

6.2 The method of sections

Perhaps the most central concept in mechanics, and thus for truss analysis, is the free-body diagram. For truss analysis we have already found it fruitful to draw free-body diagrams of the whole structure, of the bars (to see that they are two-force bodies), and of the individual joints. But you can draw a free-body diagram of anything, for example of any part of a system you are studying. Assuming static equilibrium, force and moment balance apply to that subsystem.

In the *method of sections* you find bar tensions by drawing a free-body diagram of a part of the truss that includes more than one joint and less than the whole structure.

The place where the truss is cut is called the *section*.

What's wrong with the method of joints?

The method of joints can solve any solvable truss. So why learn a different method? There are two basic reasons.

1. Sometimes one only wants to know a little and the method of joints is cumbersome.

Example: Difficulty in finding just one bar tension.

Say you are interested only in T_{KM} in the truss of fig. 6.7 on page 320. With the method of joints we could find T_{KM} by working through the joints one at a time. To get to joint K we would have to draw free-body diagrams of at least 8 other joints first. And for each we would have to solve two simultaneous equations.

2. Sometimes the method of joints doesn't best reveal basic structural ideas.

Example: Difficulty in understanding trends.

Again look at the truss of fig. 6.7. With the method of joints we would find, after all the algebra, that all the bars on the bottom (AC, CE, EH, HJ, JL, LN, NP, PR) have compression (negative tension) and that each bar has more compression than the one to its right. Similarly the top is all tension with the tension increasing with the bars more to the left. Are these trends just a consequence of lots of algebra?

The method of sections provides a shortcut, particularly for elementary textbook-like problems. And the method of sections can explain some structural trends.

The basic method of sections recipe

Say you are just trying to find one bar tension, for example T_{KM} in the truss of fig. 6.7. For simplicity we limit our attention to 2D structures.

- Find a way to cut the structure into two parts, using a *section cut* that
 - cuts the bar of interest and
 - cuts at most 3 bars in total and
 - where one of the two parts of the truss has all loads known because
 - * all loads are given applied loads, or
 - * the loads are reactions that have been found using a free-body diagram of the whole structure.
- Write and solve the equations of moment balance for one side of the structure. This should be 3 equations in 3 unknowns.
 - Either use any three independent equations (say force balance (2) and moment balance (1)), or
 - Look for a shortcut. Try to find one equation that contains the unknown of interest and no other unknowns using
 - * moment balance about the point of intersection of the lines of action of the two unknown forces that are not of interest, or
 - * if the two uninteresting unknown forces are parallel, use force balance in a direction orthogonal to them.

For a given truss and given bar tension of interest there is no guarantee that the recipe applies. You can always find a section cut through the bar of interest, but there may be too-many unknowns in the free-body diagrams of both of the resulting sub-structures.

Because 2D statics of finite objects gives three scalar equations we can generally find all three unknown bar tensions from a section cut that goes through 3 bars.

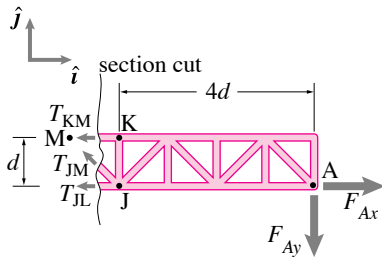


Figure 6.30: Free-body diagram of a 'section' of the structure.

Example: Three bar-forces from one FBD.

Look at the free-body diagram from a section cut in fig. 6.30. Moment balance about point J (about an axis through J in the z direction) gives:

$$\{\sum \vec{M}_J = \vec{0}\} \cdot \hat{k} \quad \Rightarrow \quad T_{KM} = 4F_{Ay}.$$

Using the FBD with this same section cut we can also find:

$$\begin{aligned} \{\sum \vec{M}_M = \vec{0}\} \cdot \hat{k} &\Rightarrow T_{JL} = -5F_{Ay} + F_{Ax}, \text{ and} \\ \{\sum \vec{F}_i = \vec{0}\} \cdot \hat{j} &\Rightarrow T_{JM} = \sqrt{2}F_{Ay}. \end{aligned}$$

Note that in the free-body diagram of fig. 6.30, moment balance about point J eliminates T_{JM} and T_{JL} and gives one equation for T_{KM} . And force balance in the $\hat{j}$ direction eliminates T_{KM} and T_{JL} , giving one equation for T_{JM} .

Using sections to gain insight

In the method of joints, as you worked your way along the structure fig. 6.7 from right to left you would have found the tensions getting bigger and bigger on the top bars and the compressions (negative tensions) getting bigger and bigger on the bottom bars. With the method of sections you can see that this comes from the lever arm of the load F being bigger and bigger for longer and longer sections of truss. The moment caused by the vertical load F_{Ay} is carried by the tension in the top bars and compression in the bottom bars.

A warning

Because of positive experiences with the method of sections for textbook-like problems and very simple structures, many people are left with the impression that the method of sections is more powerful than the method of joints. It isn't. The method of sections is of less general utility than the method of joints. And, unlike for the method of joints, there is no simple systematic way to find all of the bar tensions in all statically-determinate trusses (See fig. 6.31).

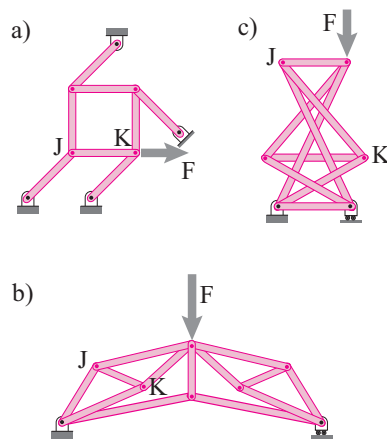


Figure 6.31: The method of sections is great when it works, but it doesn't always work. Like for the trusses here. For each of these trusses, there is no single section cut that leads to a free-body diagram from which you can find T_{JK} . Yet, these are all statically determinate trusses. For all of these trusses, to find T_{JK} one has to write the equilibrium equations for many joints and then solve these equations simultaneously. [In the truss (c), joints are marked by dots, there are no joints where the bars cross. This slightly unusual truss has no closed triangles.)]

SAMPLE 6.7 The tower truss shown in the figure is fabricated with 19 rods. All the horizontal and vertical rods are one meter long. Joint J is halfway between joints K and H. The horizontal force applied at joint K is 1 kN. Find the tensions in

1. rod GJ, and
2. rod CE.

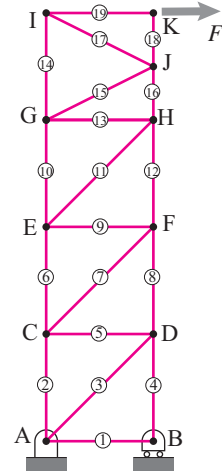


Figure 6.32

Solution To find the tension in rod GJ, numbered 15, let us make a cut through the truss as shown in fig. 6.33. The section taken here cuts rods 14, 15, and 16. The free-body diagram has only three unknown tensions acting on the part of the truss under consideration.

From the force balance in the x direction, we see at once,

$$\begin{aligned} F - T_{15} \cos \theta &= 0 \\ \Rightarrow T_{15} &= \frac{F}{\cos \theta} \\ &= \frac{1 \text{ kN}}{\cos 26.56^\circ} \\ &= 1.12 \text{ kN.} \end{aligned}$$

$$T_{GJ} \equiv T_{15} = 1.12 \text{ kN}$$

To determine the tension in rod CE, we consider a section that cuts rods CE, CF, and DF. The free-body diagram of the truss above this section is shown in fig. 6.34. Once again, we have only three unknown forces on the body under consideration (note that we will have six unknown forces that include three support reactions if we considered the lower part of the truss, below the selected section).

To find T_6 , we write the scalar moment-balance equation in the z -direction about point F:

$$\begin{aligned} aT_6 - 2aF &= 0 \\ \Rightarrow T_6 &= 2F \\ &= 2 \text{ kN.} \end{aligned}$$

$$T_{CE} \equiv T_6 = 2 \text{ kN}$$

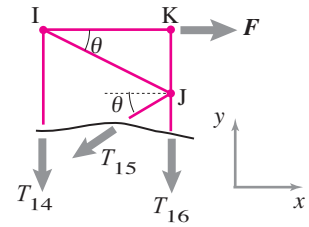


Figure 6.33

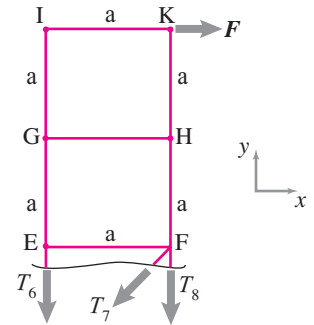


Figure 6.34

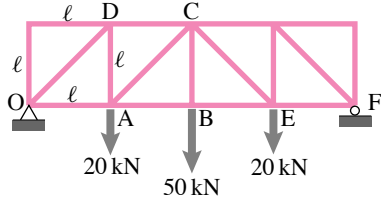


Figure 6.35

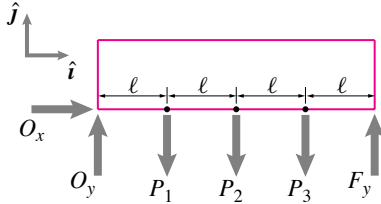


Figure 6.36

SAMPLE 6.8 A 2-D truss: The box truss shown in the figure is loaded by three vertical forces acting at joints A, B, and E. All horizontal and vertical bars in the truss are of length 2 m. Find the forces in members AB, AC, and DC.

Solution First, we need to find the support reactions at points O and F. We do this by drawing the free-body diagram of the whole truss and writing the equilibrium equations for it. Referring to Fig. 6.36, the force equilibrium, $\sum \vec{F} = \vec{0}$ implies,

$$O_x \hat{i} + (O_y + F_y - P_1 - P_2 - P_3) \hat{j} = \vec{0}. \quad (6.11)$$

Dotting eqn. (6.11) with $\hat{i}$ and $\hat{j}$, respectively, we get

$$\begin{aligned} O_x &= 0 \\ O_y + F_y &= P_1 + P_2 + P_3. \end{aligned} \quad (6.12)$$

The moment equilibrium about point O, $\vec{M}_O = \vec{0}$, gives

$$(-P_1 \ell - P_2 2\ell - P_3 3\ell + F_y 4\ell) \hat{k} = \vec{0} \quad (6.13)$$

$$\text{or} \quad F_y = \frac{1}{4}(P_1 + 2P_2 + 3P_3). \quad (6.14)$$

Solving eqns. (6.12) and (6.14), we get

$$F_y = 45 \text{ kN}, \quad \text{and} \quad O_y = 45 \text{ kN}.$$

In fact, from the symmetry of the structure and the loads, we could have guessed that the two vertical reactions must be equal, i.e., $O_y = F_y$. Then, from eqn. (6.12) it follows that $O_y = F_y = (P_1 + P_2 + P_3)/2 = 45 \text{ kN}$.

