5.5.2 In 3D, how many independent scalar equations can you write for equilibrium of a particle?

Solutions to HW set 3

4.5.2

In 3D Statics, one vector equilibrium equation can be written for a particle as

$$\sum \underline{F} = \underline{0}$$

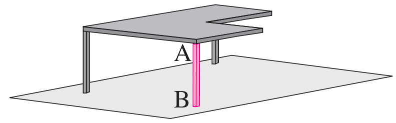
This would result in 3 independent scalar equations

$$\sum F_x = 0,$$

$$\sum F_y = 0, \text{ and}$$

$$\sum F_z = 0.$$

5.5.8 An 80 kg square table has one quarter cut away. The remaining 60 kg are supported on 3 massless legs on a level floor. Use g = 10N/kg. What is the load carried by leg AB? (State your assumptions clearly.)



Problem 5.8

Answer:
Assuming no side-loads from floor the support from leg AB is 250 N, $T_{AB} = -250$ N.

4.5.9

$m = 60$ kg

$g = 10$ N/kg

Assumption: There are no reaction forces along the floor (in $\hat{\imath}$ and $\hat{\jmath}$ directions) on the legs from the floor.

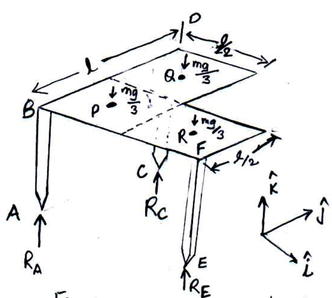


Fig 1 FBD of the table.

Assuming that the mass of the table is uniformly distributed over its surface and the legs have negligible mass, we can visualize the top surface as three squares with edge length $l/2$ and mass $\frac{m}{3}$ or 20 kg each. Center of mass of each of these squares is at the center of the square.

Now, by force balance, $\sum \vec{F} = \vec{0}$, we get

$$R_A \hat{k} + R_C \hat{k} + R_E \hat{k} - mg\hat{k} = \vec{0}$$

$$\Rightarrow \quad R_A + R_C + R_E = mg \quad \text{——— (i)}$$

Moment balance about point B gives, $\sum \vec{M}_{/B} = 0$

$$\Rightarrow \quad l\hat{\jmath} \times R_C\hat{k} + l\hat{\imath} \times R_E\hat{k} + \left(\frac{l}{4}\hat{\imath} + \frac{l}{4}\hat{\jmath}\right) \times \left(-\frac{mg}{3}\hat{k}\right) + \left(\frac{l}{4}\hat{\imath} + \frac{3l}{4}\hat{\jmath}\right) \times \left(-\frac{mg}{3}\hat{k}\right) + \ldots$$

$$\left(\frac{3l}{4}\hat{\imath} + \frac{l}{4}\hat{\jmath}\right) \times \left(-\frac{mg}{3}\hat{k}\right) = \vec{0}$$

$$\Rightarrow \quad R_C l\hat{\imath} - R_E l\hat{\jmath} + \left(\frac{5l}{4}\hat{\imath} + \frac{5l}{4}\hat{\jmath}\right) \times \left(-\frac{mg}{3}\hat{k}\right) = \vec{0}$$

$$\Rightarrow \quad R_C l\hat{\imath} - R_E l\hat{\jmath} + \left(\frac{5mgl}{12}\hat{\jmath} - \frac{5mgl}{12}\hat{\imath}\right) = \vec{0} \quad \text{——— (ii)}$$

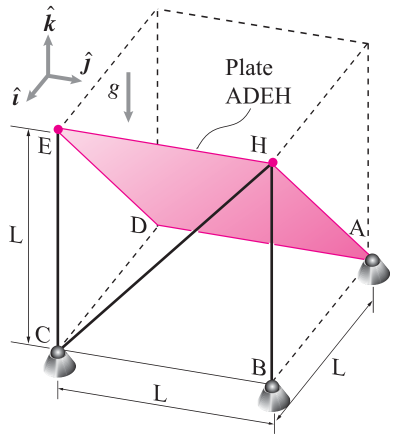
Seperating x and y components of equation (ii), we get $\boxed{R_C = R_E = \frac{5mg}{12} = 250\text{ N}}$

Substituting these values in equation (i), we get $\boxed{R_A = 100\text{ N}}$

∴ The load carried by the leg FE, $\boxed{T_{FE} = -250\text{ N}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.5.9 Uniform plate ADEH with mass m is connected to the ground with a ball and socket joint at A. It is also held by three massless bars (CE, CH and BH) that have ball and socket joints at each end, one end at the rigid ground (at C and B) and one end on the plate (at E and H). In terms of some or all of m, g, and L find the reaction at A (the force of the ground on the plate) and the three bar tensions T_{CE}, T_{CH} and T_{BH}.



