# 5.5 Advanced statics

## Preparatory Problems

**5.5.1** In 2D, the force balance and moment-balance equations for equilibrium of a body give three independent scalar equations that can be used to solve for three unknowns. How many independent scalar equations can you get from force and moment balance in 3D? Write down a set of such equations.

**5.5.2** In 3D, how many independent scalar equations can you write for equilibrium of a particle?

**5.5.3** How is moment-balance equation about an axis different from moment balance about a point? Illustrate your answer with an example.

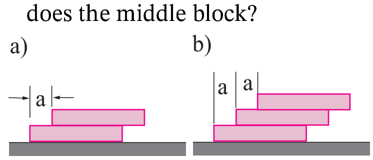
**5.5.4** How many independent scalar equations of equilibrium can you get by writing moment-balance equations about different lines or axes in 3D?

## More-Involved Problems

**5.5.5** Assume identical uniform rigid blocks with weight $W = 1\,\text{N}$, height $h = 1\,\text{cm}$, and length $\ell = 10\,\text{cm}$ are put one on top of the other. Assume there is no glue so blocks can only push against each other.

a) For two blocks what is the biggest overhang $a$ so that the top block does not tip over?

b) For three blocks what is the biggest total overhang $= 2a$ (the same overhang $a$ at each layer) so that the top block doesn't tip, nor does the middle block?




Problem 5.5.5

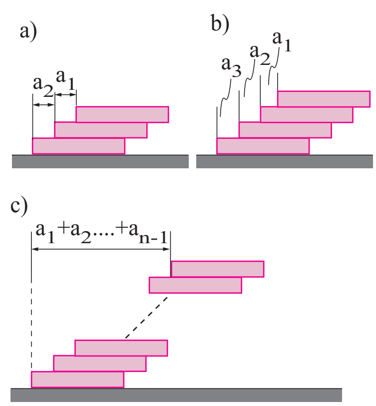
**5.5.6** See simpler problem 5.5.5. Stacking identical rigid blocks, one on top of the other, one wants to get the biggest overhang possible without the tower toppling. Each block has, say, $W = 1\,\text{N}$, height $h = 1\,\text{cm}$, and length $\ell = 10\,\text{cm}$.

a) For three blocks find the biggest $a_1$ and $a_2$ so there is no toppling. [First put the top block as far to the right as you can, $a_1$, for no toppling. Then put that pair as far to the right as possible for no toppling over the bottom block.] The total overhang is $a_1 + a_2$.

b) For 4 blocks find the largest possible overhang $a_1 + a_2 + a_3$ by placing the tower of three above as far to the right as possible relative to the bottom block. [Note that you place the center of mass of the top 3 blocks over the right edge of the fourth bottom block].

c) For $n$ blocks what is the biggest possible overhang $(= a_1 + a_2 + a_3 + \cdots + a_{n-1})$?

d) Using blocks with length $\ell = 10\,\text{cm}$ how many blocks $n$ are needed to get an overhang of 1 m? 2 m?




Problem 5.5.6

**5.5.7** Uniform plate ADEH with mass $m$ is connected to the ground with a ball and socket joint at A. It is also held by three massless bars (IE, CH and BH) that have ball and socket joints at each end, one end at the rigid ground (at I, C and B) and one end on the plate (at E and H).

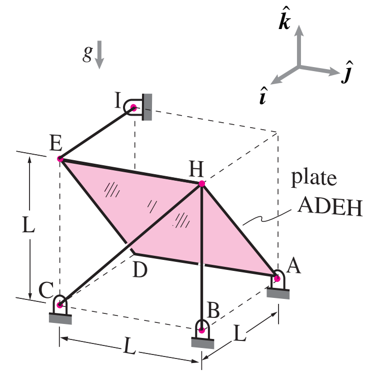
In terms of some or all of $m$, $g$, and $L$ find

a) the reaction at A (the force of the ground on the plate),

b) $T_{\text{IE}}$,

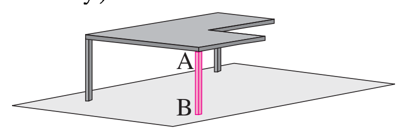
c) $T_{\text{CH}}$,

d) $T_{\text{BH}}$.



