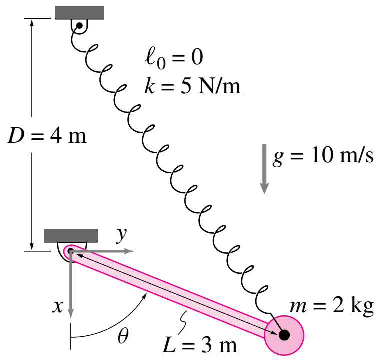
$$\therefore \quad F = K\left(\cancel{\sqrt{x^2+h^2}}\right) \cdot \frac{x}{\cancel{\left(\sqrt{x^2+h^2}\right)}}$$

$$\text{so} \quad x = \frac{F}{K}$$

$$\boxed{x = 2\,\text{cm}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

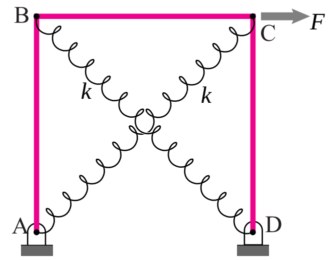
**7.1.12** A zero length spring (relaxed length $\ell_0 = 0$) with stiffness $k = 5\,\mathrm{N/m}$ supports the pendulum shown. Assume $g = 10\ \mathrm{N/\,m}$. Find $\theta$ for static equilibrium.




Problem 7.12

**Answer:**
Surprise! This pendulum is in equilibrium for all values of $\theta$.

**7.1.17** The square box mechanism shown consists of three identical bars and two identical diagonal springs in their relaxed configuration. Each bar is 0.4 m long. A horizontal force $F = 100\,\mathrm{N}$ acts at C. Find the change in length of each spring if $k = 10\,\mathrm{kN/m}$.




Problem 7.17

**Answer:**
$\Delta L_1 = -\Delta L_2 = F/(\sqrt{2}\,k)$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*