Problem 5.9

Answer:
$T_{CE} = mg/2, T_{CH} = T_{BH} = -0, A_z = mg/2, A_y = A_x = 0$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.5.12 A uniform 5 kg shelf is supported at one corner with a ball and socket joint and the other three corners with strings. At the instant of interest the shelf is at rest. Gravity acts in the $-\hat{\boldsymbol{k}}$ direction. The shelf is in the xy plane.

a) Draw a FBD of the shelf.

b) Challenge: without doing any calculations on paper can you find one of the reaction force components or the tension in any of the cables? Give yourself a few minutes of staring to try to find this force. If you can't, then come back to this question after you have done all the calculations.

c) Write down the equation of force equilibrium.

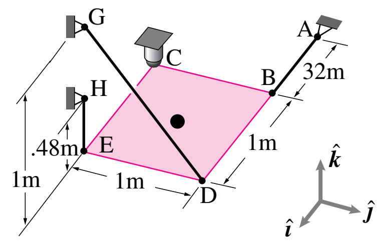
d) Write down the moment-balance equation using the center of mass as a reference point.

e) By taking components, turn (b) and (c) into six scalar equations in six unknowns.

f) Solve these equations by hand or on the computer.

g) Instead of using a system of equations try to find a single equation which can be solved for T_{EH}. Solve it and compare to your result from before.

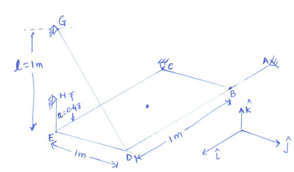
h) Challenge: For how many of the reactions can you find one equation which will tell you that particular reaction without knowing any of the other reactions? [Hint, try moment balance about an appropriate axis as well as force balance in an appropriate direction. It is possible to find five of the six unknown reaction components this way.] Must these solutions agree with (f)? Do they?



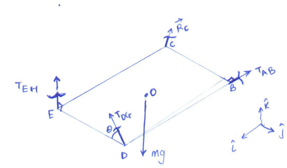
Problem 5.12

Answer:
(g) $T_{EH} = 0$ as you can find a number of ways.

Problem 4.5.13



a) FBD



b) By doing moment balance about an axis passing through point D and point C, we get,

$$\hat{\lambda}_{DC}\cdot\left(\Sigma \vec{M}_{/D}\right)=0 \Rightarrow \hat{\lambda}_{DC}\cdot\left(\ell\,\hat{\lambda}_{DC}\times\vec{R}_c + \frac{\ell}{2}\hat{\lambda}_{DC}\times(-mg\,\hat{\jmath}) + (-\ell\hat{\jmath})\times T_{EH}\hat{k}\right)=0$$

$$\Rightarrow \hat{\lambda}_{DC}\cdot(-\ell T_{EH}\hat{\imath}) = 0 \Rightarrow \boxed{T_{EH}=0}.$$

c) Force balance,

$$\Sigma\vec{F}=\vec{0}$$

$$\Rightarrow \boxed{T_{EH}\hat{k} + T_{DG}\sin\theta\,\hat{k} - T_{DG}\cos\theta\,\hat{\jmath} - T_{AB}\hat{\imath} + \vec{R}_c - mg\hat{k} = \vec{0}} \quad \text{—— (i)}$$

d) Moment balance,

$$\Sigma\vec{M}_{/O}=\vec{0}$$

$$\Rightarrow \left(\vec{r}_{OE}\times T_{EH}\hat{k}\right) + \left[\vec{r}_{OD}\times T_{DG}\left(\sin\theta\,\hat{k}-\cos\theta\,\hat{\jmath}\right)\right] + \vec{r}_{OB}\times(-T_{AB}\hat{\imath}) + \vec{r}_{OC}\times\vec{R}_c = \vec{0}$$

$$\Rightarrow \boxed{\left(-\frac{\ell}{2}T_{EH}\hat{\jmath} - \frac{\ell}{2}T_{EH}\hat{\imath}\right) + \left[-\frac{\ell}{2}T_{DG}\sin\theta\,\hat{\jmath} + \frac{\ell}{2}T_{DG}\sin\theta\,\hat{\imath} - \frac{\ell}{2}T_{DG}\cos\theta\,\hat{k}\right] + \frac{\ell}{2}T_{AB}\hat{k} + \left(\vec{r}_{OC}\times\vec{R}_c\right) = \vec{0}} \quad \text{—— (ii)}$$

e) By doing dot product of equ (i) with $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, we get

$$\vec{R}_c \cdot \hat{\imath} = T_{AB} \quad \text{——— (iii.a)}$$

$$\vec{R}_c \cdot \hat{\jmath} = T_{DG} \cos\theta \quad \text{——— (iii.b)}$$

$$T_{EH} + T_{DG} \sin\theta - mg + \vec{R}_c \cdot \hat{k} = 0 \quad \text{——— (iii.c)}$$

respectively.