Now, we proceed to find the forces in the members AB, AC, and DC. For this purpose, we make a cut in the truss such that it cuts members AD, AC, and DC, just to the right of joints A and D. Next, we draw the free-body diagram of the left (or right) portion of the truss and use the equilibrium equations to find the required forces. Referring to Fig. 6.37, the force equilibrium requires that

$$(F_{AB} + F_{DC} + F_{AC} \cos \theta) \hat{i} + (O_y - P_1 + F_{AC} \sin \theta) \hat{j} = \vec{0}. \quad (6.15)$$

Dotting eqn. (6.15) with $\hat{i}$ and $\hat{j}$, respectively, we get

$$F_{AB} + F_{DC} + F_{AC} \cos \theta = 0 \quad (6.16)$$

$$O_y - P_1 + F_{AC} \sin \theta = 0. \quad (6.17)$$

So far, we have two equations in three unknowns (F_{AB}, F_{DC}, F_{AC}). We need one more independent equation to be able to solve for the unknown forces. We now write moment equilibrium equation about point A, i.e., $\sum \vec{M}_A = \vec{0}$,

$$\begin{aligned} (-O_y \ell - F_{DC} \ell) \hat{k} &= \vec{0} \\ \Rightarrow \quad O_y + F_{DC} &= 0. \end{aligned} \quad (6.18)$$

We can now solve eqns. (6.16–6.18) any way we like, e.g., using elimination or a computer. The solution we get (see next page for details) is:

$$F_{AC} = -25\sqrt{2} \text{ kN}, \quad F_{DC} = -45 \text{ kN}, \quad \text{and} \quad F_{AB} = 70 \text{ kN}.$$

$$F_{AC} = -25\sqrt{2} \text{ kN}, \quad F_{DC} = -45 \text{ kN}, \quad F_{AB} = 70 \text{ kN}.$$

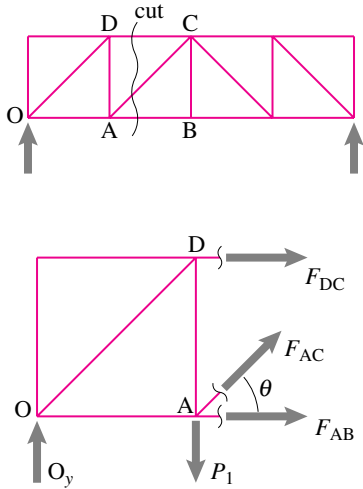


Figure 6.37

Comments:

- Note that the values of F_{AC} and F_{DC} are negative which means that bars AC and DC are in compression, not tension, as we initially assumed. Thus the solution takes care of our incorrect assumptions about the directionality of the forces.
- Short-cuts:** In the solution above, we have not used any tricks or any special points for moment equilibrium. However, with just a little bit of mechanics intuition we can solve for the required forces in five short steps as shown below.

(i) No external force in $\hat{i}$ direction implies $O_x = 0$.

(ii) Symmetry about the middle point B implies $O_y = F_y$. But,

$$O_y + F_y = \sum P_i = 90 \text{ kN} \Rightarrow O_y = F_y = 45 \text{ kN}.$$

(iii) $(\sum \vec{M}_A = \vec{0}) \cdot \hat{k}$ gives

$$O_y \ell + F_{DC} \ell = 0 \Rightarrow F_{DC} = -O_y = -45 \text{ kN}.$$

(iv) $(\sum \vec{M}_C = \vec{0}) \cdot \hat{k}$ gives

$$-O_y 2\ell + P_1 \ell + F_{AB} \ell = 0 \Rightarrow F_{AB} = 2O_y - P_1 = 70 \text{ kN}.$$

(v) $(\sum \vec{F} = \vec{0}) \cdot \hat{j}$ gives

$$O_y - P_1 + F_{AC} \sin \theta = 0 \Rightarrow F_{AC} = (P_1 - O_y) / \sin \theta = -25\sqrt{2} \text{ kN}.$$

- Solving equations:** On the previous page, we found F_{AB} , F_{DC} , and F_{AC} by solving eqns. (6.15–6.17) simultaneously. Here, we show you two ways to solve those equations.

1. *By elimination:* From eqn. (6.17), we have

$$F_{AC} = \frac{O_y - P_1}{\sin \theta} = \frac{20 \text{ kN} - 45 \text{ kN}}{1/\sqrt{2}} = -25\sqrt{2} \text{ kN}.$$

From eqn. (6.18), we get

$$F_{DC} = -O_y = -45 \text{ kN},$$

and finally, substituting the values found in eqn. (6.15), we get

$$F_{AB} = -F_{DC} - F_{AC} \cos \theta = 45 \text{ kN} + 25\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 70 \text{ kN}.$$

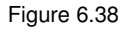
2. *On a computer:* We can write the three equations in the matrix form:

$$\underbrace{\begin{bmatrix} 1 & 1 & \cos \theta \\ 0 & 0 & \sin \theta \\ 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} F_{AB} \\ F_{DC} \\ F_{AC} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ P_1 - O_y \\ -O_y \end{bmatrix}}_{\mathbf{b}} = \underbrace{\begin{bmatrix} 0 \\ -25 \\ -45 \end{bmatrix}}_{\mathbf{b}} \text{ kN}.$$

We can now solve this matrix equation on a computer by keying in matrix A (with θ specified as $\pi/4$) and vector b as input and solving for x. ①

① Pseudocode:

```
A = [1 1 cos(pi/4)
      0 0 sin(pi/4)
      0 1 0]
b = [0 -25 -45]
solve A*x = b for x
```



② In fact, both rods BD and DE are zero-force members. Since they carry no tension, rods AD and DF must also be zero-force members for equilibrium of joint D

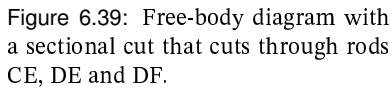


Figure 6.40: Free-body diagram with a sectional cut that cuts through rods BC, CD, DE, and DF.

SAMPLE 6.9 Consider the truss shown in the figure. Rods AB, BC, EC, EF, BD, and DE are each 2 m long, and $\angle ABD = \angle DEF = \pi/2$. Find the tensions in rods DE and CD.

Solution We do not need any analysis to find the tension in rod DE. Since DE is normal to CF, DE has to be a zero-force member for equilibrium of joint E. (2) However, let us find out the same result using the method of sections. Let us take a section just to the left of joint E that cuts through rods CE, DE and DF. The free-body diagram of the truss to the right of the section is shown in fig. 6.39.

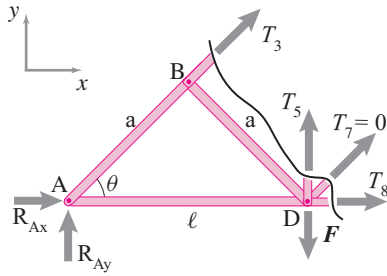
The scalar moment-balance equation, $\sum M_z = 0$, about point F gives at once,

$$aT_7 = 0 \quad \Rightarrow \quad T_7 = 0.$$

Thus rod CE is tension free. Now, we make another cut, taking the section shown in fig. 6.40 to determine the tension in rod CD. Since $T_7 = 0$, we can write the scalar moment-balance equation in the z-direction about point A to give

$$\begin{aligned}\ell T_5 - \ell F &= 0 \\ \Rightarrow T_5 &= F = 10 \text{ kN.}\end{aligned}$$

$$T_7 = 0, \quad T_5 = 10 \text{ kN}$$



6.2 Method of sections

Preparatory Problems

6.2.1 What is the method of sections?

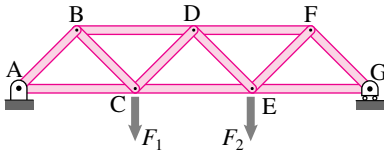
6.2.2 When is the method of sections most useful?

6.2.3 With the free-body diagram associated with one section cut how many bar tensions can you hope to find?

6.2.4 Given a truss and a particular bar in that truss

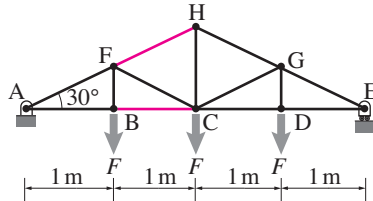
- Can you always find one section cut with which you can find the desired bar tension?
- If so, how do you find that cut? If not, why not?
- Whichever your answer above, give an example of a bar in a truss that illustrates your point.

6.2.5 This problem is exactly the same as Sample 6.3 where it was solved using method of joints. The truss is made up of five horizontal and six inclined rods. All inclined rods are 1 m long and at right angles to each other. The truss carries two vertical loads, $F_1 = 4$ kN and $F_2 = 1$ kN as shown. Find the tensions in rods CE, DE, and DF.



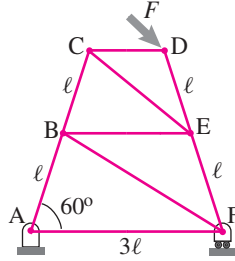
Problem 6.2.5

6.2.6 For the truss shown in the figure, find the tensions in rods BC and FH, assuming $F = 10$ kN.



Problem 6.2.6

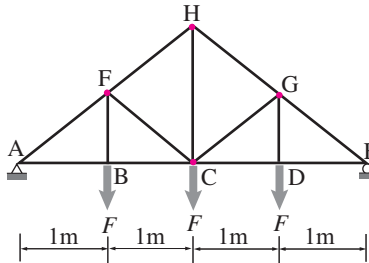
6.2.7 A force $F = 3$ kN acts at 45° with the horizontal at joint D of the truss shown in the figure. Find the tension in rod BE.



Problem 6.2.7

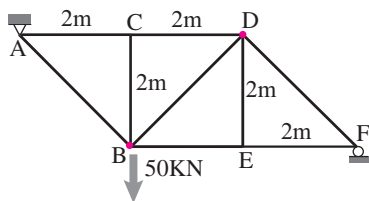
More-Involved Problems

6.2.8 For the 2m high truss shown, find the forces in bars FH, FB, and BC. Use $F = 10$ kN. Now pretend that bars FC and CG are removed and two new bars BH and HD are put in. Find the forces in bars FH, FB, and BC again. Are the forces different now? Why?



Problem 6.2.8

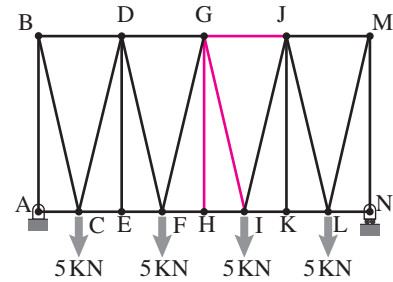
6.2.9 Find the forces in bars BC and BD in the truss shown in the figure. How does the force change in each of these bars if the load is moved to joint B from joint E?



Problem 6.2.9

6.2.10 For the truss shown in the figure, assume that $AC=CE=1$ m, and $AB=BD=2$ m. The rest of the bays are

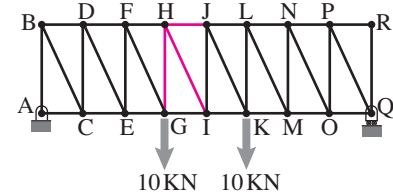
identical to bay ABDE. For the given loads, find the tensions in rods GH, GI, and GJ. [Hint: you can use information about zero-force members.]



Problem 6.2.10

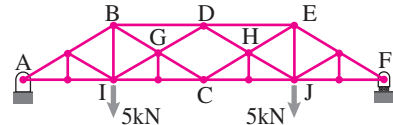
6.2.11 Consider the truss shown in Problem 6.2.6. Find the tension in rod CH. [Hint: you may have to use multiple sections or solve Problem 6.2.6 first.]

6.2.12 The truss shown in the figure consists of 8 'N' bays. In each bay, the vertical rod is 2 m long and the horizontal rod is 1 m long. For the given loads, find the tensions in rods HJ, HI, and GH.



Problem 6.2.12

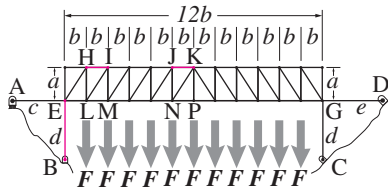
6.2.13 A complex symmetric truss spanning a length of 16 m is shown in the figure. The outermost inclined rods make an angle of 30° with the horizontal. Find the tension in rod BD. [Note: you may have to use more than one section to get the answer.]



Problem 6.2.13

6.2.14 The 2D truss shown consists of 12 diagonally braced rectangles (each a high and b wide). Thus the slope of the diagonal elements is a/b . The whole structure is supported by 4 bars (with lengths c , d , and e as marked). The loading is idealized as 11 identical loads F shown. Give your answers in terms of some or all of a , b , c , d , e and F .

