SAMPLE 6.7 The tower truss shown in the figure is fabricated with 19 rods. All the horizontal and vertical rods are one meter long. Joint J is halfway between joints K and H. The horizontal force applied at joint K is 1 kN. Find the tensions in

1. rod GJ, and
2. rod CE.

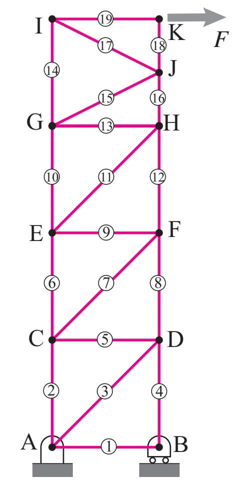

Figure 6.32

Solution To find the tension in rod GJ, numbered 15, let us make a cut through the truss as shown in fig. 6.33. The section taken here cuts rods 14, 15, and 16. The free-body diagram has only three unknown tensions acting on the part of the truss under consideration.

From the force balance in the x direction, we see at once,

$$\begin{aligned} F - T_{15}\cos\theta &= 0 \\ \Rightarrow \quad T_{15} &= \frac{F}{\cos\theta} \\ &= \frac{1\,\text{kN}}{\cos 26.56^\circ} \\ &= 1.12\,\text{kN}. \end{aligned}$$

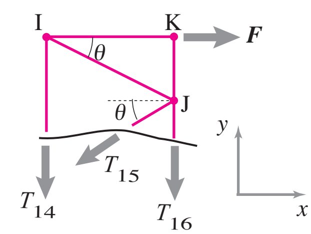

Figure 6.33

$$\boxed{T_{\text{GJ}} \equiv T_{15} = 1.12\,\text{kN}}$$

To determine the tension in rod CE, we consider a section that cuts rods CE, CF, and DF. The free-body diagram of the truss above this section is shown in fig. 6.34. Once again, we have only three unknown forces on the body under consideration (note that we will have six unknown forces that include three support reactions if we considered the lower part of the truss, below the selected section).

To find T_6, we write the scalar moment-balance equation in the z-direction about point F:

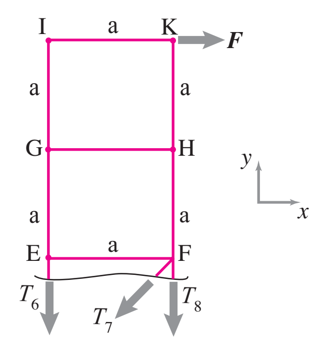

Figure 6.34

$$\begin{aligned} aT_6 - 2aF &= 0 \\ \Rightarrow \quad T_6 &= 2F \\ &= 2\,\text{kN}. \end{aligned}$$

$$\boxed{T_{\text{CE}} \equiv T_6 = 2\,\text{kN}}$$

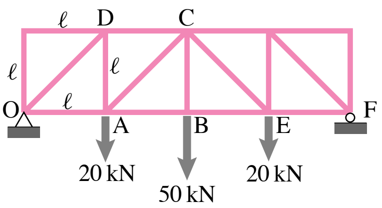

Figure 6.35

SAMPLE 6.8 A 2-D truss: The box truss shown in the figure is loaded by three vertical forces acting at joints A, B, and E. All horizontal and vertical bars in the truss are of length 2 m. Find the forces in members AB, AC, and DC.

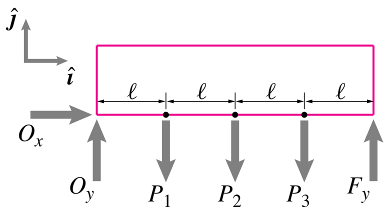

Figure 6.36

Solution First, we need to find the support reactions at points O and F. We do this by drawing the free-body diagram of the whole truss and writing the equilibrium equations for it. Referring to Fig. 6.36, the force equilibrium, $\sum \vec{F} = \vec{0}$ implies,

$$O_x\hat{\imath} + (O_y + F_y - P_1 - P_2 - P_3)\hat{\jmath} = \vec{0}. \tag{6.11}$$

Dotting eqn. (6.11) with $\hat{\imath}$ and $\hat{\jmath}$, respectively, we get

$$\begin{aligned} O_x &= 0 \\ O_y + F_y &= P_1 + P_2 + P_3. \end{aligned} \tag{6.12}$$

The moment equilibrium about point O, $\vec{M}_O = \vec{0}$, gives

$$(-P_1\ell - P_2 2\ell - P_3 3\ell + F_y 4\ell)\hat{k} = \vec{0} \tag{6.13}$$

$$\text{or} \qquad F_y = \frac{1}{4}(P_1 + 2P_2 + 3P_3). \tag{6.14}$$

Solving eqns. (6.12) and (6.14), we get

$$F_y = 45\,\text{kN}, \qquad \text{and} \qquad O_y = 45kN.$$

In fact, from the symmetry of the structure and the loads, we could have guessed that the two vertical reactions must be equal, *i.e.,* $O_y = F_y$. Then, from eqn. (6.12) it follows that $O_y = F_y = (P_1 + P_2 + P_3)/2 = 45\,\text{kN}$.