Similarly, By doing a doing a dot product of equ(ii) with $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, we get

$$(\vec{r}_{oc} \times \vec{R}_c) \cdot \hat{\imath} = \frac{T_{EH}\, l}{2} - \frac{T_{DG} \sin\theta\, l}{2} \quad \text{——— iv.a}$$

$$(\vec{r}_{oc} \times \vec{R}_c) \cdot \hat{\jmath} = \frac{T_{EH}\, l}{2} + \frac{T_{DG} \sin\theta\, l}{2} \quad \text{——— iv.b}$$

$$(\vec{r}_{oc} \times \vec{R}_c) \cdot \hat{k} = \frac{T_{DG} \cos\theta\, l}{2} - \frac{T_{AB}\, l}{2} \quad \text{——— iv.c}$$

f. From equ iv.a, iv.b and iv.c we infer,

$$\vec{r}_{oc} \times \vec{R}_c = \frac{l}{2}\left[(T_{EH} - T_{DG}\sin\theta)\hat{\imath} + (T_{EH} + T_{DG}\sin\theta)\hat{\jmath} + (T_{DG}\cos\theta - T_{AB})\hat{k} \right] \quad \text{——— (v)}$$

$$\vec{R}_c = R_{cx}\hat{\imath} + R_{cy}\hat{\jmath} + R_{cz}\hat{k} \quad \text{we get}$$

$$\vec{r}_{oc} \times \vec{R}_c = \left(-\frac{l}{2}\hat{\imath} - \frac{l}{2}\hat{\jmath}\right) \times (R_{cx}\hat{\imath} + R_{cy}\hat{\jmath} + R_{cz}\hat{k})$$

$$= \left(\frac{l R_{cx}}{2} - \frac{R_{cy}\, l}{2}\right)\hat{k} + \left(\frac{-l R_{cz}}{2}\right)\left[\hat{\imath} - \hat{\jmath}\right] \quad \text{——— (vi)}$$

dot product of equ.(v) and equ(vi) with $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ gives

$$-\frac{l}{2} R_{cz} = \frac{l}{2}(T_{EH} - T_{DG}\sin\theta) \quad \text{—— (vii.a)}$$

$$+\frac{l}{2} R_{cz} = \frac{l}{2}(T_{EH} + T_{DG}\sin\theta) \quad \text{—— (vii.b)}$$

$$\frac{l}{2}(R_{cx} - R_{cy}) = \frac{l}{2}(T_{DG}\cos\theta - T_{AB}) \quad \text{——— (vii.c)}$$

Adding equ (vii.a) and (vii.b) we get $\boxed{T_{EH} = 0}$

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Subtracting (vii. a) and (vii. b) we get

$$R_{cz} = T_{DG} \sin\theta$$

Now eliminating R_{cx}, R_{cy}, R_{cz} from equ set (iii) and (vii) and putting $T_{EH} = 0$ we get,

$$T_{AB} - T_{DG}\cos\theta = T_{DG}\cos\theta - T_{AB} \quad \text{——— (viii).a}$$

and

$$T_{DG}\sin\theta - mg = -T_{DG}\sin\theta \quad \text{——— (viii.b)}$$

Solving equ (viii)a and (viii).b we get

$$\boxed{T_{DG} = \frac{mg}{2\sin\theta} \qquad T_{AB} = \frac{mg}{2\tan\theta}}$$

Hence the complete solution is :

$$\boxed{T_{DG} = \frac{mg}{2\sin\theta}\ ; \quad T_{AB} = \frac{mg}{2\tan\theta}\ ; \quad T_{EH} = 0\ ; \quad \vec{R}_c = \frac{mg}{2\tan\theta}\hat{i} + \frac{mg}{2\tan\theta}\hat{j} + \frac{mg}{2}\hat{k}}$$

Substituting, $mg \simeq 5\times 9.81$ N $\simeq 50$N

and $\theta = 45^\circ$

we get

$$\boxed{\vec{R}_c = 25(\hat{i}+\hat{j}+\hat{k})\text{ N} \quad T_{DG} = 35.35\text{ N} \quad T_{AB} = 25\text{ N} \quad T_{EH} = 0\text{ N}}$$

Ans.

