5.4.1 For the bar shown which ones of the following statements are true?

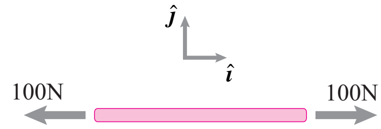
a) The two forces cancel so the tension is zero.

b) The two forces add so the tension is 200 N.

c) The tension is $100\,\mathrm{N}\hat{\imath}$.

d) The tension is $-100\,\mathrm{N}\hat{\imath}$.

e) The tension is $100\,\mathrm{N}\hat{\imath}$ on the right end and $-100\,\mathrm{N}\hat{\imath}$ on the left end.

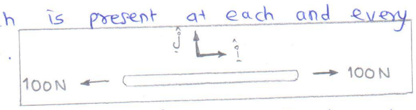


Problem 5.1

Answer:
None are true. The tension is 100 N.

Answers to 4.4.1

a) False.
The two forces don't cancel each other. Tension is an 'internal force' which is present at each and every point inside the bar. Tension is a scalar.



b) False.
The two forces don't add. The tension at any point inside the bar is 100 N.

c) False
Tension is a scalar and given statement says that it is a vector

d) False.
Tension is a scalar and given statement says that it is a vector.

e) False

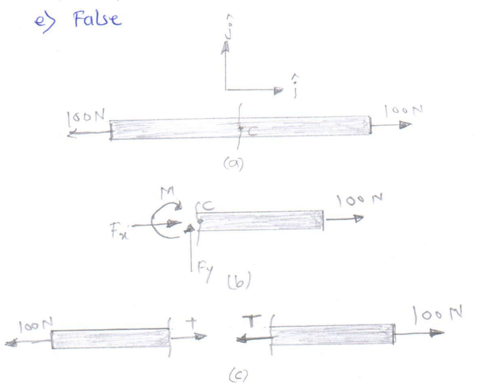


Fig (a) : Given bar

Fig (b) : FBD of right half with force distribution at the cut replaced by an equivalent force couple system.

Fig (c) : Further simplified FBD for both parts using laws of ~~mechanics~~ mechanics

Let us cut the bar at any arbitrary point C inside the bar. (Here, for simplicity, cut at half).

From fig (b), for right half,

sum of vertical forces is zero $\Rightarrow \vec{F}_y = 0\hat{j}$

sum of horizontal " " " $\Rightarrow \vec{F}_x = -100\,\mathrm{N}\,\hat{i}$

sum of moments about the cut is zero $\Rightarrow \vec{M} = 0\,\hat{k}$

Hence, it is clear that tension is 100 N at any point inside the bar. Note that tension is a scalar.

And, the tension force vector is $-T\hat{i}$ on the left end of the right half and $T\hat{i}$ on the right end of the left half. Hence, the force vector at the cut is

$$\vec{F} = T\hat{\lambda}$$

where, $\hat{\lambda}$ is a unit vector pointing out from the FBD cut, and it switches direction depending on the half rod you are looking at. The same scalar T works for both the cases.

Hence, given statement is false.

5.4.4 A string is conceptually cut in half by making free-body diagrams of the left and right halves of the string. At the cut on the left half of the string acts the force $50\,\mathrm{N}\hat{\imath}$. At the cut on the right half of the string acts the force $-50\,\mathrm{N}\hat{\imath}$. With two different forces acting on the two halves how can one define a single 'tension'?

Q4.4.4 A string is conceptually cut in half by making free-body diagrams of the left and right halves of the string. At the cut on the left half of the string acts the force 50Ni.At the cut on the right half of the string acts the force -50Ni. With two different forces acting on the two halves how can one define a single 'tension'?

<u>Solution:</u>

Tension (T) is scalar quantity!! The force vector at the cut can be expressed as

$$\vec{F} = T\hat{\boldsymbol{\lambda}}$$

it is $\hat{\boldsymbol{\lambda}}$ (the unit vector) which switches direction depending on which half rope we are looking at.

Even though we have two different forces (-50N***i*** and 50N***i***) acting on two halve, we can define single tension i.e. 50N in this case.