- On a sketch of the figure below clearly mark all the zero-force members (put a '0' on the middle of each bar that has a 'bar force' of zero).
- Find the 'bar-force' in bar EB. *
- Find the 'bar-force' in bar HI. *
- Find the 'bar-force' in bar JK.
[Hint: Use the method of sections and, to reduce calculations, replace a group of the F forces with a single equivalent force.] *



Problem 6.2.14

6.2.1 What is the method of sections?

5.2.1 The method of sections is a tool used for determining tensions in the bars of a truss system. It involves passing an imaginary line through the truss, cutting it into sections, each of which includes more than one joint. For the truss to be in equilibrium, each sub-section thus formed should also be in equilibrium.

The method of sections is a method through which a certain bar or set of bars tensions can be found through one or more joints and less than the whole structure.

6.2.2 When is the method of sections most useful?

The method of sections is most useful in the case where the tension of only a few bars of the truss has to be found. Additionally it is useful in simple trusses.

6.2.3 With the free-body diagram associated with one section cut how many bar tensions can you hope to find?

With a single section cut, the tensions of up to 3 bars can be found.

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

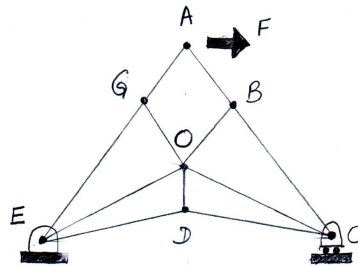
6.2.4 Given a truss and a particular bar in that truss

- a) Can you always find one section cut with which you can find the desired bar tension?
- b) If so, how do you find that cut? If not, why not?
- c) Whichever your answer above, give an example of a bar in a truss that illustrates your point.

5.2.4 a) No, it is not always possible to find one section cut with which the desired bar tension can be found.

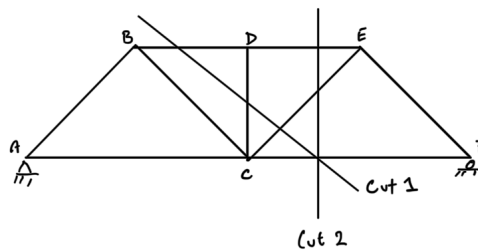
b) It is not always possible to find a section cut because the method of sections has an upper limit of 3 bars through which it can pass. In complex geometries it can often be impossible to find a section cut that passes through 3 bars or less.

c)



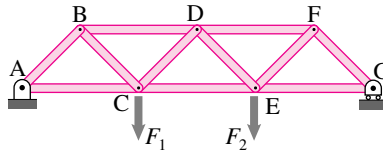
In the given truss system, it is not possible to cut a section which can help us find the tension in member OD.

- (a) Not necessarily, in certain cases more than one cut may be needed.
- (b) The first cut would be such that it cuts the desired bar and the least number of total bars. The next set of cuts then cut one of the bars cut by the first cut, cutting the least number of bars. This process continues until only 3 bars are cut.
- (c) Example Truss :



6.2.5 This problem is exactly the same as Sample ?? where it was solved using method of joints. The truss is made up of five horizontal and six inclined rods. All inclined rods are 1 m long and at right angles to each other. The truss carries two vertical loads, $F_1 = 4 \text{ kN}$ and $F_2 = 1 \text{ kN}$ as shown. Find the tensions in rods CE,

DE, and DF.



Problem 6.5

5.2.5

Given that all the inclined rods are 1 m long and right angles to each other.

$F_1 = 4 \text{ kN}$ and $F_2 = 1 \text{ kN}$.

Find the tensions in rods CE, DE & DF.

Consider the FBD of the truss.

Apply force balance for truss, $\sum \mathbf{F} = \mathbf{0}$

$$\Rightarrow \{ R_1 \hat{i} + R_2 \hat{j} + R_3 \hat{j} - F_1 \hat{j} - F_2 \hat{j} = \mathbf{0} \}$$

$$\{ \} \cdot \hat{i} \Rightarrow R_1 = 0$$

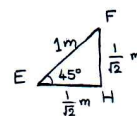
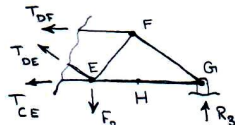
$$\{ \} \cdot \hat{j} \Rightarrow R_2 + R_3 = F_1 + F_2 = 5 \text{ kN}$$

Applying Moment Balance about A, $\sum M_A = 0$

$$\Rightarrow \{ \sqrt{2} \text{ m} \hat{i} \times -F_1 \hat{j} + 2\sqrt{2} \text{ m} \hat{i} \times -F_2 \hat{j} + 3\sqrt{2} \text{ m} \hat{i} \times R_3 \hat{j} = 0 \}$$

$$\Rightarrow \{ \} \cdot \hat{k} \Rightarrow R_3 = \frac{F_1 + 2F_2}{3} \Rightarrow R_3 = 2 \text{ kN} \Rightarrow R_2 = 3 \text{ kN}$$

To find the tensions in rods CE, DE and DF, we use method of sections. The section is cut through CE, DE and DF. Consider the FBD of the cut truss consisting of E, F and G joints as shown.



Applying Moment Balance about E, $\sum M_E = 0$

$$\Rightarrow \{ \frac{1}{\sqrt{2}} \text{ m} \hat{j} \times -T_{DF} \hat{i} + \sqrt{2} \text{ m} \hat{i} \times R_3 \hat{j} = 0 \}$$

$$\{ \} \cdot \hat{k} \Rightarrow T_{DF} = -4 \text{ kN}$$

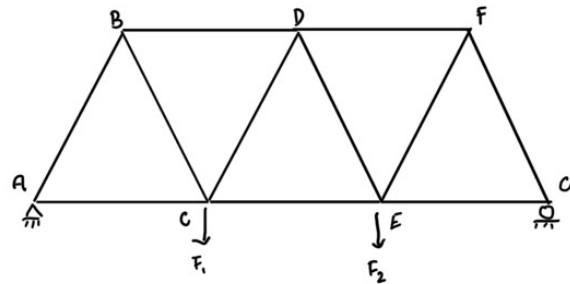
Applying force balance, $\sum \mathbf{F} = \mathbf{0}$

$$\Rightarrow \{ -T_{CE} \hat{i} - T_{DF} \hat{i} - T_{DE} \cos 45^\circ \hat{i} + T_{DE} \sin 45^\circ \hat{j} - F_2 \hat{j} + R_3 \hat{j} = \mathbf{0} \}$$

$$\{ \} \cdot \hat{j} \Rightarrow T_{DE} = \frac{(F_2 - R_3)}{\sin 45^\circ} \Rightarrow T_{DE} = -\sqrt{2} \text{ kN}$$

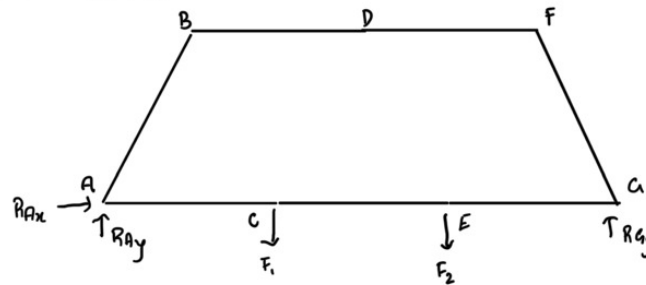
$$\{ \} \cdot \hat{i} \Rightarrow -T_{CE} - T_{DF} - \frac{T_{DE}}{\sqrt{2}} = 0 \Rightarrow T_{CE} = 5 \text{ kN}$$

Sketch:



To Find: T_{CE}, T_{CD}, T_{DF}

FBD (Whole structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow R_{Ax} \hat{i} + (R_{Ay} - 4\text{ kN} - 1\text{ kN} + R_{gy}) = 0$$

$$\Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_{Ay} + R_{gy} = 5\text{ kN}$$

$$\sum \vec{M}_A = (\vec{r}_{AC} \times (-F_1)\hat{j}) + (\vec{r}_{AE} \times (-F_2)\hat{j}) + (\vec{r}_{AG} \times R_{gy}\hat{j}) = \vec{0}$$

$$\Rightarrow (1\text{ m}\hat{i} \times -4\text{ kN}\hat{j}) + (2\text{ m}\hat{i} \times -1\text{ kN}\hat{j}) + (3\text{ m}\hat{i} \times R_{gy}\hat{j}) = \vec{0}$$

$$\Rightarrow -4\text{ m kN}\hat{k} - 2\text{ m kN}\hat{k} + 3R_{gy}\text{ m}\hat{k} = \vec{0}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\Rightarrow 3 R_{Ay} \text{ m } \hat{k} = 6 \text{ m kN } \hat{k}$$

$$\Rightarrow R_{Ay} = 2 \text{ kN}$$

$$\Rightarrow R_{Ax} = 3 \text{ kN}$$

Find T_{CE} , T_{CD} , T_{DF} ,

$$\sum F_x = 0$$

$$\Rightarrow -T_{EC} - T_{ED} \cos 60 - T_{FD} = 0$$

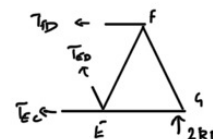
$$\sum F_y = 0$$

$$\Rightarrow T_{ED} \sin 60 + 2 \text{ kN} = 0$$

$$\Rightarrow T_{ED} = -\frac{4}{\sqrt{3}} \text{ kN}$$

$$T_{DE} = \frac{4}{\sqrt{3}} \text{ kN}$$

FBD (section):



$$\sum \vec{M}_E = (\vec{r}_{EF} \times R_{Ay} \hat{j}) + (\vec{r}_{ED} \times \vec{T}_{FD}) = \vec{0}$$

$$\Rightarrow (2 \text{ kN m} + T_{FD} \text{ m}) \hat{k} = \vec{0}$$

$$\Rightarrow T_{FD} = -2 \text{ kN}$$

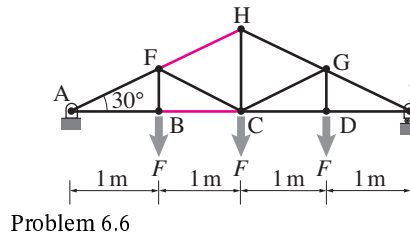
$$T_{DF} = 2 \text{ kN}$$

$$\Rightarrow T_{EC} = 2 \left(1 + \frac{1}{\sqrt{3}} \right) \text{ kN}$$

$$T_{CE} = -2 \left(1 + \frac{1}{\sqrt{3}} \right) \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