Now, we proceed to find the forces in the members AB, AC, and DC. For this purpose, we make a cut in the truss such that it cuts members AD, AC, and DC, just to the right of joints A and D. Next, we draw the free-body diagram of the left (or right) portion of the truss and use the equilibrium equations to find the required forces. Referring to Fig. 6.37, the force equilibrium requires that

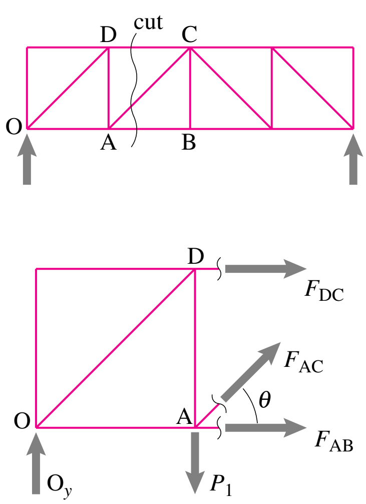

Figure 6.37

$$(F_{AB} + F_{DC} + F_{AC}\cos\theta)\hat{\imath} + (O_y - P_1 + F_{AC}\sin\theta)\hat{\jmath} = \vec{0}. \tag{6.15}$$

Dotting eqn. (6.15) with $\hat{\imath}$ and $\hat{\jmath}$, respectively, we get

$$F_{AB} + F_{DC} + F_{AC}\cos\theta = 0 \tag{6.16}$$

$$O_y - P_1 + F_{AC}\sin\theta = 0. \tag{6.17}$$

So far, we have two equations in three unknowns (F_{AB}, F_{DC}, F_{AC}). We need one more independent equation to be able to solve for the unknown forces. We now write moment equilibrium equation about point A, *i.e.,* $\sum \vec{M}_A = \vec{0}$,

$$\begin{aligned} (-O_y\ell - F_{DC}\ell)\hat{k} &= \vec{0} \\ \Rightarrow \qquad O_y + F_{DC} &= 0. \end{aligned} \tag{6.18}$$

We can now solve eqns. (6.16–6.18) any way we like, *e.g.,* using elimination or a computer. The solution we get (see next page for details) is:

$$F_{AC} = -25\sqrt{2}\,\text{kN}, \quad F_{DC} = -45\,\text{kN}, \qquad \text{and} \qquad F_{AB} = 70\,\text{kN}.$$

$$F_{AC} = -25\sqrt{2}\,\text{kN}, \quad F_{DC} = -45\,\text{kN}, \quad F_{AB} = 70\,\text{kN}.$$

Comments:

