**SAMPLE 6.10** For the truss shown in the figure, the coordinates of the three joints are: A(0,0), B(2m,2m), and C(4m,0). Find all reactions and bar forces using computer analysis. Show the input data to the program used and the matrices [A] and [L] generated by the program.

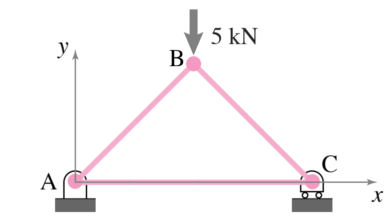


Figure 6.48

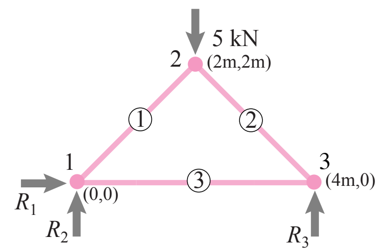


Figure 6.49

**Solution** The free-body diagram of the truss with the unknown reactions serially numbered is shown in fig. 6.49. We have also numbered the bars and joints for preparing the input data file as described in the text. Here, we have three bars and three joints, three unknown reactions, and one externally applied load. Therefore, the input matrices [B] for bar data, [J] for joint data, [R] for support reaction data, and [F] for applied load data are as follows (see page 341 for row and column descriptions).

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}, \quad J = \begin{bmatrix} 1 & 0.0 & 0.0 \\ 2 & 2.0 & 2.0 \\ 3 & 4.0 & 0.0 \end{bmatrix},$$

$$R = \begin{bmatrix} 1 & 1 & 1.0 & 0.0 \\ 2 & 1 & 0.0 & 1.0 \\ 3 & 3 & 0.0 & 1.0 \end{bmatrix}, \quad F = \begin{bmatrix} 2 & 0.0 & -5.0 \end{bmatrix}$$

The computer program based on the pseudocode described in the text generates the following matrices [A] and [L], before solving for the tensions and reactions:

```
A = [ 0.7071         0    1.0000    1.0000         0         0
      0.7071         0         0         0    1.0000         0
     -0.7071    0.7071         0         0         0         0
     -0.7071   -0.7071         0         0         0         0
           0   -0.7071   -1.0000         0         0         0
           0    0.7071         0         0         0    1.0000 ]
L = [ 0
      0
      0
      5
      0
      0 ]
```

The final step, `Solve {A T = F} for T,` gives the following output

```
T = [ -3.5355
      -3.5355
       2.5000
            0
       2.5000
       2.5000 ]
```

which means, $T_1 = T_2 = -3.5355\,\mathrm{N}$, $T_3 = 2.5\,\mathrm{N}$, $R_1 = 0$, and $R_2 = R_3 = 2.5\,\mathrm{N}$.

$$\boxed{T_1 = T_2 = -3.5355\,\mathrm{N}, T_3 = 2.5\,\mathrm{N}, R_1 = 0, R_2 = R_3 = 2.5\,\mathrm{N}}$$

**Note:** If you write a truss code, you can use this sample to check your code.

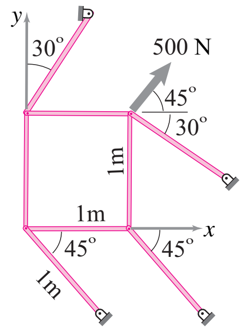


Figure 6.50

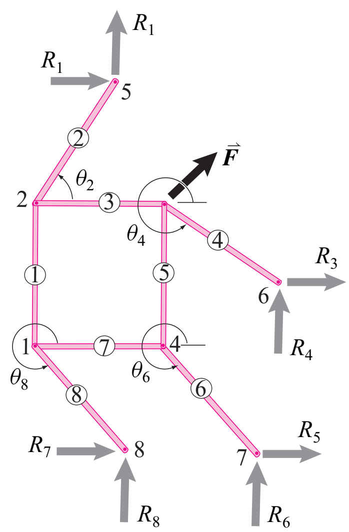


Figure 6.51: Free-body diagram of the truss along with bar and joint numbers. From the given geometry, $\theta_1 = \pi/3, \theta_2 = 11\pi/6 = -\pi/6$, and $\theta_3 = \theta_4 = 7\pi/4$. Assuming the bar length to be $\ell$, we can write the coordinates of all joints in terms of $\ell$ and $\theta$'s. For example, $x_5 = \ell \cos\theta_1, y_5 = \ell + \ell \sin\theta_1$.