<u>The other way of looking at the problem:</u>

Let us conceptually cut a string in half and make FBDs,

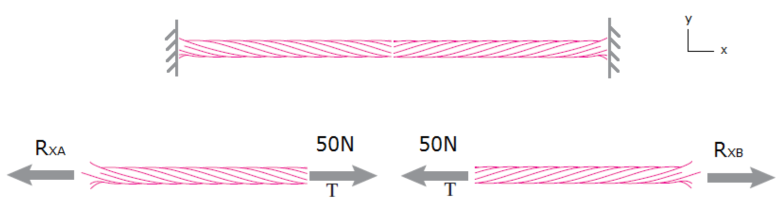


figure (Sol 4.4.4A)

Applying static force balance equation on both halves separately, we have

for left half: $\Sigma \mathbf{F}=\mathbf{0}$ → $50\mathrm{N}\boldsymbol{i} + \mathbf{R}_{XA} = \mathbf{0}$ → $\mathbf{R}_{XA} = -50\mathrm{N}\boldsymbol{i}$

Similarly, for right half: $\Sigma \mathbf{F}=\mathbf{0}$ → $-50\mathrm{N}\boldsymbol{i} + \mathbf{R}_{XB} = \mathbf{0}$ → $\mathbf{R}_{XB} = 50\mathrm{N}\boldsymbol{i}$

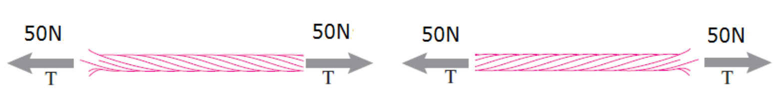


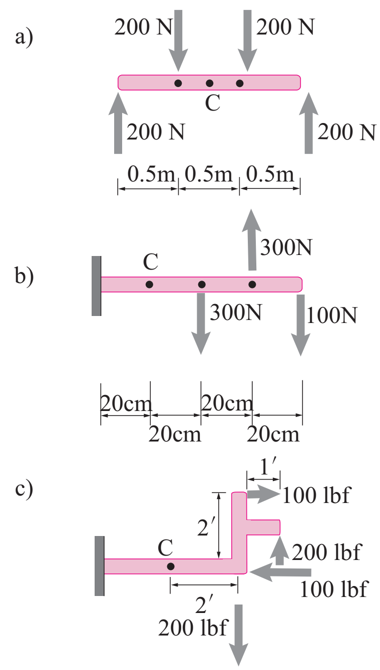
Figure (Sol 4.4.4B)

Now, the external loading on the both halves is exactly same as seen from Figure(Sol 4.4.4B)

Thus we can say same tension exist in both halves.

<u>Disclaimer:</u> *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

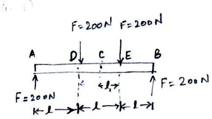
5.4.7 Find the tension, shear force and bending moment at C for each of the structures below. There is no gravity. Assume dimensions if needed.



Problem 5.7

4.4.7

a)



$l = 0.5$ m

Fig (i)

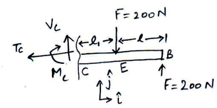


Fig (ii) FBD of the structure cut at 'C'.

Let us assume that point 'c' is distance 'l_1' to the left of point E.

Considering the FBD of the portion of the structure cut at point 'c', as shown in fig (ii),

Force balance, $\Sigma \vec{F} = \vec{0}$, gives

$$V_c \hat{j} - T_c \hat{i} - F\hat{j} + F\hat{j} = \vec{0} \quad \text{—— (i)}$$

$\{\text{equ(i)}\}\cdot\hat{i} \Rightarrow \boxed{T_c = 0N}$

$\{\text{equ(i)}\}\cdot\hat{j} \Rightarrow \boxed{V_c = 0N}$

Moment balance about point C gives

$$-M_c\hat{k} + l_1\hat{i}\times(-F\hat{j}) + (l_1+l)\hat{i}\times(F\hat{j}) = \vec{0}$$

$$\Rightarrow (-M_c + lF)\hat{k} = \vec{0}$$

$$\Rightarrow \boxed{M_c = Fl} \Rightarrow \boxed{\begin{array}{l} T_c = 0 \text{ N} \\ V_c = 0 \text{ N} \\ M_c = Fl = 100 \text{ N}\cdot\text{m} \end{array}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

4.4.7 (b) →

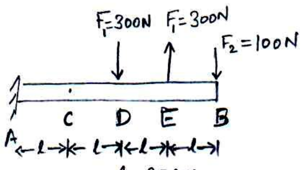
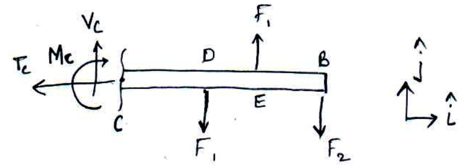


Fig (i)



Fig(ii) FBD of the structure cut at 'C'.