6.2.6 For the truss shown in the figure, find the tensions in rods BC and FH, assuming $F = 10$ kN.



Problem 6.6

5.2.6 To find reactions at A & E

In fig (1)

$$[\sum \vec{M}_A = \vec{0}] \cdot \hat{k}$$

$$\Rightarrow -F - 2F - 3F + R_E \times 4 = 0$$

$$\Rightarrow \boxed{R_E = \frac{3F}{2}}$$

$$[\sum \vec{F} = \vec{0}] \cdot \hat{i}$$

$$\Rightarrow \boxed{R_{Ax} = 0}$$

$$[\sum \vec{F} = \vec{0}] \cdot \hat{j}$$

$$\Rightarrow R_{Ay} + R_E - 3F = 0$$

$$\Rightarrow \boxed{R_{Ay} = \frac{3F}{2}}$$

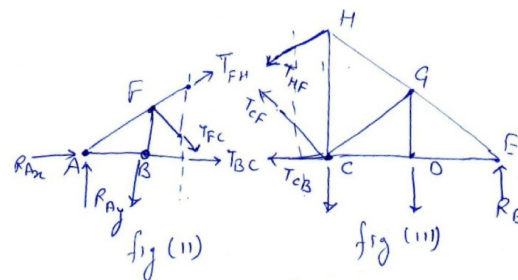
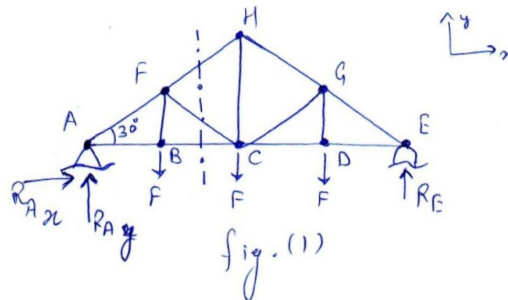
Now In figure (11)

$$[\sum \vec{M}_F = \vec{0}] \cdot \hat{k}$$

$$\Rightarrow \left[-\frac{3F}{2} + T_{BC} \tan 30^\circ \right] = 0$$

$$T_{BC} = \frac{3F}{2 \tan 30^\circ} = 25.98 \text{ kN}$$

$$\boxed{T_{BC} = 25.98 \text{ kN}}$$



In figure (111)

$$[\sum \vec{M}_C = \vec{0}] \cdot \hat{k}$$

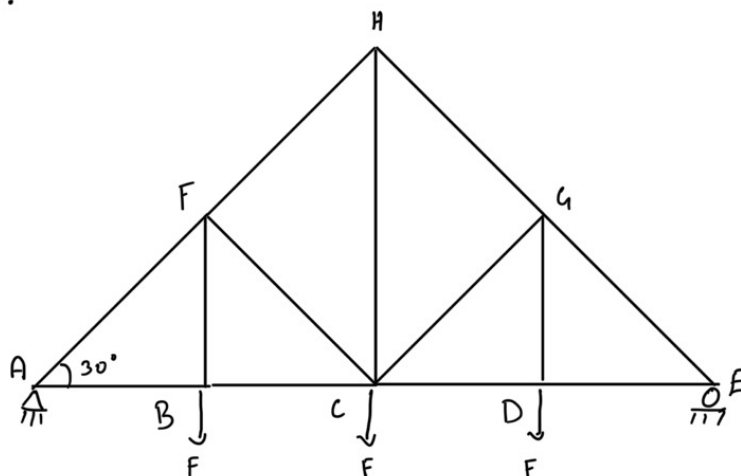
$$\Rightarrow 2 \sin 30^\circ \cdot T_{FH} - F + \frac{3F}{2} \times 2 = 0$$

$$\Rightarrow T_{FH} = \frac{-F}{\sin 30^\circ}$$

$$T_{FH} = -2F$$

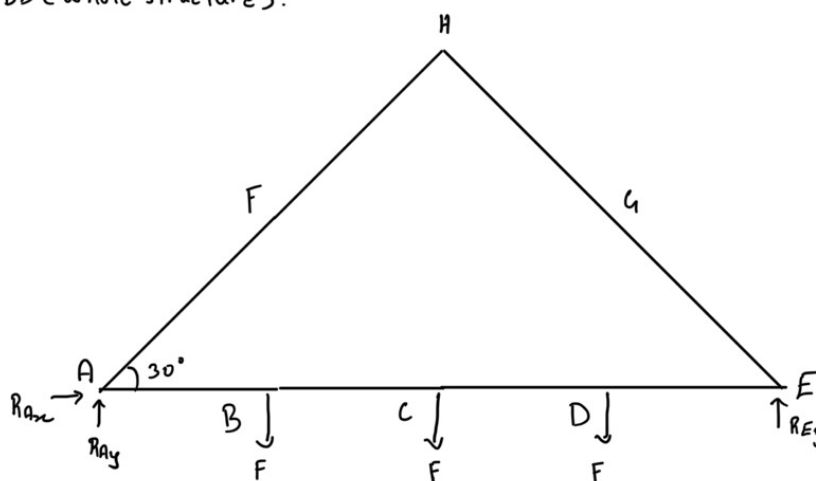
$$\boxed{T_{FH} = -20 \text{ kN}}$$

Sketch :



To Find : T_{BC} , T_{FH}

FBD (Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_{Ay} - 10\text{ kN} - 10\text{ kN} - 10\text{ kN} + R_{Ey} = 0$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\Rightarrow R_{Ay} + R_{Ey} = 30 \text{ kN}$$

$$\sum \vec{M}_A = (\vec{r}_{AB} \times (-F)\hat{j}) + (\vec{r}_{AC} \times (-F)\hat{j}) + (\vec{r}_{AD} \times (-F)\hat{j}) + (\vec{r}_{AE} \times R_{Ey}\hat{j}) = \vec{0}$$

$$\Rightarrow (10 \text{ kN} - 20 \text{ kN} - 30 \text{ kN} + 4 R_{Ey})\hat{k} = \vec{0}$$

$$\Rightarrow 4 R_{Ey} = 60 \text{ kN}$$

$$\Rightarrow R_{Ey} = 15 \text{ kN}$$

$$\Rightarrow R_{Ay} = 15 \text{ kN}$$

Find T_{FH} , T_{BC} ,

$$\sum F_x = 0$$

$$\Rightarrow T_{BC} + T_{FH} \cos 30 + T_{FC} \cos 30 = 0 \quad - (i)$$

$$\sum F_y = 0$$

$$= T_{BC} + T_{FH} \sin 30 - T_{FC} \sin 30 = -30 \text{ kN} \quad - (ii)$$

$$\sum \vec{M}_F = (\vec{r}_{FA} \times (R_{Ax}\hat{i} + R_{Ay}\hat{j})) + (\vec{r}_{FB} \times T_{BC}\hat{i}) = \vec{0}$$

$$= (-\hat{i} - \tan 30\hat{j}) \times R_{Ay}\hat{j} + (-\tan 30\hat{j} \times T_{BC}\hat{i}) = \vec{0}$$

$$\Rightarrow R_{Ay}\hat{k} - T_{BC} \tan 30\hat{k} = \vec{0}$$

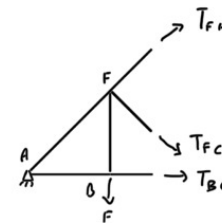
$$T_{BC} = 15\sqrt{3} \text{ kN}$$

$$\sin 30 (i) + \cos 30 (ii)$$

$$30\sqrt{3} \text{ kN} + 2T_{FH} \cos 30 \sin 30 = 30\frac{\sqrt{3}}{2} \text{ kN}$$

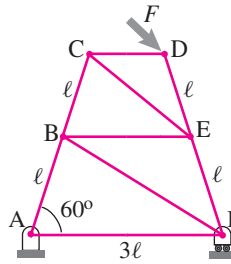
$$T_{FH} = -30 \text{ kN}$$

FBD (Section):



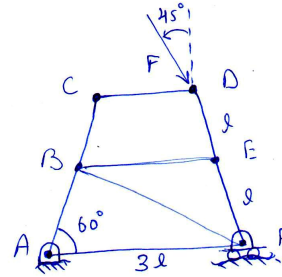
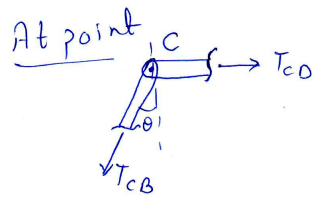
Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

6.2.7 A force $F = 3 \text{ kN}$ acts at 45° with the horizontal at joint D of the truss shown in the figure. Find the tension in rod BE.



Problem 6.7

S.2.7



$$\sum \vec{F} = \vec{0}$$

$$\Rightarrow \{ T_{CD} \hat{i} + T_{BC} \cos \theta (-\hat{j}) + T_{BC} \sin \theta (-\hat{i}) = \vec{0} \}$$

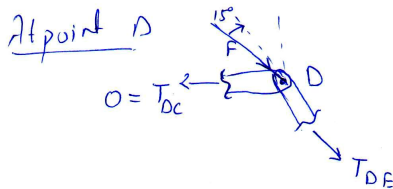
$$\Rightarrow [\sum \vec{F} = \vec{0}] \cdot \hat{j} \Rightarrow -T_{BC} \sin \theta = 0$$

$$\therefore \boxed{T_{BC} = 0}$$

$$\Rightarrow [\sum \vec{F} = \vec{0}] \cdot \hat{i}$$

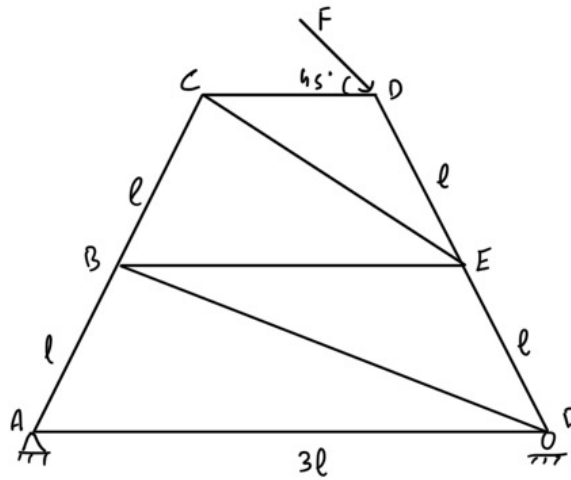
$$\Rightarrow T_{CD} - T_{BC} \sin \theta = 0$$

$$\Rightarrow \boxed{T_{CD} = 0}$$



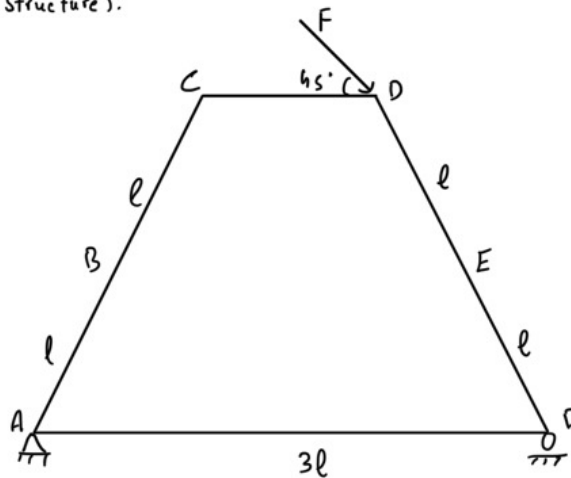
There is no bar tension to balance the part of F that is perpendicular to DE .
So problem is indeterminate, can't be solved.

Sketch:



To Find: T_{BE}

FBD (Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow 3 \text{ kN} \cos 45^\circ + R_{Ax} = 0$$

$$\Rightarrow R_{Ax} = -\frac{3}{\sqrt{2}} \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\sum F_y = 0$$

$$\Rightarrow 3 \text{ kN} \sin 45^\circ + R_{Ay} + R_{Fy} = 0$$

$$\Rightarrow R_{Ay} + R_{Fy} = -\frac{3}{\sqrt{2}} \text{ kN}$$

$$\sum \vec{M}_A = \left(\vec{r}_{AD} \times \left(\frac{1}{\sqrt{2}} \hat{i} - \frac{1}{\sqrt{2}} \hat{j} \right) F \right) + \left(\vec{r}_{AD} \times R_{Dy} \hat{j} \right) = \vec{0}$$

$$= \left((2\ell \cos 60^\circ + \ell) \hat{i} + (2\ell \sin 60^\circ) \hat{j} \right) \times \left(-\frac{3}{\sqrt{2}} \text{ kN} \hat{i} - \frac{3}{\sqrt{2}} \text{ kN} \hat{j} \right) \\ + (3\ell \hat{i} \times R_{Ey} \hat{j}) = \vec{0}$$

$$\Rightarrow 3 \left(\frac{3}{\sqrt{2}} - \sqrt{2} \right) \ell \text{ kN} \hat{k} + 3 R_{Ey} \ell \hat{k} = \vec{0}$$

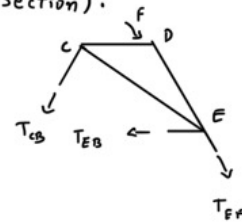
$$\Rightarrow R_{Ey} = \left(3 \sqrt{\frac{3}{2}} - \sqrt{2} \right) \ell \text{ kN}$$

Find T_{EB} ,

$$\sum F_x = 0$$

$$\Rightarrow F \cos 45^\circ - T_{EB} - T_{CB} \cos 75^\circ \\ + T_{EF} \cos 75^\circ = 0 \quad - (i)$$