g) Doing moment balance about an axis passing through points D and C we get

$$\hat{\lambda}_{DC}\cdot\left(\Sigma \vec{M}_{/D}\right) = 0\ ; \Rightarrow \hat{\lambda}_{DC}\cdot\left[\vec{DC}\times\vec{R}_c + \vec{DO}\times(-mg\hat{j}) + \vec{DE}\times(T_{EH}\hat{k})\right] = 0$$

$\Rightarrow$

since $\vec{DC} \parallel \hat{\lambda}_{DC}$ and $\vec{DO} \parallel \hat{\lambda}_{DC}$ but $\vec{DE}$ is not $\parallel$ to $\hat{\lambda}_{DC}$ we get

$$\Rightarrow \boxed{T_{EH} = 0}$$

h) Taking an axis passing from D to E

$$\hat{\lambda}_{DE} \cdot (\Sigma \vec{M}_{DE}) = 0$$

$$\Rightarrow \hat{\lambda}_{DE} \cdot \left[(\vec{DE} \times \vec{T}_{EH}) + \{ \vec{DO} \times m\vec{g}(-\hat{k}) \} + (\vec{DC} \times \vec{C}_z) \right] = 0$$

$$\Rightarrow \hat{\lambda}_{DE} \left[(\vec{DE} \times \vec{T}_{EH}) + \left\{ \left(-\frac{L}{2}\hat{\imath} - \frac{L}{2}\hat{\jmath}\right) \times mg\hat{k} \right\} + \left\{ (-L\hat{\imath} - L\hat{\jmath}) \times C_z \hat{k} \right\} \right] = 0$$

$\hat{\lambda}_{DE}$ lies along $-\hat{\jmath}$ and is $\parallel$ to $\vec{DE}$

$$\Rightarrow \quad 0 + \hat{\lambda}_{DE} \cdot \left\{ -\frac{L}{2}\hat{\jmath} - \frac{L}{2}\hat{\imath} \right\} mg + \hat{\lambda}_{DE} \cdot \left\{ -L\hat{\jmath} - L\hat{\imath} \right\} C_z = 0$$

$$\Rightarrow \quad +\frac{Lmg}{2} + 0 - C_z L - 0 = 0$$

$$\Rightarrow \boxed{C_z = +\frac{mg}{2}} \quad \text{or} \quad \boxed{R_{C_z} = \frac{mg}{2}}$$

Taking an axis passing from C to D, solved in part (g), $\boxed{\vec{T}_{EH} = \vec{0}}$

Taking an axis passing from C to E,

$$\hat{\lambda}_{CE} \cdot \left(\cancel{\vec{CE} \times \vec{T}_{EH}}^{\,0} + \vec{CO} \times (-mg\hat{k}) + \vec{CD} \times \vec{T}_{DC} + \cancel{\vec{CB} \times \vec{T}_{AB}}^{\,0} \right) = 0$$

$$\hat{\lambda}_{CE} \cdot \left(\frac{mgL}{2}\hat{\jmath} - \frac{mgL}{2}\hat{\imath} + (L\hat{\imath} + L\hat{\jmath}) \times \left(-T_{DC}\cos\theta\,\hat{\jmath} + T_{DC}\sin\theta\,\hat{k}\right) \right) = 0$$

$\because$ $\hat{\lambda}_{CE}$ is along $\hat{\imath}$, solving we get $\boxed{T_{DC} = \dfrac{mg}{2\sin\theta}}$

Taking an axis passing from C to G.

$$\hat{\lambda}_{CG} \cdot \left[\vec{CE} \times \vec{T}_{EH} + \vec{CD} \times \vec{T}_{DG} + \vec{CO} \times (-mg\hat{k}) + \vec{CB} \times \vec{T}_{AB} \right] = \vec{0}$$

$$\Rightarrow \hat{\lambda}_{CG} \cdot \left[(L\hat{i} \times \vec{0}) + L(-T_{DC}\cos\theta\,\hat{k} - T_{DC}\sin\theta\,\hat{j} + T_{DC}\sin\theta\,\hat{i}) + \left(\tfrac{L}{2}\hat{i} + \tfrac{L}{2}\hat{j}\right) \times (-mg\hat{k}) + (L\hat{j} \times T_{AB}(-\hat{i})) \right] = \vec{0}$$

$$\Rightarrow \frac{(\cancel{L}\hat{i} + \cancel{L}\hat{k})}{\cancel{L}\sqrt{2}} \left[0 + L(-T_{DC}\cos\theta\,\hat{k} - T_{DG}\sin\theta\,\hat{j} + T_{DC}\sin\theta\,\hat{i}) - mg\left(\tfrac{L}{2}\hat{i} + \tfrac{L}{2}\hat{j}\right) \times \hat{k} + T_{AB}L\hat{k} \right] = \vec{0}$$