Considering the FBD of the portion of the structure right to point 'C', we get by force balance, $\sum \vec{F} = \vec{0}$

$$\Rightarrow \quad +V_c\hat{j} - T_c\hat{i} - F_1\hat{j} + F_1\hat{j} - F_2\hat{j} = \vec{0} \quad \text{— (i)}$$

$\{\text{eqn}(i)\}\cdot\hat{j} \Rightarrow \boxed{V_c = F_2 = 100\text{ N}}$

$\{\text{eqn}(i)\}\cdot\hat{i} \Rightarrow \boxed{T_c = 0\text{ N}}$

By doing moment balance about point 'C' we get

$$-M_c\hat{k} + (l\hat{i} \times -F_1\hat{j}) + (2l\hat{i} \times F_1\hat{j}) + (3l\hat{i} \times -F_2\hat{j}) = \vec{0}$$

$$\Rightarrow \quad (-M_c + F_1 l - 3F_2 l)\hat{k} = \vec{0}$$

In this case $F_1 = 300N$ and $F_2 = 100N$

Hence, we get $\boxed{M_c = 0\text{ N}\cdot\text{m}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Ans 4.4.7 (C) →

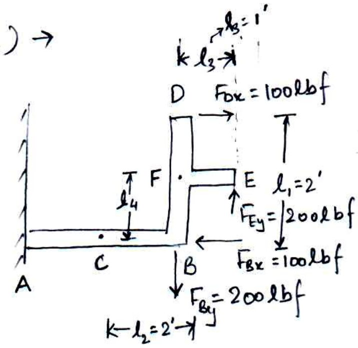


Fig (i)

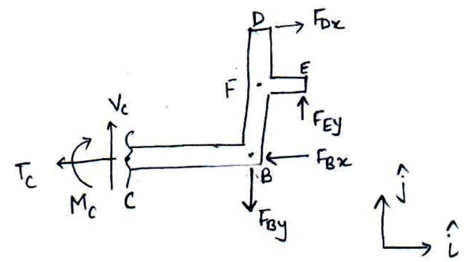


Fig (ii) FBD

Consider the section of the structure to the right of point 'c', [Fig(ii)].

Force balance, $\sum \vec{F} = \vec{0}$, gives

$$-T_c\hat{i} + V_c\hat{j} - F_{By}\hat{j} - F_{Bx}\hat{i} + F_{Ey}\hat{j} + F_{Dx}\hat{i} = \vec{0} \qquad \text{(i)}$$

$$\{\text{eqn (i)}\}\cdot\hat{i} \Rightarrow \boxed{T_c = F_{Dx} - F_{Bx} = 100\,\text{lbf} - 100\,\text{lbf} = 0\,\text{lbf}}$$

$$\{\text{eqn (i)}\}\cdot\hat{j} \Rightarrow \boxed{V_c = F_{By} - F_{Ey} = 200\,\text{lbf} - 200\,\text{lbf} = 0\,\text{lbf}}$$

Doing moment balance about point 'c', we get, $\sum \vec{M}_{/c} = \vec{0}$

$$-M_c\hat{k} + \vec{r}_{B/C}\times(-F_{By}\hat{j} - F_{Bx}\hat{i}) + \vec{r}_{E/C}\times(F_{Ey}\hat{j}) + \vec{r}_{D/C}\times(F_{Dx}\hat{i}) = \vec{0}$$

$$\Rightarrow -M_c\hat{k} + l_2\hat{i}\times(-F_{By}\hat{j} - F_{Bx}\hat{i}) + \left[(l_2+l_3)\hat{i} + l_4\hat{j}\right]\times F_{Ey}\hat{j} + (l_2\hat{i} + l_1\hat{j})\times F_{Dx}\hat{i} = \vec{0}$$