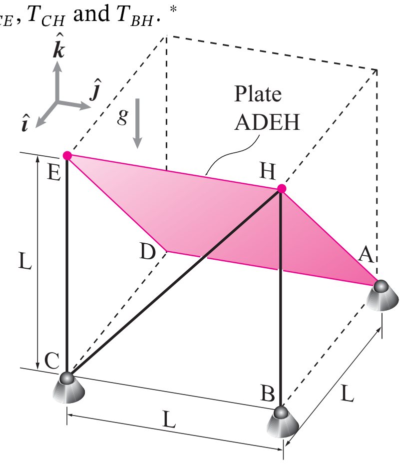

Problem 5.5.7

**5.5.8** An 80 kg square table has one quarter cut away. The remaining 60 kg are supported on 3 massless legs on a level floor. Use $g = 10\text{N/kg}$. What is the load carried by leg AB? (State your assumptions clearly.) *



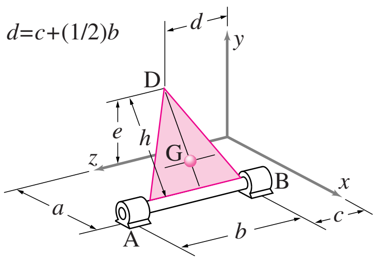

Problem 5.5.8

**5.5.9** Uniform plate ADEH with mass $m$ is connected to the ground with a ball and socket joint at A. It is also held by three massless bars (CE, CH and BH) that have ball and socket joints at each end, one end at the rigid ground (at C and B) and one end on the plate (at E and H). In terms of some or all of $m$, $g$, and $L$ find the reaction at A (the force of the ground on the plate) and the three bar tensions $T_{CE}$, $T_{CH}$ and $T_{BH}$. *




Problem 5.5.9

**5.5.10** A massless triangular plate rests against a frictionless wall at point $D$ and is rigidly attached to a massless rod supported by two ideal bearings fixed to the floor. A ball of mass $m$ is fixed to the centroid of the plate. There is gravity and the system is at rest. What is the reaction at point $D$ on the plate?

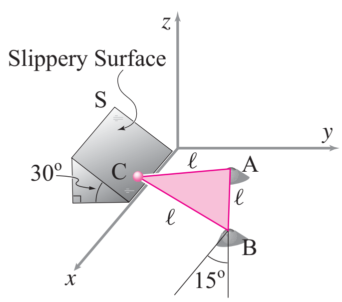



Problem 5.5.10

**5.5.11** A uniform equilateral triangular plate with weight $W = 1000N$ and sides $\ell = 2\,\mathrm{m}$ rests against a slippery plane S. Point C is negligibly above the $xy$ plane. The bottom edge of the triangle has ball-and-socket joints at A and B, with the line AB on the $xy$ plane making an angle of $15^\circ$ with the $x$ direction.

a) Find the reaction at C

b) Find all you can about the reactions at A and B.




Problem 5.5.11

**5.5.12** A uniform 5 kg shelf is supported at one corner with a ball and socket joint and the other three corners with strings. At the instant of interest the shelf is at rest. Gravity acts in the $-\hat{\boldsymbol{k}}$ direction. The shelf is in the $xy$ plane.

a) Draw a FBD of the shelf.

b) Challenge: without doing any calculations on paper can you find one of the reaction force components or the tension in any of the cables? Give yourself a few minutes of staring to try to find this force. If you can't, then come back to this question after you have done all the calculations.

c) Write down the equation of force equilibrium.

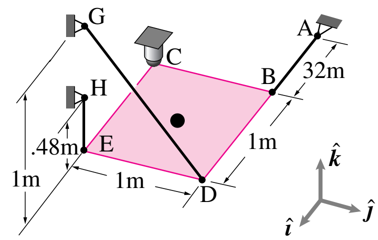
d) Write down the moment-balance equation using the center of mass as a reference point.

e) By taking components, turn (b) and (c) into six scalar equations in six unknowns.

f) Solve these equations by hand or on the computer.

g) Instead of using a system of equations try to find a single equation which can be solved for $T_{EH}$. Solve it and compare to your result from before. *

h) Challenge: For how many of the reactions can you find one equation which will tell you that particular reaction without knowing any of the other reactions? [Hint, try moment balance about an appropriate axis as well as force balance in an appropriate direction. It is possible to find five of the six unknown reaction components this way.] Must these solutions agree with (f)? Do they?




Problem 5.5.12

**5.5.13** The sign is held up by 6 rods. Find the tension in bars

a) BH *

b) EB *

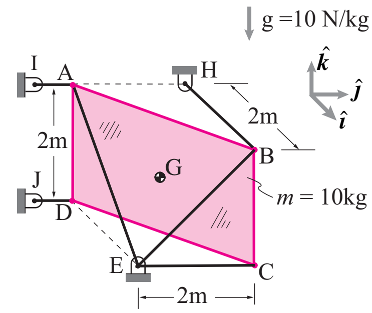
c) AE *

d) IA *

e) JD *

f) EC *

[One game you can play is to see how many of the tensions you can find *without* knowing any of the others. Another approach is to set up and solve 6 equations in 6 unknowns.]