FBD(Section):



$$\sum F_y = 0$$

$$\Rightarrow -F \sin 45^\circ - T_{CB} \sin 75^\circ - T_{EF} \sin 75^\circ = 0 \quad - (ii)$$

$$\sum \vec{M}_{/E} = \left(\vec{r}_{EC} \times \vec{T}_{CB} \right) + \left(\vec{r}_{ED} \times \vec{F} \right) = \vec{0}$$

$$= \left((-\ell \cos 75^\circ - \ell) \hat{i} + (\ell \sin 75^\circ) \hat{j} \right) \times T_{CB}$$

$$+ (-\ell \cos 75^\circ \hat{i} + \ell \sin 75^\circ \hat{j}) \times (F \cos 45^\circ \hat{i} - F \sin 45^\circ \hat{j}) = \vec{0}$$

$$\Rightarrow [\ell T_{CB} \sin 75^\circ + \ell F (\sin 45^\circ \cos 75^\circ - \sin 75^\circ \cos 45^\circ)] \hat{k} = \vec{0}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\Rightarrow T_{CB} = -F(\sin 45 \cot 75 - \cos 45)$$

$$\sin 75 \text{ (i)} + \cos 75 \text{ (ii)}$$

$$F(\cos 45 \sin 75 - \sin 45 \cos 75) - T_{EB} \sin 75 - T_{CB}(\cos 75 + \sin 75) = 0$$

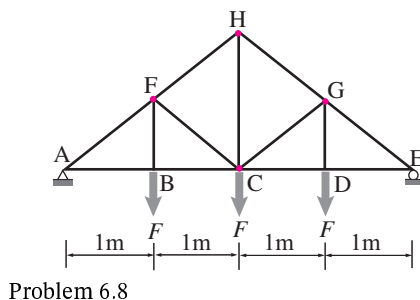
$$\Rightarrow F(\cos 45 \sin 75 - \sin 45 \cos 75 + \sin 45 \cot 75 \cdot \cos 45)(\cos 75 + \sin 75) - T_{EB} \sin 75 = 0$$

$$\Rightarrow T_{EB} = F \left(\cos 45 - \sin 45 \cot 75 + \frac{\sin 45 \cot 75}{\sin 75} - \frac{\cos 45}{\sin 75} \right) (\cot 75 + 1)$$

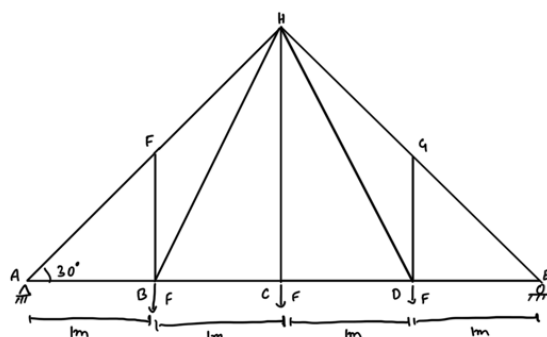
$$T_{BE} = -F \left(\cos 45 - \sin 45 \cot 75 + \frac{\sin 45 \cot 75}{\sin 75} - \frac{\cos 45}{\sin 75} \right) (\cot 75 + 1)$$

$$= -0.023F$$

6.2.8 For the 2m high truss shown, find the forces in bars FH, FB, and BC. Use $F = 10\text{ kN}$. Now pretend that bars FC and CG are removed and two new bars BH and HD are put in. Find the forces in bars FH, FB, and BC again. Are the forces different now? Why?

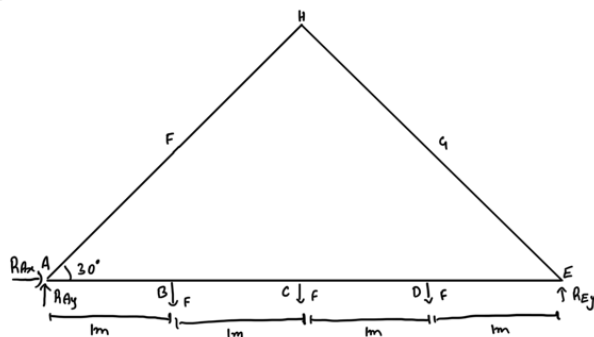


Sketch:



To Find: T_{FH}, T_{FB}, T_{BC}

FBD (Whole Structure):



Finding Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_{Ay} - 30\text{ kN} + R_{Ey} = 0$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\Rightarrow R_{Ay} + R_{Ey} = 30 \text{ kN}$$

$$\sum \vec{M}_A = (\vec{r}_{AB} \times (-F)\hat{j}) + (\vec{r}_{AC} \times (-F)\hat{j}) + (\vec{r}_{AD} \times (-F)\hat{j}) + (\vec{r}_{AE} \times R_{Ey}\hat{j}) = \vec{0}$$

$$\Rightarrow (10 \text{ m kN} + 20 \text{ m kN} + 30 \text{ kN} + 4 \text{ m } R_{Ey})\hat{k} = \vec{0}$$

$$\} \hat{k} \Rightarrow R_{Ey} = -15 \text{ kN}$$

$$\Rightarrow R_{Ay} = 45 \text{ kN}$$

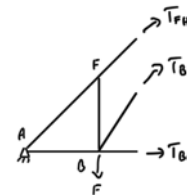
Find T_{BC} and T_{FH} ,

$$\sum F_x = 0$$

FBD (Section):

$$T_{BC} + \frac{T_{BH}}{\sqrt{3}} + \frac{T_{FH}}{\sqrt{2}} = 0 \quad - (i)$$

$$\sum F_y = 0$$



$$\frac{2 T_{BH}}{\sqrt{3}} + \frac{T_{FH}}{\sqrt{2}} + 45 \text{ kN} = 0 \quad - (ii)$$

$$\sum \vec{M}_A = (\vec{r}_{AB} \times (-F)\hat{j}) + (\vec{r}_{AB} \times (\frac{1}{\sqrt{3}}\hat{i} + \frac{2}{\sqrt{3}}\hat{j})T_{BH})$$

$$= (1 \text{ m } \hat{i} \times -10 \text{ kN } \hat{j}) + (1 \text{ m } \hat{i} \times (\frac{1}{\sqrt{3}} T_{BH} \hat{i} + \frac{2}{\sqrt{3}} T_{BH} \hat{j}))$$

$$\Rightarrow -10 \text{ kN m } \hat{k} + \frac{2}{\sqrt{3}} T_{BH} \hat{k} = \vec{0}$$

$$\Rightarrow T_{BH} = 5\sqrt{3} \text{ kN}$$

(i) - (ii)

$$T_{BC} - T_{BH} \left(\frac{-1}{\sqrt{3}} \right) = 45 \text{ kN}$$

$$T_{BC} = 40 \text{ kN}$$

Substituting in (ii)

$$T_{FH} = -55\sqrt{2} \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Find T_{BF} ,

$$\sum F_x = 0$$

$$\Rightarrow T_{BC} + \frac{T_{BH}}{\sqrt{3}} + \frac{T_{AH}}{\sqrt{2}} = 0$$

$$\Rightarrow T_{AH} = -55\sqrt{2} \text{ kN}$$

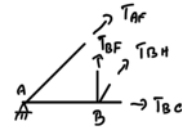
$$\sum F_y = 0$$

$$\Rightarrow \frac{2T_{BH}}{\sqrt{3}} + \frac{T_{AH}}{\sqrt{2}} + T_{BF} = 0$$

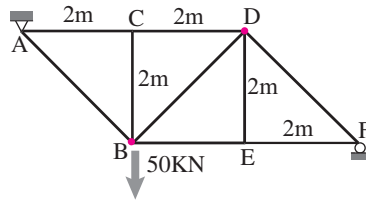
$$\Rightarrow T_{BF} = 45 \text{ kN}$$

$T_{FB} = -45 \text{ kN}$

FBD (Section) :



6.2.9 Find the forces in bars BC and BD in the truss shown in the figure. How does the force change in each of these bars if the load is moved to joint B from joint E?



Problem 6.9

5.2.9

Case 1: Find tension in bars BC & BD when the load is at E.

Applying force balance to the truss,

$$\sum \mathbf{F} = \mathbf{0} \Rightarrow \{R_1 \hat{i} - R_2 \hat{j} - 50 \text{ kN} \hat{j} + R_3 \hat{j} = \mathbf{0}\}$$

$$\{ \cdot \hat{i} \Rightarrow R_1 = 0 \quad (1)$$

$$\{ \cdot \hat{j} \Rightarrow R_3 - R_2 = 50 \text{ kN} \quad (2)$$

Applying Moment balance about A, $\sum M_A = 0$

$$\Rightarrow \{ 4 \text{ m} \hat{i} \times -50 \text{ kN} \hat{j} + 6 \text{ m} \hat{i} \times R_3 \hat{j} = \mathbf{0} \}$$

$$\{ \cdot \hat{k} \Rightarrow R_3 = 33.33 \text{ kN} \quad (3)$$

$$\text{From (2) and (3), } R_2 = -16.67 \text{ kN} \quad (4)$$

Consider the FBD with the section cut through AB, BC and CD.

Moment balance about C, $\sum M_C = 0$

$$\Rightarrow -2 \text{ m} \hat{i} \times -R_2 \hat{j} + [\sqrt{2} \text{ m} \cos 45^\circ (-\hat{i}) + \sqrt{2} \text{ m} \sin 45^\circ (-\hat{j})] \times (T_{AB} \cos 45^\circ \hat{i} - T_{AB} \sin 45^\circ \hat{j}) = 0$$

$$\Rightarrow 2 \text{ m} R_2 \hat{k} + \sqrt{2} \text{ m} \frac{1}{2} T_{AB} \hat{k} + \sqrt{2} \text{ m} \frac{1}{2} T_{AB} \hat{k} = 0$$

$$\Rightarrow T_{AB} = -\sqrt{2} R_2 \Rightarrow T_{AB} = 23.57 \text{ kN} \quad (5)$$

Force balance, $\sum \mathbf{F} = \mathbf{0} \Rightarrow \{ T_{AB} \cos 45^\circ \hat{i} - T_{AB} \sin 45^\circ \hat{j} - T_{BC} \hat{i} + T_{CD} \hat{i} - R_2 \hat{j} = \mathbf{0} \}$

$$\{ \cdot \hat{i} \Rightarrow T_{CD} = -\frac{T_{AB}}{\sqrt{2}} = R_2 \Rightarrow T_{CD} = -16.67 \text{ kN}$$

$$\{ \cdot \hat{j} \Rightarrow T_{BC} = -\frac{T_{AB}}{\sqrt{2}} - R_2 \Rightarrow \boxed{T_{BC} = 0 \text{ N}}$$

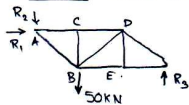
To find T_{BD} , Consider the FBD with the section cut through

CD, BD and BE.

Applying Force balance in $\hat{j}$ direction, $T_{BD} \sin 45^\circ - R_2 = 0$

$$\Rightarrow \boxed{T_{BD} = -23.57 \text{ kN}}$$

Case 2: Load is moved to 'B'



Apply Moment balance about A, $\sum M_A = 0 \Rightarrow \{ 2 \text{ m} \hat{i} \times -50 \text{ kN} \hat{j} + 6 \text{ m} \hat{i} \times R_3 \hat{j} = \mathbf{0} \} \cdot \hat{k}$

$$\Rightarrow R_3 = 16.67 \text{ kN}$$

Using LMB (2) $R_2 = -33.33 \text{ kN}$.