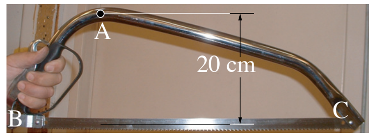
$$\Rightarrow -M_c\hat{k} - F_{By}l_2\hat{k} + (l_2+l_3)F_{Ey}\hat{k} - l_1F_{Dx}\hat{k} = \vec{0}$$

$$\Rightarrow M_c = F_{Ey}(l_2+l_3) - F_{By}l_2 - F_{Dx}l_1$$

$$\Rightarrow \boxed{M_c = (600 - 400 - 200)\,\text{lbf}\cdot\text{ft} = 0\,\text{lbf}\cdot\text{ft}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.4.9 The tension in the bow-saw blade BC is 250 N. Find the tension, shear, and bending moment at A.



Problem 5.9

4.4.9

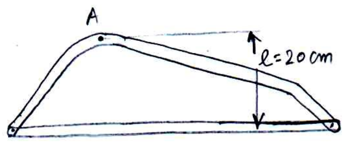


Fig. (i)

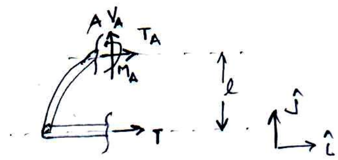


Fig (ii) FBD of the bow-Saw cut at point A and somewhere along the blade.

Force balance, considering the FBD in fig (ii) gives,

$$T_A\hat{i} + T\hat{i} + V_A\hat{j} = \vec{0} \quad \text{———} \quad (i)$$

Separating, x and y components of equ(i) we get

$$\boxed{T_A = -T = -250\,\text{N}} \quad \text{———} \quad (ii)$$

and

$$\boxed{V_A = 0\,\text{N}} \quad \text{———} \quad (ii)$$

Doing moment balance about point A we get

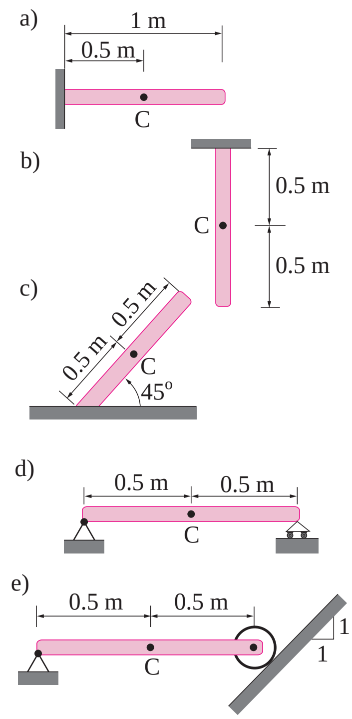
$$M_A\hat{k} + -l\hat{j}\times T\hat{i} = \vec{0}$$

$$\Rightarrow \quad M_A = -Tl = -\frac{250\times 20}{100}\ \text{N}\cdot\text{m}$$

$$\Rightarrow \quad \boxed{M_A = -50\ \text{N}\cdot\text{m}} \quad \text{———} \quad (iii)$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.4.10 Find the tension, shear and bending moment at section C for each of the structures below. And also at D, if marked. Include gravity, assume all bars are uniform with density of (100 N/m). Assume reasonable dimensions as needed.



Problem 5.10

Problem 4.4.10 solution:

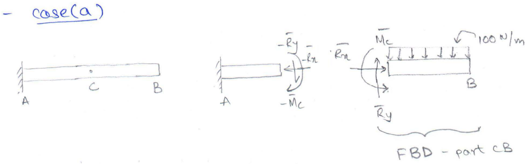
- It is given that all bars are uniform with density 100 (N/m)
- On any differential element of length ds (meters), the gravitational force ($d\bar{G}$) acting on it is given by

$$d\bar{G} = 100\ (N/m)\ ds\ (m)\ (\hat{\lambda})$$

where $\hat{\lambda}$ is unit vector along vertically downward direction.

- For calculating internal forces and moment i.e tension (T), shear force (V), and bending moment (BM) let us conceptually cut the bar at point C, for all the given cases.