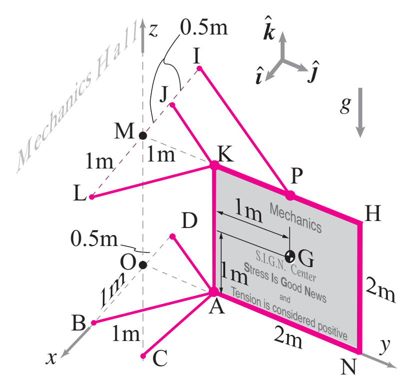

Problem 5.5.13

**5.5.14** The 100 kg, 2 m square, uniform sign KHNA is held up by 6 bars.
**Structure and geometry clarifications:** The sign is held vertically, 1m in front of, and orthogonal-to a vertical wall. Each bar holding the sign has a ball-and-socket joint both where it attaches to the sign and where it attaches to the wall. The points L, M, J, I, K, P and H lie in the same horizontal plane that includes the top edge of the sign. The points M, O, and C lie on a vertical line that is coplanar with the sign. Points B, O, D, A, and N lie in a horizontal plane shared with the bottom edge of the sign. The center of mass of the sign is at G. $g = 10\mathrm{N/kg}$.

a) Find the "bar force" in bar AC.
[hint: $\Sigma F_z = \left\{\Sigma \vec{\boldsymbol{F}}\right\} \cdot \hat{\boldsymbol{k}} = 0$]. *

b) Find the "bar force" in bar IP.
[hint: $\Sigma M_{AK} = \left\{\Sigma \vec{\boldsymbol{M}}_{/A}\right\} \cdot \hat{\boldsymbol{k}} = 0$]. *

c) Find the "bar force" in bar KL. *




Problem 5.5.14

**5.5.15** Below is a highly schematic picture of a tricycle. The wheels are at C, B and A. The person-trike system has center of mass at G directly over the rear axle. The wheels at C and A are good free-turning, high friction wheels. The wheel at B is in a small ditch and can't move. Assume no slip and that $F, m, g, w, \ell$, and $h$ are given.

a) Of the 9 possible reaction components at A, B, and C, which do you know are zero *a priori*.

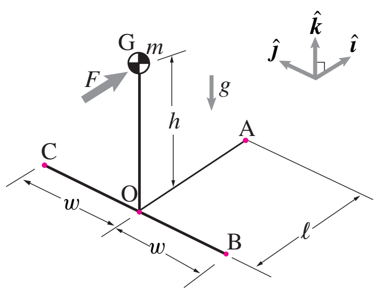
b) Find all the reaction components (the full reaction force) at A.

c) Find the vertical component of the reaction at C.

d) Find the $x$ and $z$ reaction components at B.

e) Find the sum of the $y$ components of the reactions at B and C.

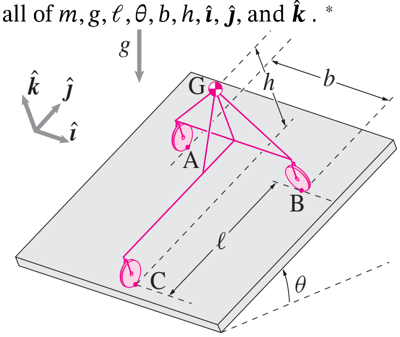
f) Can you find the $y$ component of the reaction at C? Why or why not?




Problem 5.5.15

**5.5.16** A 3-wheeled robot with mass $m$ is parked on a hill with slope $\theta$. The ideal massless robot wheels are free to roll but not to slip sideways. The robot steering mechanism has turned the wheels so that wheels at A and C are free to roll in the $\hat{\boldsymbol{\jmath}}$ direction and the wheel at B is free to roll in the $\hat{\boldsymbol{\imath}}$ direction. The center of mass of the robot at G is $h$ above (normal to the slope) the trailer bed and symmetrically above the axle connecting wheels A and B. The wheels A and B are a distance $b$ apart. The length of the robot is $\ell$.

Find the force vector $\vec{\boldsymbol{F}}_A$ of the ground on the robot at A in terms of some or all of $m, g, \ell, \theta, b, h, \hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$. *




Problem 5.5.16