$$\Rightarrow (\hat{i} + \hat{k}) \left[-T_{DG}\cos\theta\,\hat{k} - \cancel{T_{DG}\sin\theta\,\hat{j}} + T_{DG}\sin\theta\,\hat{i} + \cancel{\frac{mg}{2}\hat{j}} - \frac{mg}{2}\hat{i} + T_{AB}\hat{k} \right] = \vec{0}$$

$$T_{DG}\sin\theta - \frac{mg}{2} - T_{DG}\cos\theta + T_{AB} = \vec{0}$$

From previous, $T_{DG} = mg/2\sin\theta$

$$\Rightarrow T_{DG}\cos\theta = T_{AB}$$

$$\boxed{\Rightarrow \frac{mg}{2\tan\theta} = T_{AB}}$$

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Taking an axis passing through G along $\hat{\imath}$

$$\hat{\lambda}_{Gi} \cdot \left[\cancelto{0}{(\vec{GE} \times \vec{T}_{HE})} + (\vec{GC} \times \vec{C}) + (\vec{GB} \times \vec{T}_{AB}) + (\vec{GO} \times mg(-\hat{k})) + \cancelto{0}{(\vec{GD} \times \vec{T}_{DC})} \right] = \vec{0}$$

$$\Rightarrow \hat{\lambda}_{Gi} \cdot \left[\{(-l\hat{\imath} - l\hat{k}) \times (R_{Cx}\hat{\imath} + R_{Cy}\hat{\jmath} + R_{Cz}\hat{k})\} + \{(l\hat{\imath} - l\hat{\jmath} + l\hat{k}) \times (T_{AB}\hat{\imath})\} + \{(\tfrac{l}{2}\hat{\imath} - \tfrac{l}{2}\hat{\jmath} + l\hat{k}) \times mg\hat{k}\} \right]$$

$$\Rightarrow \hat{\lambda}_{Gi} \cdot \left[-lR_{Cx}\hat{\jmath} - lR_{Cy}\hat{k} + lR_{Cy}\hat{\imath} + lR_{Cz}\hat{\jmath} + lT_{AB}\hat{k} + lT_{AB}\hat{\jmath} - \frac{mgl}{2}\hat{\jmath} - \frac{mgl}{2}\hat{\imath} \right] = \vec{0}$$

$\hat{\lambda}_{Gi}$ lies along $\hat{\imath}$

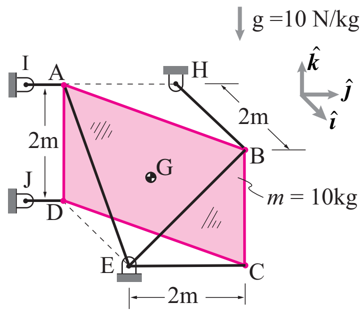
$$\Rightarrow lR_{Cy} - \frac{mgl}{2} = 0 \quad \Rightarrow \boxed{R_{Cy} = \frac{mg}{2}}$$

No axis could be found to directly determine R_{Cx} by a single equation. All the solutions thus obtained match with those obtained in the previous parts. This necessarily has to be so, since we are solving for the same system.

5.5.13 The sign is held up by 6 rods. Find the tension in bars

a) BH
b) EB
c) AE
d) IA
e) JD
f) EC

[One game you can play is to see how many of the tensions you can find *without* knowing any of the others. Another approach is to set up and solve 6 equations in 6 unknowns.]



Problem 5.13

Answer:
(a) Use axis EC.
(b) Use axis AH.
(c) Use $\hat{\jmath}$ axis through B.
(d) Use axis DE.
(e) Use axis EH.
(f) Can't do in one shot.

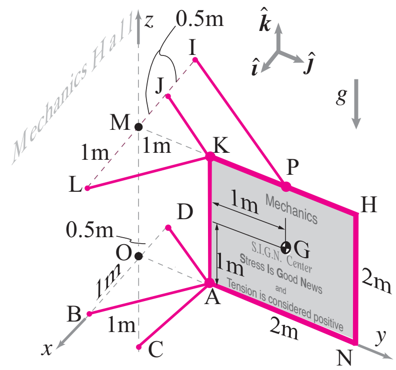
5.5.14 The 100 kg, 2 m square, uniform sign KHNA is held up by 6 bars.