- case (a)



* Applying force and moment balance equation on FBD

$$\Sigma \bar{F} = \bar{0} \Rightarrow \bar{R}_x + \bar{R}_y + \int_0^{0.5} d\bar{G} = \bar{0}$$

$$\Rightarrow T\hat{i} + V\hat{j} + 100\ (N/m) \cdot 0.5\ (m)\ (-\hat{j}) = \bar{0}$$

$$\Rightarrow \boxed{T = 0\ N} \text{ and } \boxed{V = 50 (N)}$$

$$\Sigma \bar{M} = \bar{0} \Rightarrow \Sigma M_{z/c} = 0$$

$$\Rightarrow \bar{M}_c + \int_0^{0.5} \bar{r} \times d\bar{G} = \bar{0}$$

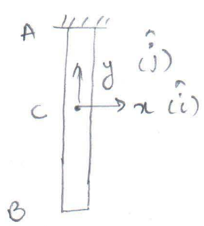
$$\Rightarrow (BM)\hat{k} + \int_0^{0.5} x(m)\,\hat{i} \times 100(N/m)\,dx(m)(-\hat{j}) = \bar{0}$$

$$\Rightarrow BM\,\hat{k} = 100\,(N/m)\,\frac{x^2}{2}(m^2)\Big|_0^{0.5}\,\hat{k}$$

$$\Rightarrow BM\,\hat{k} = 12.5\,Nm\,\hat{k}$$

$$\Rightarrow \boxed{BM = 12.5\,Nm}$$

case (b)

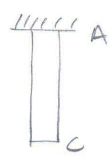


Applying force and moment balance eq^n on FBD - part BC

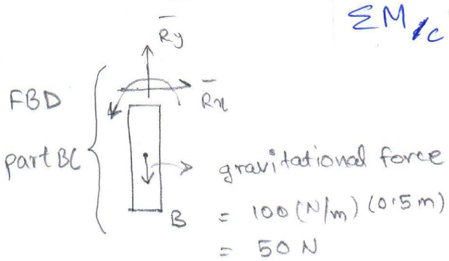
$$\Sigma \bar{F} = \bar{0} \Rightarrow \bar{R}_x + \bar{R}_y + \int_0^{-0.5} d\bar{G} = \bar{0}$$

$$\Rightarrow V\hat{i} + T\hat{j} + 100\,(N/m)\cdot 0.5\,(m)\,(-\hat{j}) = \bar{0}$$

$$\Rightarrow \boxed{V = 0\,N} \text{ and } \boxed{T = 50\,(N)}$$



$$\Sigma \bar{M}_{/c} = \bar{0} \Rightarrow \bar{M}_c + \int_0^{-0.5} \bar{r} \times d\bar{G} = \bar{0}$$

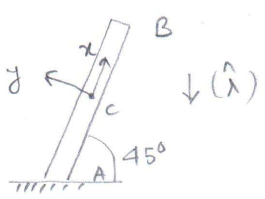


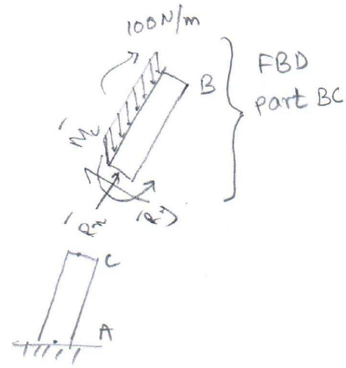
$$\Rightarrow BM\,\hat{k} = 0 \quad (\because \bar{r} \times d\bar{G} = \bar{0})$$

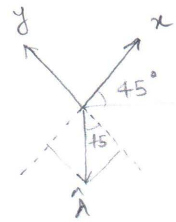
$$\Rightarrow \boxed{BM = 0\,N\cdot m}$$

Sign Convention: Counter-clockwise bending moment acting on the left face of the bar is assumed to be positive in all the cases.