**SAMPLE 6.11** The truss shown in the figure has no triangles, yet it is rigid in the configuration shown as discussed in the text. It is also an example of a truss where you cannot find a sequence of joints that will let you solve for the bar forces 'locally', that is, without solving all joint equations simultaneously. Assume all bars to be 1 m long. Find all reactions and bar forces. Show the input data to the program used.

**Solution** The free-body diagram of the truss with the unknown reactions serially numbered is shown in fig. 6.51. Note that support reactions have been taken as unknown $x$ and $y$ components of the reaction at each support point. We could have, alternatively, taken the reaction components to be along and normal to the bars at each support point.

The bars and joints are numbered as shown. Here, we have eight bars and eight joints, eight unknown reactions, and one externally applied load. Let the length of each bar be $\ell = 1\,\mathrm{m}$. The angle of outer bars with the $x$-axis are $\theta_2 = \pi/3$, $\theta_4 = -\pi/6$, $\theta_6 = \theta_8 = -\pi/4$. Therefore, the input matrices [B] (bar data), [J] (joint data), [R] (support reaction data), and [F] (applied load data) are as follows (see page 341 for row and column descriptions).

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 5 \\ 3 & 2 & 3 \\ 4 & 3 & 6 \\ 5 & 3 & 4 \\ 6 & 4 & 7 \\ 7 & 1 & 4 \\ 8 & 1 & 8 \end{bmatrix}, \quad J = \begin{bmatrix} 1 & 0.0 & 0.0 \\ 2 & 0.0 & \ell \\ 3 & \ell & \ell \\ 4 & \ell & 0.0 \\ 5 & \ell\cos\theta_2 & \ell + \ell\sin\theta_2 \\ 6 & \ell + \ell\cos\theta_4 & \ell + \ell\sin\theta_4 \\ 7 & \ell + \ell\cos\theta_6 & \ell\sin\theta_6 \\ 8 & \ell\cos\theta_8 & \ell\sin\theta_8 \end{bmatrix},$$

$$R = \begin{bmatrix} 1 & 5 & 1.0 & 0.0 \\ 2 & 5 & 0.0 & 1.0 \\ 3 & 6 & 1.0 & 0.0 \\ 4 & 6 & 0.0 & 1.0 \\ 5 & 7 & 1.0 & 0.0 \\ 6 & 7 & 0.0 & 1.0 \\ 7 & 8 & 1.0 & 0.0 \\ 8 & 8 & 0.0 & 1.0 \end{bmatrix}, \quad F = \begin{bmatrix} 3 & 500/\sqrt{2} & 500/\sqrt{2} \end{bmatrix}$$

The computer program based on the pseudocode described in the text generates the following output, [T], for the tensions and reactions: The final step, `Solve {A T = F} for T,` gives the following result for bar tensions and reactions.

$$\begin{array}{rcrrcr} T_1 & = & -418.26\,\mathrm{N}, & R_1 & = & -241.48\,\mathrm{N}, \\ T_2 & = & -482.96\,\mathrm{N}, & R_2 & = & -418.26\,\mathrm{N}, \\ T_3 & = & 241.48\,\mathrm{N}, & R_3 & = & -112.07\,\mathrm{N}, \\ T_4 & = & -129.41\,\mathrm{N}, & R_4 & = & 64.70\,\mathrm{N}, \\ T_5 & = & 418.26\,\mathrm{N}, & R_5 & = & 418.26\,\mathrm{N}, \\ T_6 & = & 591.51\,\mathrm{N}, & R_6 & = & -418.26\,\mathrm{N}, \\ T_7 & = & 418.26\,\mathrm{N}, & R_7 & = & -418.26\,\mathrm{N}, \\ T_8 & = & -591.51\,\mathrm{N}, & R_8 & = & 418.26\,\mathrm{N}. \end{array}$$

**Note:** It is easy to check that $R_1 + R_3 + R_5 + R_7 = -F\cos(45^\circ)$ and $R_2 + R_4 + R_6 + R_8 = -F\sin(45^\circ)$ where $F = 500\,\mathrm{N}$. That is, for the free-body diagram of the truss, $\sum F_x = 0$ and $\sum F_y = 0$.