As the analysis of the section cut through AB, BC & CD done in case 1 is

still applicable, $T_{AB} = -\sqrt{2} R_2 = 47.13 \text{ kN}$ and $\boxed{T_{BC} = 0 \text{ N}}$

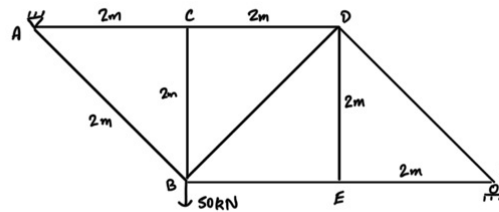
Consider the FBD, applying force balance in $\hat{j}$ direction, $T_{BD} \sin 45^\circ - R_2 - 50 \text{ kN} = 0$

$$\Rightarrow \boxed{T_{BD} = 23.57 \text{ kN}}$$

When load is moved to B, compression in BD changes to tension of same magnitude.

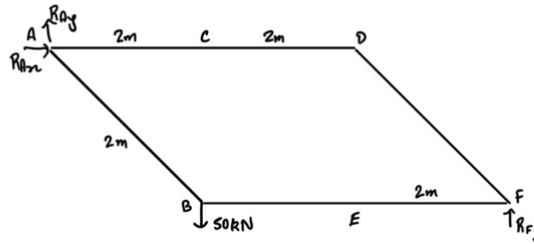
Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Sketch:



To Find : T_{BC}, T_{BD}

FBD (Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_{Ay} + R_{Fy} = 50 \text{ kN}$$

$$\sum \vec{M}_A = \vec{0}$$

$$= (2\hat{i} - 2\hat{j}) \times (-50\text{kN}\hat{j}) + (6\hat{i} - 2\hat{j}) \times R_{Fy}$$

$$= (-100 \text{ kNm} + 6R_{Fy})\hat{k} = \vec{0}$$

$$\Rightarrow R_{Fy} = \frac{50}{3} \text{ kN}$$

$$\Rightarrow R_{Ay} = \frac{100}{3} \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Find T_{CB} ,

$$\sum F_x = 0$$

$$\Rightarrow T_{CD} + \frac{T_{AB}}{\sqrt{2}} = 0$$

$$\sum F_y = 0$$

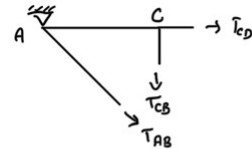
$$\Rightarrow -T_{CB} - \frac{T_{AB}}{\sqrt{2}} + \frac{100}{\sqrt{3}} \text{ kN} = 0$$

$$\begin{aligned} \sum \vec{M}_A &= \vec{r}_{AC} \times T_{CB} \hat{j} \\ &= 2\text{m} \hat{i} \times T_{CB} \hat{j} = 0 \end{aligned}$$

$$\Rightarrow T_{CB} = 0$$

$$T_{BC} = 0 \text{ N}$$

FBD (Section):

Find T_{DB} ,

$$\sum F_x = 0$$

$$\Rightarrow -T_{EB} - \frac{T_{DB}}{\sqrt{2}} - T_{DC} = 0$$

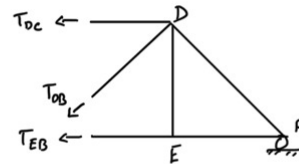
$$\sum F_y = 0$$

$$\Rightarrow -\frac{T_{DB}}{\sqrt{2}} + R_{fy} = 0$$

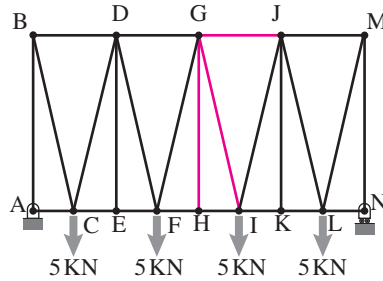
$$\Rightarrow T_{DB} = \frac{50\sqrt{2}}{3} \text{ kN}$$

$$T_{BD} = -\frac{50\sqrt{3}}{3} \text{ kN}$$

FBD (Section):

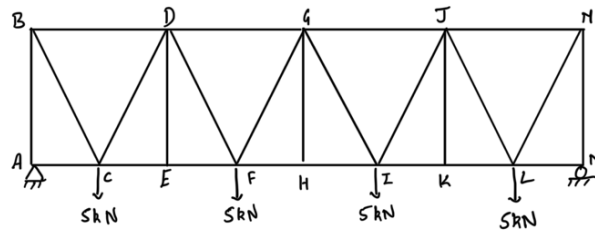


6.2.10 For the truss shown in the figure, assume that $AC=CE=1\text{ m}$, and $AB=BD=2\text{ m}$. The rest of the bays are identical to bay ABDE. For the given loads, find the tensions in rods GH, GI, and GJ. [Hint: you can use information about zero-force members.]



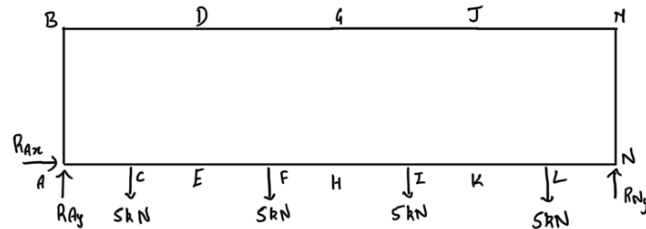
Problem 6.10

Sketch:



To Find : T_{GH}, T_{GI}, T_{GJ}

FBD (Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$\Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_{Ay} - 20\text{ kN} + R_{Ny} = 0$$

$$\Rightarrow R_{Ay} + R_{Ny} = 20\text{ kN}$$

$$\begin{aligned} \sum \vec{M}_A &= (\vec{r}_{AC} \times (-5\text{ kN})\hat{j}) + (\vec{r}_{AF} \times (-5\text{ kN})\hat{j}) + (\vec{r}_{AI} \times (-5\text{ kN})\hat{j}) \\ &\quad + (\vec{r}_{AL} \times (-5\text{ kN})\hat{j}) + (\vec{r}_{AN} \times R_{Ny}\hat{j}) = \vec{0} \\ &= (1\text{ m}\hat{i} \times (-5\text{ kN})\hat{j}) + (3\text{ m}\hat{i} \times (-5\text{ kN})\hat{j}) + (5\text{ m}\hat{i} \times (-5\text{ kN})\hat{j}) \end{aligned}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$+ (7m\hat{i} \times (-5kN)\hat{j}) + (8m\hat{i} \times R_{Ny}\hat{j}) = \vec{0}$$

$$= ((-5 - 15 \cdot 25 - 35) kN + 8 R_{Ny}) m \hat{k} = \vec{0}$$

$$\Rightarrow R_{Ny} = 10 kN$$

$$\Rightarrow R_{Ay} = 10 kN$$

Find T_{QH} ,

Since Vertical bars are zero force members,

$$T_{QH} = 0 N$$

Finding T_{I4} , T_{J4}

$$\sum F_x = 0$$

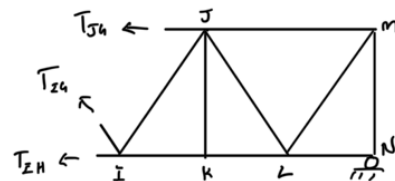
$$\Rightarrow -T_{IH} - \frac{T_{I4}}{\sqrt{3}} - T_{J4} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow \frac{2T_{I4}}{\sqrt{3}} + R_{Ny} = 0$$

$$T_{I4} = -5\sqrt{3} kN$$

FBD (Section):



$$\sum \vec{M}_{/I} = (\vec{r}_{IN} \times R_{Ny}\hat{j}) + (\vec{r}_{LJ} \times (-T_{J4})\hat{i})$$

$$= (3m\hat{i} \times 10kN\hat{j}) + ((1m\hat{i} + 2m\hat{j}) \times (-T_{J4})\hat{i})$$

$$\Rightarrow (30kN + 2T_{J4})m\hat{k} = \vec{0}$$

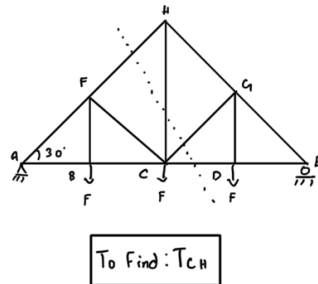
$$\Rightarrow T_{J4} = -15kN$$

$$T_{4J} = 15kN$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

6.2.11 Consider the truss shown in Problem 6.2.6. Find the tension in rod CH. [Hint: you may have to use multiple sections or solve Problem 6.2.6 first.]

Sketch:

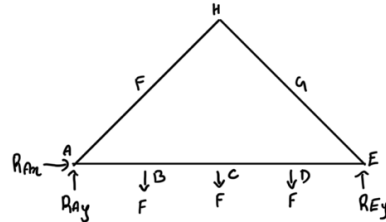


From 6.2.6,

$$R_{Ey} = 15 \text{ kN}$$

$$T_{HF} = 30 \text{ kN}$$

FBD (Whole Structure):



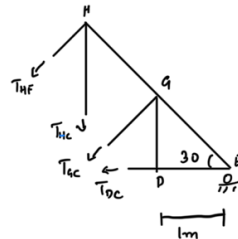
Finding T_{HC} ,

$$\sum F_x = 0$$

$$T_{HF} \cos 60 - T_{HC} \cos 60 - T_{DC} = 0$$

$$\frac{T_{HC}}{2} + T_{DC} = -15 \text{ kN} \quad (i)$$

FBD (Section):



$$\sum F_y = 0$$

$$-T_{HC} - T_{HF} \sin 60 - T_{GC} \sin 60 + 30 \text{ kN} = 0$$

$$-T_{HC} - 15\sqrt{3} \text{ kN} - \sqrt{3} \frac{T_{HC}}{2} + 30 \text{ kN} = 0$$

$$T_{HC} + \frac{\sqrt{3}}{2} T_{HC} = 15(\sqrt{3} - 2) \text{ kN}$$

$$\sum \vec{M}_H = \vec{0}$$

$$= (\vec{r}_{HG} \times \vec{T}_{GC}) + (\vec{r}_{HD} \times \vec{T}_{DC}) + (\vec{r}_{HE} \times R_{Ey} \hat{j}) = \vec{0}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\begin{aligned}
 &= (1\text{m} \cos 60^\circ \hat{i} - 1\text{m} \sin 60^\circ \hat{j}) \times T_{AC} + (\cos 60^\circ \hat{i} - (1\text{m} \sin 60^\circ + \frac{1}{2}\text{m}) \hat{j}) \times T_{DC} \\
 &\quad + (2\text{m} \cos 60^\circ \hat{i} - 2\text{m} \sin 60^\circ \hat{j}) \times R_{E_y} \hat{j} = \vec{0} \\
 &= (-\cos 60^\circ \sin 60^\circ T_{AC} - \sin 60^\circ \cos 60^\circ T_{AC} - \sin 60^\circ T_{DC} + \frac{T_{DC}}{2} + 15\text{m kN}) \hat{k} = \vec{0} \\
 &\Rightarrow -2 \sin 60^\circ \cos 60^\circ T_{AC} - T_{DC} (\sin 60^\circ + \frac{1}{2}) = -15 \text{ kNm} \\
 &\Rightarrow -\frac{\sqrt{3}}{2} T_{AC} - \frac{1+\sqrt{3}}{2} T_{DC} = -15 \text{ kNm} \quad - (ii)
 \end{aligned}$$

$$\sqrt{3} (i) + (ii)$$

$$T_{DC} \left(\sqrt{3} - \frac{1+\sqrt{3}}{2} \right) = -15\sqrt{3} \text{ kNm} - 15 \text{ kNm}$$

$$\Rightarrow T_{DC} (1+\sqrt{3}) = -15(1+\sqrt{3}) \text{ kNm}$$

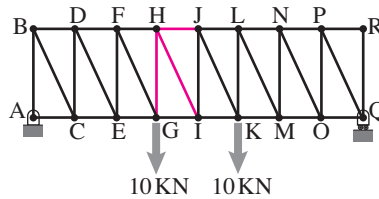
$$\Rightarrow T_{DC} = -15 \text{ kNm}$$

$$\Rightarrow T_{AC} = -60 \text{ kNm}$$

$$\begin{aligned}
 \Rightarrow T_{HC} &= (15(\sqrt{3}-2) - 30\sqrt{3}) \text{ kN} \\
 &= 15(\sqrt{3}-2-2\sqrt{3}) \text{ kN}
 \end{aligned}$$

$$T_{CH} = -15(2+\sqrt{3}) \text{ kN}$$

6.2.12 The truss shown in the figure consists of 8 'N' bays. In each bay, the vertical rod is 2 m long and the horizontal rod is 1 m long. For the given loads, find the tensions in rods HJ, HI, and GH.