case (c)







$$(\hat{\lambda}) = -\cos(45^\circ)\hat{i} - \sin(45^\circ)\hat{j}$$

$$\hat{\lambda} = -\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

gravitational force $d\bar{G} = 100(\mathrm{N/m})\, dx\,(\mathrm{m})\,(\hat{\lambda})$

$$= 100(\mathrm{N})dx \; -\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

applying force and moment balance eq^n on the FBD - part BC

$$\Sigma\bar{F} = \bar{0} \Rightarrow \bar{R}_x + \bar{R}_y + \int_0^{0.5} d\bar{G} = \bar{0}$$

$$\Rightarrow T\hat{i} + V\hat{j} - \frac{100}{\sqrt{2}}(\mathrm{N/m})\; 0.5\,(\mathrm{m})\; (\hat{i}+\hat{j}) = \bar{0}$$

$$\Rightarrow \boxed{T = \frac{50}{\sqrt{2}}\,\mathrm{N}} \quad \text{and} \quad \boxed{V = \frac{50}{\sqrt{2}}\,\mathrm{N}}$$

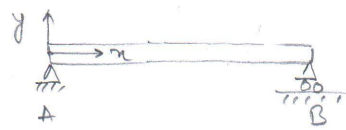
$$\Sigma\bar{M}_{/C} = \bar{0} \Rightarrow \bar{M}_C + \int_0^{0.5} \bar{r}\times d\bar{G} = \bar{0}$$

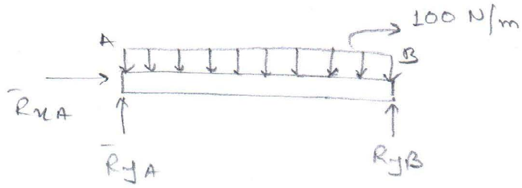
$$\rightarrow BM\,\hat{k} = \int_0^{0.5} x\,(\mathrm{m})\,\hat{i} \times \frac{100}{\sqrt{2}}(\mathrm{N/m})\,(\hat{i}+\hat{j})\,dx$$

$$BM\,\hat{k} = \frac{100}{\sqrt{2}}\frac{x^2}{2}\,(\mathrm{N\cdot m})\Big|_0^{0.5}\hat{k}$$

$$\boxed{BM = \frac{12.5}{\sqrt{2}}\,\mathrm{N\cdot m}}$$

case (d)





(FBD - part AB)

- For finding the internal forces and moment at point C, first we need to calculate, the unknown reaction acting at point B.

- Applying moment balance eq^n to FBD-part AB

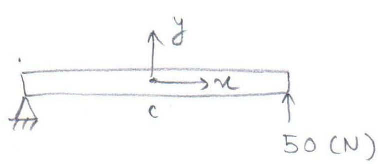
$$\Sigma \bar{M}_{/A} = \bar{0} \Rightarrow 1(m)\,\hat{i} \times |\bar{R}_{yB}|\,\hat{j} + \int_0^1 \bar{r} \times d\bar{G} = \bar{0}$$

$$\Rightarrow |\bar{R}_{yB}|(m)\,\hat{k} = \int_{0.5}^{1} x(m)\,\hat{i} \times 100\,(N/m)\,dx(m)\,(\hat{j})$$

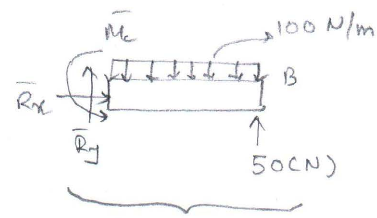
$$\Rightarrow |\bar{R}_{yB}|(m)\,\hat{k} = \frac{100}{2}x^2\Big|_0^1 (N)\,\hat{k}$$

Note: The quantity $|R_{yB}|$ is with units, so the unit (N) is NOT mentioned explicitly

$$\Rightarrow |\bar{R}_{yB}|(m) = 50\,N\cdot m \Rightarrow \boxed{|\bar{R}_{yB}| = 50\,N}$$



*Notice that the co-ordiante system has been changed



FBD part BC

applying force and moment balance eq^n to FBD. part BC

$$\Sigma \bar{F} = \bar{0} \Rightarrow \bar{R}_y + \bar{R}_x + 50\,(N)\,\hat{j} + \int_0^{0.5} d\bar{G} = \bar{0}$$

$$\Rightarrow T\,\hat{i} + V\,\hat{j} = 100\,(N/m)\;0.5(m)\,\hat{j} - 50\,(N)\,\hat{j}$$

$\boxed{T = 0\,N}$ and $\boxed{V = 0\,N}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