Structure and geometry clarifications: The sign is held vertically, 1m in front of, and orthogonal-to a vertical wall. Each bar holding the sign has a ball-and-socket joint both where it attaches to the sign and where it attaches to the wall. The points L, M, J, I, K, P and H lie in the same horizontal plane that includes the top edge of the sign. The points M, O, and C lie on a vertical line that is coplanar with the sign. Points B, O, D, A, and N lie in a horizontal plane shared with the bottom edge of the sign. The center of mass of the sign is at G. $g = 10\text{N/kg}$.

a) Find the "bar force" in bar AC. [hint: $\Sigma F_z = \{\Sigma \vec{F}\} \cdot \hat{\boldsymbol{k}} = 0$].

b) Find the "bar force" in bar IP. [hint: $\Sigma M_{AK} = \left\{\Sigma \vec{\boldsymbol{M}}_{/A}\right\} \cdot \hat{\boldsymbol{k}} = 0$].

c) Find the "bar force" in bar KL.



Problem 5.14

Answer:
$T_{AC} = -\sqrt{2}mg = -1000\sqrt{2}\,\text{N} \approx -1410\,\text{N}$ (the bar is in compression)
$T_{IP} = 0$
$T_{KL} = \sqrt{2}mg/6 = \left(1000\sqrt{2}/6\right)N \approx 408\,\text{N}$ (the bar is in tension)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.5.15 Below is a highly schematic picture of a tricycle. The wheels are at C, B and A. The person-trike system has center of mass at G directly over the rear axle. The wheels at C and A are good free-turning, high friction wheels. The wheel at B is in a small ditch and can't move. Assume no slip and that F, m, g, w, ℓ, and h are given.

a) Of the 9 possible reaction components at A, B, and C, which do you know are zero *a priori*.

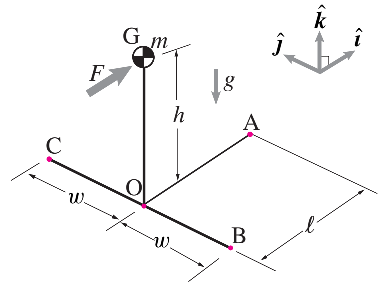
b) Find all the reaction components (the full reaction force) at A.

c) Find the vertical component of the reaction at C.

d) Find the x and z reaction components at B.

e) Find the sum of the y components of the reactions at B and C.

f) Can you find the y component of the reaction at C? Why or why not?



Problem 5.15

Ans 4.5.16 :

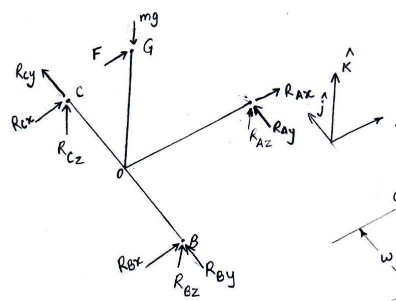
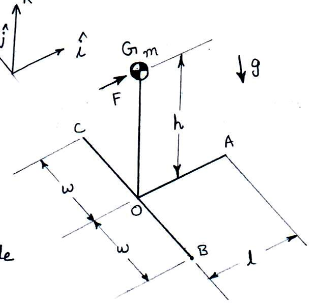


Fig (i) FBD of the tricycle



a) Since all wheels are "good free-turning" wheels, they cannot stop rotating unless moment along their axis of rotation about their center is zero. Hence, $R_{cx} = R_{Ax} = 0$. R_{By} need not be zero because the corresponding wheel is "stuck".

b) Now, considering the FBD in fig (i), force balance, $\Sigma \vec{F} = \vec{0}$ gives,

$$(R_{Bx} + F)\hat{\imath} + (R_{Az} + R_{Bz} + R_{Cz} - mg)\hat{k} + (R_{Ay} + R_{By} + R_{cy})\hat{\jmath} = \vec{0}$$

Seperating, x, y and z components, we get

$$R_{Bx} = -F \; ; \quad R_{Az} + R_{Bz} + R_{Cz} = mg \; ; \quad R_{Ay} + R_{By} + R_{cy} = 0$$

— (i.a) — (i.b) — (i.c)

Now, the moment balance of all forces about point 'O' gives, $\vec{M}_{/O} = 0$

$$\Rightarrow (h\hat{k} \times F\hat{\imath}) + l\hat{\imath} \times (R_{Az}\hat{k} + R_{Ay}\hat{\jmath}) + (-w\hat{\jmath}) \times (R_{Bx}\hat{\imath} + R_{By}\hat{\jmath} + R_{Bz}\hat{k}) + (w\hat{\jmath}) \times (R_{cy}\hat{\jmath} + R_{cz}\hat{k}) = \vec{0}$$

$$\Rightarrow hF\hat{\jmath} + (-R_{Az}l\hat{\jmath} + R_{Ay}l\hat{k}) + (R_{Bx}w\hat{k} - R_{Bz}w\hat{\imath}) + (R_{cz}w\hat{\imath}) = \vec{0} \quad \text{—— (ii)}$$

