6.2 The method of sections

Perhaps the most central concept in mechanics, and thus for truss analysis, is the free-body diagram. For truss analysis we have already found it fruitful to draw free-body diagrams of the whole structure, of the bars (to see that they are two-force bodies), and of the individual joints. But you can draw a free-body diagram of anything, for example of any part of a system you are studying. Assuming static equilibrium, force and moment balance apply to that subsystem.

In *the method of sections* you find bar tensions by drawing a free-body diagram of a part of the truss that includes more than one joint and less than the whole structure.

The place where the truss is cut is called the *section*.

What's wrong with the method of joints?

The method of joints can solve any solvable truss. So why learn a different method? There are two basic reasons.

1. Sometimes one only wants to know a little and the method of joints is cumbersome.

   Example: Difficulty in finding just one bar tension.
   Say you are interested only in T_{KM} in the truss of fig. 6.7 on page 320. With the method of joints we could find T_{KM} by working through the joints one at a time. To get to joint K we would have to draw free-body diagrams of at least 8 other joints first. And for each we would have to solve two simultaneous equations.

2. Sometimes the method of joints doesn't best reveal basic structural ideas.

   Example: Difficulty in understanding trends.
   Again look at the truss of fig. 6.7. With the method of joints we would find, after all the algebra, that all the bars on the bottom (AC, CE, EH, HJ, JL, LN, NP, PR) have compression (negative tension) and that each bar has more compression than the one to its right. Similarly the top is all tension with the tension increasing with the bars more to the left. Are these trends just a consequence of lots of algebra?

The method of sections provides a shortcut, particularly for elementary textbook-like problems. And the method of sections can explain some structural trends.

The basic method of sections recipe

Say you are just trying to find one bar tension, for example T_{KM} in the truss of fig. 6.7. For simplicity we limit our attention to 2D structures.

- Find a way to cut the structure into two parts, using a *section cut* that
  - cuts the bar of interest and
  - cuts at most 3 bars in total and
  - where one of the two parts of the truss has all loads known because
    - ∗ all loads are given applied loads, or
    - ∗ the loads are reactions that have been found using a free-body diagram of the whole structure.
- Write and solve the equations of moment balance for one side of the structure. This should be 3 equations in 3 unknowns.
  - Either use any three independent equations (say force balance (2) and moment balance (1)), or
  - Look for a shortcut. Try to find one equation that contains the unknown of interest and no other unknowns using
    - ∗ moment balance about the point of intersection of the lines of action of the two unknown forces that are not of interest, or
    - ∗ if the two uninteresting unknown forces are parallel, use force balance in a direction orthogonal to them.

For a given truss and given bar tension of interest there is no guarantee that the recipe applies. You can always find a section cut through the bar of interest, but there may be too-many unknowns in the free-body diagrams of both of the resulting sub-structures.

Because 2D statics of finite objects gives three scalar equations we can generally find all three unknown bar tensions from a section cut that goes through 3 bars.

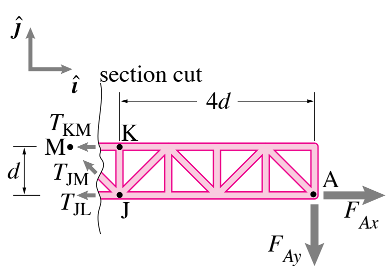

Figure 6.30: Free-body diagram of a 'section' of the structure.

Example: Three bar-forces from one FBD.
Look at the free-body diagram from a section cut in fig. 6.30. Moment balance about point J (about an axis through J in the z direction) gives:

$$\left\{\sum \vec{\boldsymbol{M}}_{\mathrm{J}} = \vec{\boldsymbol{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad T_{\mathrm{KM}} = 4F_{Ay}.$$

Using the FBD with this same section cut we can also find:

$$\left\{\sum \vec{\boldsymbol{M}}_{\mathrm{M}} = \vec{\boldsymbol{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad T_{\mathrm{JL}} = -5F_{Ay} + F_{Ax}, \text{ and}$$
$$\left\{\sum \vec{\boldsymbol{F}}_{\mathrm{i}} = \vec{\boldsymbol{0}}\right\} \cdot \hat{\boldsymbol{\jmath}} \quad \Rightarrow \quad T_{\mathrm{JM}} = \sqrt{2}F_{Ay}.$$

Note that in the free-body diagram of fig. 6.30, moment balance about point J eliminates T_{JM} and T_{JL} and gives one equation for T_{KM}. And force balance in the $\hat{\boldsymbol{\jmath}}$ direction eliminates T_{KM} and T_{JL}, giving one equation for T_{JM}.

Using sections to gain insight

In the method of joints, as you worked your way along the structure fig. 6.7 from right to left you would have found the tensions getting bigger and bigger on the top bars and the compressions (negative tensions) getting bigger and bigger on the bottom bars. With the method of sections you can see that this comes from the lever arm of the load F being bigger and bigger for longer and longer sections of truss. The moment caused by the vertical load F_{Ay} is carried by the tension in the top bars and compression in the bottom bars.

A warning

Because of positive experiences with the method of sections for textbook-like problems and very simple structures, many people are left with the impression that the method of sections is more powerful than the method of joints. It isn't. The method of sections is of less general utility than the method of joints. And, unlike for the method of joints, there is no simple systematic way to find all of the bar tensions in all statically-determinate trusses (See fig. 6.31).

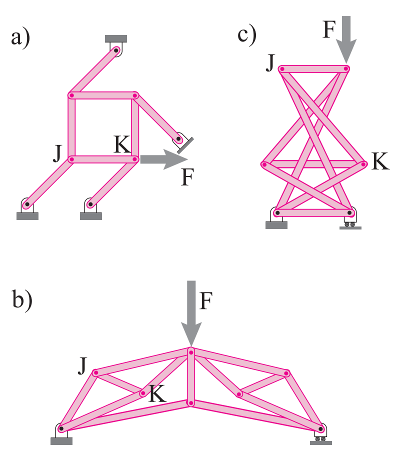

Figure 6.31: The method of sections is great when it works, but it doesn't always work. Like for the trusses here. For each of these trusses, there is no single section cut that leads to a free-body diagram from which you can find T_{JK}. Yet, these are all statically determinate trusses. For all of these trusses, to find T_{JK} one has to write the equilibrium equations for many joints and then solve these equations simultaneously. [In the truss (c), joints are marked by dots, there are no joints where the bars cross. This slightly unusual truss has no closed triangles).]