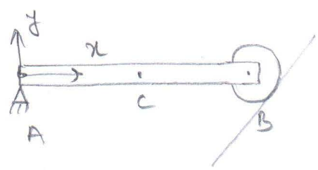
$$\Sigma \bar{M} = \bar{0} \Rightarrow \bar{M}_c + \int_0^{0.5} \bar{r} \times d\bar{G} + 0.5\,(m)\,\hat{i} \times 50\,(N)\,\hat{j} = \bar{0}$$

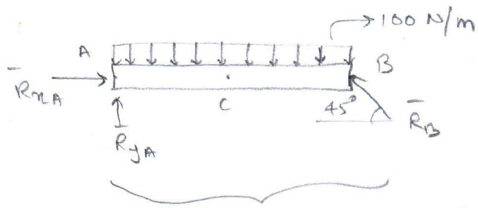
$$\Rightarrow \bar{M}_c + 25(N\cdot m)\,\hat{k} = \int_0^{0.5} x\,(m)\,\hat{i} \times 100\,(N/m)\,\hat{j}$$

$$BM\,\hat{k} = \left[100\frac{x^2}{2}\Big|_0^{0.5} (N\cdot m) - 25\,(N\cdot m) \right]\hat{k}$$

$$\boxed{BM = -12.5\,Nm}$$

case (e)





- for finding the internal forces and moment at point C, first we need to calculate, the unknown reaction acting at point B.

- Applying moment balance eq^n to FBD - part AB

$$\Sigma \bar{M}_{/A} = \bar{0} \qquad 1\,(m)\,\hat{i} \times \bar{R}_B + \int_0^1 \bar{r} \times d\bar{G} = \bar{0}$$

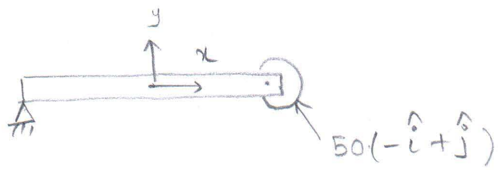
$$\Rightarrow 1\,(m)\,\hat{i} \times |\bar{R}_B| \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j}) + \int_0^1 x\,(m)\,\hat{i} \times 100\,(N/m)\,dx\,(m)(-\hat{j}) = \bar{0}$$

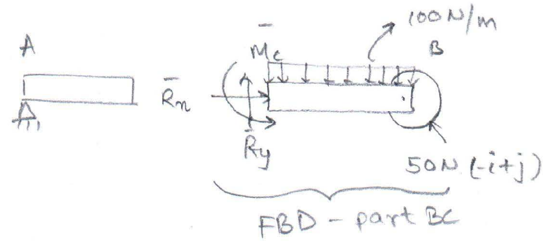
$$\Rightarrow \frac{|\bar{R}_B|}{\sqrt{2}}(m)\,\hat{k} = 100\frac{x^2}{2}\Big|_0^1 (N)\,\hat{k}$$

$$|\bar{R}_B|\,(m) = 50\sqrt{2}\ N\cdot m$$

$$\boxed{|\bar{R}_B| = 50\sqrt{2}\ N}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*





applying force and moment balance eqn to FBD part BC

$$\sum \bar{F} = 0 \Rightarrow \bar{R}_x + \bar{R}_y + 50(N)(-\hat{i}+\hat{j}) + \int_0^{0.5} d\bar{G} = 0$$

$$\Rightarrow T\hat{i} + V\hat{j} = 50N(\hat{i}-\hat{j}) + 100(N/m)\,0.5(m)\,\hat{j}$$

$$\Rightarrow \boxed{T = 50\,N} \quad \text{and} \quad \boxed{V = 0\,N}$$

$$\sum \bar{M} = 0 \Rightarrow \bar{M}_c + \int_0^{0.5} \bar{r}\times d\bar{G} + 0.5(m)\hat{i} \times 50(N)(-\hat{i}+\hat{j}) = 0$$

$$\bar{M}_c = \int_0^{0.5} x(cm)\hat{i} \times 100(N/m)\hat{j} - 25(N\cdot m)\hat{k}$$

$$BM\,\hat{k} = \left[100\left.\frac{x^2}{2}\right|_0^{0.5}(N\cdot m) - 25(N\cdot m)\right]\hat{k}$$

$$\boxed{BM = -12.5\,Nm}$$