Dot product of equation (ii) with $\hat{\jmath}$ gives,

$$hF = R_{Az}l \Rightarrow \boxed{R_{Az} = \frac{hF}{l}}$$

Dot product of equation (ii) with $\hat{k}$ gives $\Rightarrow R_{Ay}l = -R_{Bx}w$

$\Rightarrow R_{Ay} = Fw/l$ because $R_{Bx} = -F$ (from (i.a)). Hence, we get $\Rightarrow$ $\boxed{R_{Ax} = 0\text{ N},\; R_{Ay} = \frac{wF}{l},\; R_{Az} = \frac{hF}{l}}$

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c) By doing dot product of equation (ii) with $\hat{\imath}$, we get

$$-R_{Bz}w + R_{Cz}w = 0 \Rightarrow R_{Cz} = R_{Bz}$$

Now using this result with equation (i.b) and using the value of R_{Az} from the solution of part (b) of this problem, we get

$$R_{Az} + 2R_{Cz} = mg$$

$$\Rightarrow \boxed{R_{Cz} = \frac{1}{2}\left(mg - \frac{hF}{\ell}\right)}$$

d) From equation (i.a), $\boxed{R_{Bx} = -F}$

From the solution of part (c) we have, $\boxed{R_{Bz} = R_{Cz} = \frac{1}{2}\left(mg - \frac{hF}{\ell}\right)}$

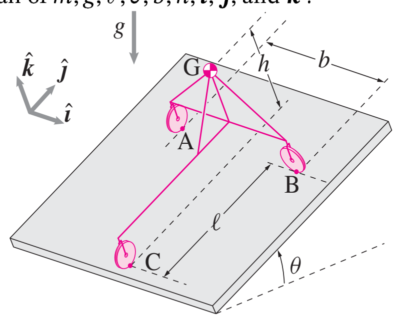
e) From equation (i.c), $R_{By} + R_{Cy} = -R_{Ay}$

$$\Rightarrow \boxed{R_{By} + R_{Cy} = -\frac{wF}{\ell}}$$

f). We cannot find the y component of the reaction at 'C', because we have 9 variables of which 2 are known a priori (part (a)) leaving behind 7 variables. But we get only 6 independent equations from static equillibrium of the tricycle. Hence, R_{By} and R_{Cy} cannot be solved for individually.

5.5.16 A 3-wheeled robot with mass m is parked on a hill with slope θ. The ideal massless robot wheels are free to roll but not to slip sideways. The robot steering mechanism has turned the wheels so that wheels at A and C are free to roll in the $\hat{\jmath}$ direction and the wheel at B is free to roll in the $\hat{\imath}$ direction. The center of mass of the robot at G is h above (normal to the slope) the trailer bed and symmetrically above the axle connecting wheels A and B. The wheels A and B are a distance b apart. The length of the robot is ℓ.

Find the force vector $\vec{F}_A$ of the ground on the robot at A in terms of some or all of $m, g, \ell, \theta, b, h, \hat{\imath}, \hat{\jmath}$, and $\hat{k}$.



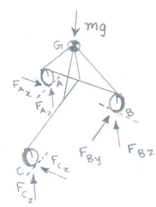
Problem 5.16

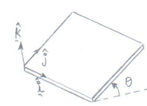
Answer:
Hint: With reference to a free-body diagram of the robot, use moment balance about axis BC.

4.5.17

The 3-wheeled robot with mass m is parked on a hill with slope 'θ'.

The wheels at A and C can roll in $\hat{j}$ direction and the wheel at B can roll in $\hat{i}$ direction.





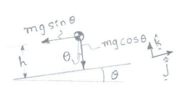
As the wheels can not slip sideways, the reactions at A, B, & C have 2 components each.

$\underline{F}_A = F_{Ax}\hat{\underline{i}} + F_{Az}\hat{\underline{k}}$

$\underline{F}_B = F_{By}\hat{\underline{j}} + F_{Bz}\hat{\underline{k}}$

$\underline{F}_C = -F_{Cx}\hat{\underline{i}} + F_{Cz}\hat{\underline{k}}$

The force due to the gravitational acceleration acts in the $\hat{j}$ - $\hat{k}$ plane.