Problem 6.12

5.2.12

Find the tensions in rods HJ, HI & GH.
Consider the FBD of the truss shown in the figure.

Applying force balance, $\sum F = 0$

$$\Rightarrow \{R_1 \hat{i} + R_2 \hat{j} - 10\text{ kN} \hat{j} - 10\text{ kN} \hat{j} + R_3 \hat{j} = 0\}$$

$$\{ \cdot \hat{i} \Rightarrow R_1 = 0 \quad \text{--- (1)}$$

$$\{ \cdot \hat{j} \Rightarrow R_2 + R_3 = 20\text{ kN} \quad \text{--- (2)}$$

Applying Moment balance, $\sum M_A = 0$

$$\Rightarrow \{3\text{ m} \hat{i} \times 10\text{ kN} \hat{j} + 5\text{ m} \hat{i} \times 10\text{ kN} \hat{j} + 8\text{ m} \hat{i} \times R_3 \hat{j} = 0\}$$

$$\{ \cdot \hat{k} \Rightarrow -30\text{ kNm} - 50\text{ kNm} + 8\text{ m} R_3 = 0 \Rightarrow R_3 = 10\text{ kN} \quad \text{--- (3)}$$

From (2) and (3), $R_2 = 10\text{ kN}$.

As the loading is symmetric, a quick verification reveals that the vertical reactions at A and Q are 10 kN each.

Consider the FBD of a portion of the truss with the section cut through HJ, HI, and GI as shown in the figure.

Applying Moment balance about H, $\sum M_H = 0$

$$\Rightarrow \{-2\text{ m} \hat{j} \times T_{GI} \hat{i} - 3\text{ m} \hat{i} \times R_2 \hat{j} = 0\}$$

$$\Rightarrow T_{GI} = \frac{3}{2} R_2 = 15\text{ kN} \quad \text{--- (4)}$$

Applying force balance, $\sum F = 0$

$$\Rightarrow \{R_2 \hat{j} - 10\text{ kN} \hat{j} + T_{GI} \hat{i} + T_{HI} \cos \theta \hat{i} - T_{HI} \sin \theta \hat{j} + T_{HJ} \hat{i} = 0\}$$

$$\{ \cdot \hat{j} \Rightarrow R_2 - 10\text{ kN} - T_{HI} \sin \theta = 0 \Rightarrow T_{HI} = 0 \quad \text{--- (5)}$$

$$\{ \cdot \hat{i} \Rightarrow T_{GI} + T_{HJ} \cos \theta + T_{HI} = 0 \Rightarrow T_{HJ} = -15\text{ kN} \quad \text{--- (6) [from (4) & (5)]}$$

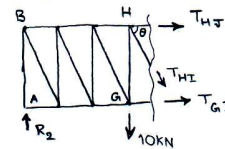
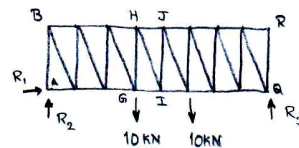
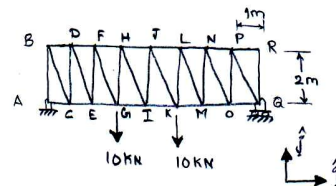
To find the force in GH, Consider the following FBD.

Applying force balance in $\hat{j}$ direction,

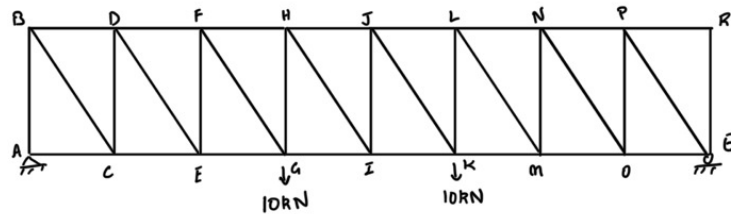
$$\Rightarrow \{R_2 \hat{j} - 10\text{ kN} \hat{j} + T_{GH} \hat{j} = 0\}$$

$$\{ \cdot \hat{j} \Rightarrow 10\text{ kN} - 10\text{ kN} + T_{GH} = 0$$

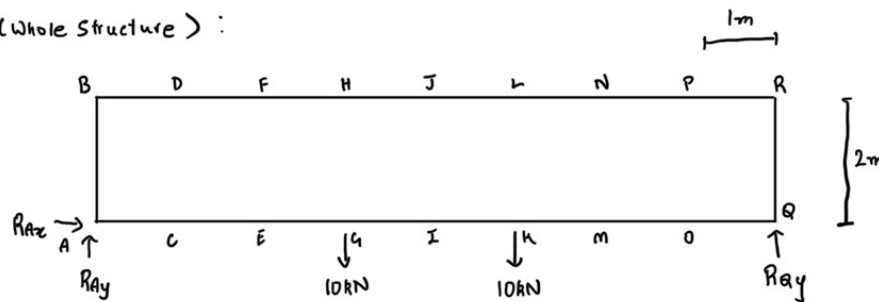
$$\Rightarrow T_{GH} = 0$$



Sketch :

To Find : T_{HI} , T_{HJ} , T_{GH}

FBD (Whole Structure) :



Find Reaction Forces,

$$\sum F_x = 0 \quad \Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Qy} - 20 \text{ kN} = 0$$

$$R_{Ay} + R_{Qy} = 20 \text{ kN}$$

$$\sum \vec{M}_A = \vec{0}$$

$$\sum \vec{M}_A = (\vec{r}_{AG} \times (-10 \text{ kN } \hat{j})) + (\vec{r}_{AK} \times (-10 \text{ kN } \hat{j})) + (\vec{r}_{AQ} \times R_{Qy} \hat{j}) = \vec{0}$$

$$= (3 \text{ m } \hat{i} \times -10 \text{ kN } \hat{j}) + (5 \text{ m } \hat{i} \times -10 \text{ kN } \hat{j}) + (8 \text{ m } \hat{i} \times R_{Qy} \hat{j}) = \vec{0}$$

$$= (-30 \text{ m kN} - 50 \text{ m kN} + 8 R_{Qy} \text{ m}) \hat{k} = \vec{0}$$

$$\Rightarrow R_{Qy} = 10 \text{ kN} \quad R_{Ay} = 10 \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Find T_{HJ} and T_{HI} ,

$$\sum F_x = 0$$

$$T_{HJ} + \frac{T_{HI}}{\sqrt{3}} + T_{GI} = 0$$

$$\sum F_y = 0$$

$$-\frac{2 T_{HI}}{\sqrt{3}} - 10 \text{ kN} + 10 \text{ kN} = 0$$

$$T_{HI} = 0 \text{ kN}$$

$$\sum \vec{M}_H = \vec{0}$$

$$\sum \vec{M}_H = (\vec{r}_{HG} \times T_{GI} \hat{i}) + (\vec{r}_{HA} \times (R_{Ax} \hat{i} + R_{Ay} \hat{j}))$$

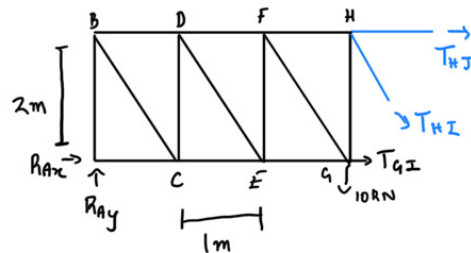
$$= (-2\text{m} \hat{j} \times T_{GI} \hat{i}) + (-3\text{m} \hat{i} - 2\text{m} \hat{j}) \times 10 \text{ kN} \hat{j}$$

$$= (-2\text{m} T_{GI} - 30 \text{ m kN}) \hat{k} = \vec{0}$$

$$\Rightarrow T_{GI} = -15 \text{ kN}$$

$$T_{HJ} = 15 \text{ kN}$$

FBD (Section):

Find T_{GH} ,

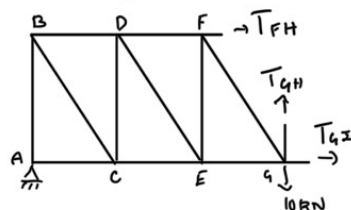
$$\sum F_y = 0$$

$$T_{GH} - 10 \text{ kN} + R_{Ay} = 0$$

$$T_{GH} - 10 \text{ kN} + 10 \text{ kN} = 0$$

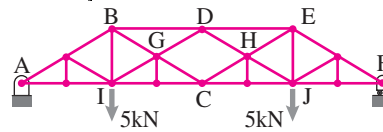
$$T_{GH} = 0 \text{ kN}$$

FBD (Section):



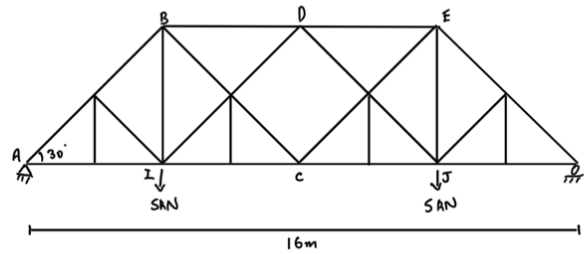
6.2.13 A complex symmetric truss spanning a length of 16 m is shown in the figure. The outermost inclined rods make an angle of 30° with the horizontal. Find the tension in rod BD. [Note: you may have to use more than one section to get

the answer.]



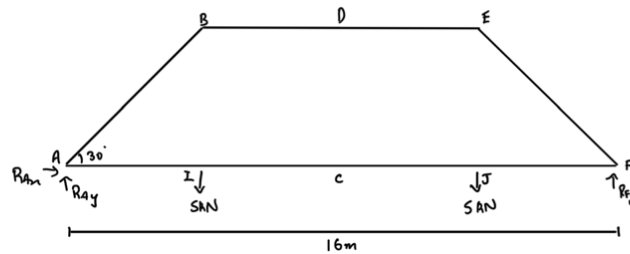
Problem 6.13

Sketch:



To Find : T_{BD}

FBD (Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$= R_{Ax} = 0$$

$$\sum F_y = 0$$

$$= R_{Ay} + R_{Fy} = 10 \text{ kN}$$

$$\sum \vec{M}_A = (\vec{r}_{AI} \times -5\text{kN}\hat{j}) + (\vec{r}_{AJ} \times -5\text{kN}\hat{j}) + (\vec{r}_{AF} \times R_{Fy}\hat{j}) = \vec{0}$$

$$: (4\text{m}\hat{i} \times -5\text{kN}\hat{j}) + (12\text{m}\hat{i} \times -5\text{kN}\hat{j}) + (16\text{m}\hat{i} \times R_{Fy}\hat{j}) = \vec{0}$$

$$: (-20\text{m kN} - 60\text{m kN} + 16\text{m} R_{Fy})\hat{k} = \vec{0}$$

$$\Rightarrow R_{Fy} = 5 \text{ kN}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\Rightarrow R_{Ay} = 5 \text{ kN}$$

Find $T_{x'z}$,

$$\sum F_x = 0$$

$$= T_{x'z} + T_{A'B} \cos 30^\circ + T_{A'z} \cos 30^\circ + T_{BD} = 0$$

$$\sum F_y = 0$$

$$= T_{A'B} \sin 30^\circ + T_{A'z} \sin 30^\circ + 5 \text{ kN} = 0$$

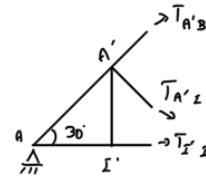
$$\sum \vec{M}_{A'} = (\vec{r}_{AA'} \times R_{Ay}) + (\vec{r}_{A'z} \times T_{x'z}) = \vec{0}$$

$$= (-2\text{m} \hat{i} - 2/\sqrt{3} \hat{j}) \times (5 \text{ kN} \hat{j}) + (2/\sqrt{3} \hat{j} \times T_{x'z}) = \vec{0}$$

$$= (-10 \text{ m kN} - 2T_{x'z}) \hat{k} = \vec{0}$$

$$\Rightarrow T_{x'z} = -5\sqrt{3} \text{ m kN}$$

FBD (Section):