- Note that the values of F_{AC} and F_{DC} are negative which means that bars AC and DC are in compression, not tension, as we initially assumed. Thus the solution takes care of our incorrect assumptions about the directionality of the forces.
- **Short-cuts:** In the solution above, we have not used any tricks or any special points for moment equilibrium. However, with just a little bit of mechanics intuition we can solve for the required forces in five short steps as shown below.
  (i) No external force in $\hat{\imath}$ direction implies $O_x = 0$.
  (ii) Symmetry about the middle point B implies $O_y = F_y$. But,
  $$O_y + F_y = \sum P_i = 90\,\text{kN} \quad \Rightarrow \quad O_y = F_y = 45\,\text{kN}.$$
  (iii) $(\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\boldsymbol{0}}) \cdot \hat{\boldsymbol{k}}$ gives
  $$O_y \ell + F_{DC}\ell = 0 \quad \Rightarrow \quad F_{DC} = -O_y = -45\,\text{kN}.$$
  (iv) $(\sum \vec{\boldsymbol{M}}_{\text{C}} = \vec{\boldsymbol{0}}) \cdot \hat{\boldsymbol{k}}$ gives
  $$-O_y\, 2\ell + P_1\ell + F_{AB}\ell = 0 \quad \Rightarrow \quad F_{AB} = 2O_y - P_1 = 70\,\text{kN}.$$
  (v) $(\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}) \cdot \hat{\boldsymbol{\jmath}}$ gives
  $$O_y - P_1 + F_{AC}\sin\theta = 0 \quad \Rightarrow \quad F_{AC} = (P_1 - O_y)/\sin\theta = -25\sqrt{2}\,\text{kN}.$$
- **Solving equations:** On the previous page, we found F_{AB}, F_{DC}, and F_{AC} by solving eqns. (6.15–6.17) simultaneously. Here, we show you two ways to solve those equations.
  1. *By elimination:* From eqn. (6.17), we have
  $$F_{AC} = \frac{O_y - P_1}{\sin\theta} = \frac{20\,\text{kN} - 45\,\text{kN}}{1/\sqrt{2}} = -25\sqrt{2}\,\text{kN}.$$
  From eqn. (6.18), we get
  $$F_{DC} = -O_y = -45\,\text{kN},$$
  and finally, substituting the values found in eqn. (6.15), we get
  $$F_{AB} = -F_{DC} - F_{AC}\cos\theta = 45\,\text{kN} + 25\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 70\,\text{kN}.$$
  2. *On a computer:* We can write the three equations in the matrix form:
  $$\underbrace{\begin{bmatrix} 1 & 1 & \cos\theta \\ 0 & 0 & \sin\theta \\ 0 & 1 & 0 \end{bmatrix}}_{\texttt{A}} \underbrace{\begin{Bmatrix} F_{AB} \\ F_{DC} \\ F_{AC} \end{Bmatrix}}_{\texttt{x}} = \begin{Bmatrix} 0 \\ P_1 - O_y \\ -O_y \end{Bmatrix} = \underbrace{\begin{Bmatrix} 0 \\ -25 \\ -45 \end{Bmatrix}}_{\texttt{b}} \text{kN}.$$
  We can now solve this matrix equation on a computer by keying in matrix `A` (with θ specified as $\pi/4$) and vector `b` as input and solving for `x`.(1)

(1) Pseudocode:

```
A = [1 1 cos(pi/4)
     0 0 sin(pi/4)
     0 1 0]
b = [0 -25 -45]
solve A*x = b for x
```

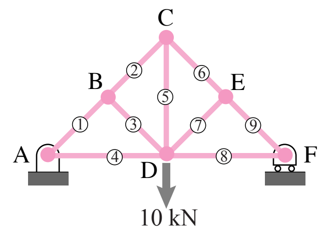

Figure 6.38

SAMPLE 6.9 Consider the truss shown in the figure. Rods AB, BC, EC, EF, BD, and DE are each 2 m long, and $\angle$ ABD = $\angle$DEF = $\pi/2$. Find the tensions in rods DE and CD.

Solution We do not need any analysis to find the tension in rod DE. Since DE is normal to CF, DE has to be a zero-force member for equilibrium of joint E.(2) However, let us find out the same result using the method of sections. Let us take a section just to the left of joint E that cuts through rods CE, DE and DF. The free-body diagram of the truss to the right of the section is shown in fig. 6.39.

The scalar moment-balance equation, $\sum M_z = 0$, about point F gives at once,

$$aT_7 = 0 \quad \Rightarrow \quad T_7 = 0.$$

Thus rod CE is tension free. Now, we make another cut, taking the section shown in fig. 6.40 to determine the tension in rod CD. Since $T_7 = 0$, we can write the scalar moment-balance equation in the z-direction about point A to give

$$\begin{aligned} \ell T_5 - \ell F &= 0 \\ \Rightarrow \quad T_5 &= F = 10\,\text{kN}. \end{aligned}$$

$$\boxed{T_7 = 0, \quad T_5 = 10\,\text{kN}}$$

(2) In fact, both rods BD and DE are zero-force members. Since they carry no tension, rods AD and DF must also be zero-force members for equilibrium of joint D

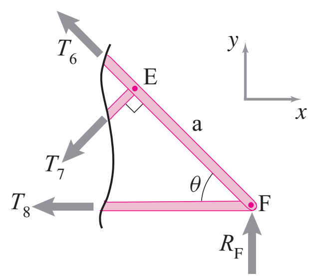

Figure 6.39: Free-body diagram with a sectional cut that cuts through rods CE, DE and DF.

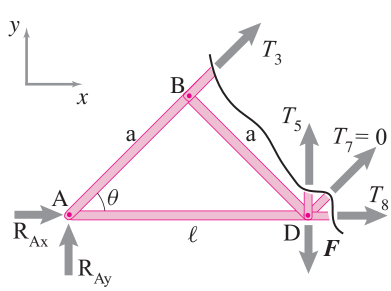

Figure 6.40: Free-body diagram with a sectional cut that cuts through rods BC, CD, DE, and DF.