As the robot is in static equilibrium, applying Linear Momentum Balance, $\Sigma \underline{F} = \underline{0}$

$$\Rightarrow \quad \underline{F}_A + \underline{F}_B + \underline{F}_C - mg\cos\theta\,\hat{\underline{k}} - mg\sin\theta\,\hat{\underline{j}} = \underline{0}$$

$$\Rightarrow \left\{ F_{Ax}\hat{\underline{i}} + F_{Az}\hat{\underline{k}} + F_{By}\hat{\underline{j}} + F_{Bz}\hat{\underline{k}} - F_{Cx}\hat{\underline{i}} + F_{Cz}\hat{\underline{k}} - mg\cos\theta\,\hat{\underline{k}} - mg\sin\theta\,\hat{\underline{j}} = \underline{0} \right\}$$

$$\{\ \}\cdot\hat{\underline{i}} \Rightarrow F_{Ax} - F_{Cx} = 0 \Rightarrow F_{Cx} = F_{Ax} \qquad -①$$

$$\{\ \}\cdot\hat{\underline{j}} \Rightarrow F_{By} - mg\sin\theta = 0 \Rightarrow F_{By} = mg\sin\theta \qquad -②$$

$$\{\ \}\cdot\hat{\underline{k}} \Rightarrow F_{Az} + F_{Bz} + F_{Cz} - mg\cos\theta = 0 \Rightarrow F_{Az} + F_{Bz} + F_{Cz} = mg\cos\theta \qquad -③$$

Taking moment balance about an axis parallel to $\hat{k}$ and passing through 'C',

$$\Rightarrow \underline{r}_{G/C} \times (-mg\sin\theta\,\hat{\underline{j}} - mg\cos\theta\,\hat{\underline{k}}) + \underline{r}_{B/C} \times \underline{F}_B + \underline{r}_{A/C} \times \underline{F}_A = \underline{0} \qquad (\because \underline{r}_{G/C} = l\hat{\underline{j}} + h\hat{\underline{k}})$$

$$\Rightarrow -mgl\cos\theta\,\hat{\underline{i}} + mgh\sin\theta\,\hat{\underline{i}} + \left(\frac{b}{2}\hat{\underline{i}} + l\hat{\underline{j}}\right) \times (F_{By}\hat{\underline{j}} + F_{Bz}\hat{\underline{k}}) + \left(-\frac{b}{2}\hat{\underline{i}} + l\hat{\underline{j}}\right) \times (F_{Ax}\hat{\underline{i}} + F_{Az}\hat{\underline{k}}) = \underline{0}$$

$$\Rightarrow \left\{ \begin{array}{l} -mgl\cos\theta\,\hat{i} \\ +mgh\sin\theta\,\hat{i} \end{array} + \frac{b}{2}F_{By}\hat{\underline{k}} + \frac{b}{2}F_{Bz}(-\hat{\underline{j}}) + lF_{Bz}\hat{\underline{i}} + \frac{b}{2}F_{Az}\hat{\underline{j}} - lF_{Ax}\hat{\underline{k}} + lF_{Az}\hat{\underline{i}} = \underline{0} \right\}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$\{\ \}\cdot\hat{k}$

$\Rightarrow \quad \frac{b}{2} F_{By} - l F_{Ax} = 0 \quad \Rightarrow \quad F_{Ax} = \frac{b}{2l} F_{By} \qquad [\because F_{By} = mg \sin\theta]$

$$\boxed{F_{Ax} = \frac{b}{2l} mg \sin\theta} \quad — ④$$

$\{\ \}\cdot\hat{j}$

$\Rightarrow \quad -\frac{b}{2} F_{Bz} + \frac{b}{2} F_{Az} = 0 \quad \Rightarrow \quad F_{Az} = F_{Bz} \quad — ⑤$

$\{\ \}\cdot\hat{i}$

$\Rightarrow \quad -mgl\cos\theta + mgh\sin\theta + l F_{Bz} + l F_{Az} = 0$

$$\Rightarrow \quad F_{Bz} = \boxed{F_{Az} = \frac{mgl\cos\theta - mgh\sin\theta}{2l}} \quad — ⑥$$

$$\Rightarrow \boxed{\underline{F}_A = mg\frac{b}{2l}\sin\theta\,\hat{\underline{i}} + \left(\frac{mg}{2}\cos\theta - mg\frac{h}{2l}\sin\theta\right)\hat{\underline{k}}}$$

From ② and ⑥

$$\underline{F}_B = mg\sin\theta\,\hat{\underline{j}} + \left(\frac{mg}{2}\cos\theta - mg\frac{h}{2l}\sin\theta\right)\hat{\underline{k}} \quad //$$

From ①, ③ and ⑥

$$\underline{F}_C = -F_{Ax}\hat{\underline{i}} + \left[mg\cos\theta - 2\frac{(mgl\cos\theta - mgh\sin\theta)}{2l}\right]\hat{\underline{k}}$$

$$\Rightarrow \underline{F}_C = -mg\frac{b}{2l}\sin\theta\,\hat{\underline{i}} + mg\frac{h}{2l}\sin\theta\,\hat{\underline{k}} \quad //$$

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