Find T_{BD} ,

$$\sum F_x = 0$$

$$\Rightarrow T_{A'z} \cos 30^\circ + T_{BD} = 5\sqrt{3} \text{ kN}$$

$$\sum F_y = 0$$

$$\Rightarrow -T_{Bz} - T_{A'z} \sin 30^\circ + 5 \text{ kN} = 0$$

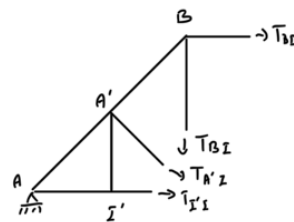
$$\sum \vec{M}_B = (\vec{r}_{BA'} \times (\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) T_{A'z}) + (\vec{r}_{BA} \times (R_{Ax} \hat{i} + R_{Ay} \hat{j})) + (\vec{r}_{Bz} \times T_{x'z} \hat{i})$$

$$= (-2\text{m} \hat{i} - 2\text{m} \tan 30^\circ \hat{j}) \times (\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) T_{A'z}$$

$$+ (-4\text{m} \hat{i} - 4\text{m} \tan 30^\circ \hat{j}) \times (5 \text{ kN} \hat{j})$$

$$+ (-2\text{m} (\hat{i} + 2 \tan 30^\circ \hat{j}) \times (T_{x'z} \hat{i})) = \vec{0}$$

FBD (Section):

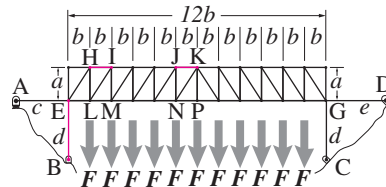


$$= \left(\frac{5T_{B1}}{4} - 20 \text{ kN} - \frac{4}{\sqrt{3}} T_{B2} \right) \hat{k} = \vec{0}$$

$$\Rightarrow T_{B1} = 0 \text{ kN}$$

$$T_{B2} = -5\sqrt{3} \text{ kN}$$

6.2.14 The 2D truss shown consists of 12 diagonally braced rectangles (each a high and b wide). Thus the slope of the diagonal elements is a/b . The whole structure is supported by 4 bars (with lengths c , d , and e as marked). The loading is idealized as 11 identical loads F shown. Give your answers in terms of some or all of a , b , c , d , e and F .



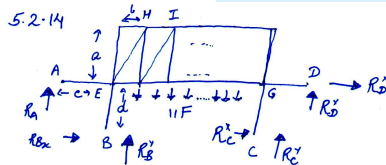
Problem 6.14

Answer:

$$T_{EB} = -11F/2$$

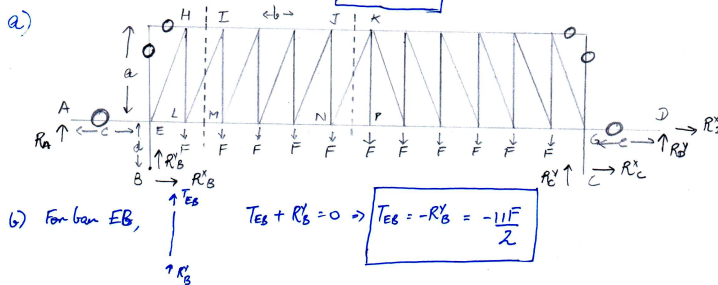
$$T_{HI} = -11bF/2a$$

$$T_{JK} = -35bF/2a, \text{ (more than 3 times the compression of HI)}$$



$$\begin{aligned} \sum \vec{F}_x = \vec{0} &\Rightarrow R_B^x + R_C^x + R_D^x = \vec{0}; R_B^x = 0 \text{ \& } R_C^x = 0 \Rightarrow R_D^x = 0 \\ \sum \vec{F}_y = \vec{0} &\Rightarrow -11F + R_A^y + R_B^y + R_C^y + R_D^y = \vec{0}; R_A^y = 0 \text{ \& } R_D^y = 0 \Rightarrow +11F = R_B^y + R_C^y \\ \sum \vec{M}_B = \vec{0} &\Rightarrow -R_A^x c + 12b R_C^y + (12b + e) R_D^y - R_B^y (d) = (\sum (11)) 6F \\ &\Rightarrow 12b R_C^y = 66Fb \quad \therefore R_C^y = \frac{11F}{2} \end{aligned}$$

$$\begin{aligned} \sum \vec{M}_C = \vec{0} &\Rightarrow (12b + c) (-R_A^x) + 12b (-R_B^y) + e R_D^y + d (-R_D^x) + b (\sum (11)) F = \vec{0} \\ &\Rightarrow 12b R_B^y = 66bF \quad \therefore R_B^y = \frac{11F}{2} \end{aligned}$$



c) For bar HI, we cut a section as shown in the figure

$$\begin{aligned} \sum \vec{F}_x = \vec{0} &\Rightarrow T_{HI} + T_{HI} + T_{HI} \cos \theta = \vec{0} \\ \sum \vec{M}_L = \vec{0} &\Rightarrow T_{HI} a + R_B^y b = \vec{0} \\ &\Rightarrow T_{HI} = -\frac{b}{a} R_B^y \quad \therefore T_{HI} = -\frac{11bF}{2a} \end{aligned}$$

d) For bar JK, we use the method of sections again

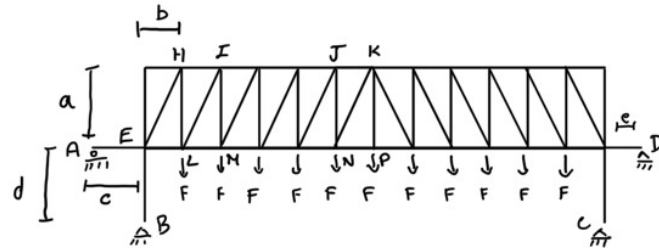
$$\begin{aligned} \sum \vec{M}_N = \vec{0} &\Rightarrow -T_{JK}(a) - R_B^y(5b) + F(4+3+2+1)b = 0 \\ &\Rightarrow -T_{JK}(a) - R_B^y(5b) + F(10b) = 0 \end{aligned}$$

Solving and substituting the value of R_B^y , we get

$$T_{JK} = -\frac{35bF}{2a}$$

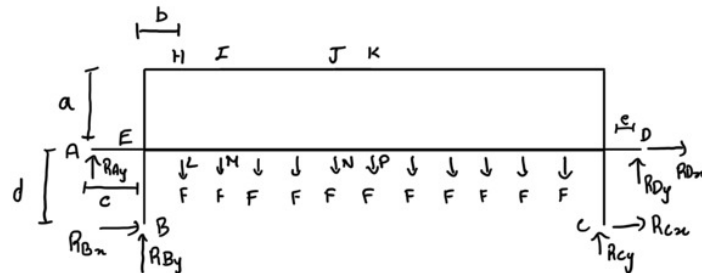
Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Sketch:



To Find: T_{EB} , T_{HI} , T_{JK}

FBD(Whole Structure):



Find Reaction Forces,

$$\sum F_x = 0$$

$$R_{Bx} + R_{Cx} + R_{Dx} = 0$$

$$R_{Bx} = R_{Cx} = 0 \text{ N (} \sum F \text{ at B and C)}$$

$$\Rightarrow R_{Dx} = 0 \quad R_{Bx} = R_{Cx} = R_{Dx} = 0$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{By} + R_{Cy} + R_{Dy} = 11F$$

$$\sum \vec{M}_D = \vec{0}$$

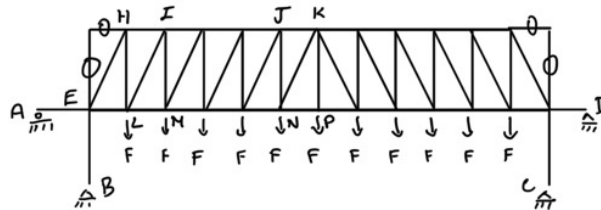
$$= (-e\hat{i} - d\hat{j}) \times R_{Ay}\hat{j} + (-e-b)\hat{i} \times (-F)\hat{j} + (-e-2b)\hat{i} \times (-F)\hat{j} + (-e-3b)\hat{i} \times (-F)\hat{j}$$

$$+ (-e-4b)\hat{i} \times (-F)\hat{j} + (-e-5b)\hat{i} \times (-F)\hat{j} + (-e-6b)\hat{i} \times (-F)\hat{j} + (-e-7b)\hat{i} \times (-F)\hat{j}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

$$\begin{aligned}
 & + (-e-8b)\hat{i} \times (-F)\hat{j} + (-e-9b)\hat{i} \times (-F)\hat{j} + (-e-10b)\hat{i} \times (-F)\hat{j} + (-e-11b)\hat{i} \times (-F)\hat{j} \\
 & + ((-e-12b)\hat{i} + (-d)\hat{j}) \times R_{By}\hat{j} + (-e-12b-c)\hat{i} \times R_{Ay}\hat{j} = \vec{0} \\
 & = (-e(R_{Ay}+R_{By}+R_{Cy})-12b(R_{By}+R_{By})-cR_{Ay}-F(11e+65b))\hat{k} = \vec{0} \\
 & = (-e(R_{Ay}+R_{By}+R_{Cy}+11F)-b(12R_{Ay}+12R_{By}+65F)-cR_{Ay})\hat{k} = \vec{0}
 \end{aligned}$$

a)



b) Since EB is a vertical bar,

$$T_{EB} = 0N$$

c) Find T_{HI} ,

$$\sum F_x = 0$$

$$T_{HI} + T_{LI} + T_{LH} \cos(\tan^{-1}(a/b)) = 0$$

$$\sum F_y = 0$$

$$T_{LH} \sin(\tan^{-1}(a/b)) - F + R_{Ay} + R_{By} = 0$$

$$T_{LH} = \sec(\tan^{-1}(a/b)) [F - R_{Ay} - R_{By}]$$

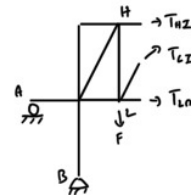
$$\sum \vec{r}_{i/L} \times \vec{T}_{i/L} = (\vec{r}_{LH} \times T_{LH} \hat{i}) + (\vec{r}_{LA} \times R_{Ay} \hat{j}) + (\vec{r}_{LB} \times R_{By} \hat{j})$$

$$= a\hat{j} \times T_{LH} \hat{i} + (-b-c)\hat{i} \times R_{Ay} \hat{j} + (-b\hat{i} - d\hat{j}) \times R_{By} \hat{j}$$

$$= (aT_{LH} - bR_{Ay} - cR_{Ay} - bR_{By})\hat{k} = \vec{0}$$

$$T_{HI} = \frac{b}{a}(R_{Ay} + R_{By}) + \frac{c}{a}R_{Ay}$$

FBD (Section):



d) Find T_{JK} ,

$$\sum \vec{M}_{/N} = (\vec{r}_{NJ} \times T_{JK} \hat{i}) + (\vec{r}_{NA} \times R_{Ay} \hat{j})$$

$$= a \hat{j} \times T_{JK} \hat{i} + (-sb - c) \hat{i} \times R_{Ay} \hat{j}$$

$$+ (-sb \hat{i} - d \hat{j}) \times R_{By} \hat{j}$$

$$= (a T_{JK} - sb R_{Ay} - c R_{Ay} - sb R_{By}) \hat{k} = \vec{0}$$

$$T_{JK} = \frac{1}{a} [-sb(R_{Ay} + R_{By}) - c R_{Ay}]$$

FBD (Section):

