SAMPLE 5.18 A structure is made up of two bars – a thick bent bar ABC and a thin rod CE. Point C is halfway between B and D, $\ell = 0.8\,\text{m}$ and $\theta = 60^\circ$. Bar ABC is pulled up by a force $F = 500\,\text{N}$ at point A.

1. Find the internal forces in the bar ABC just to the left of point B.
2. Find the force in bar CE at the section s-s shown in the figure.

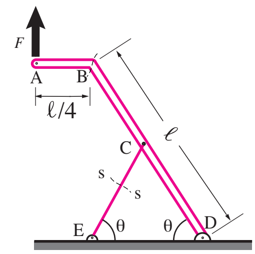

Figure 5.70

Solution We cut the bar ABC at point B. The free-body diagram of the left part AB is shown in fig. 5.71. The internal forces acting at the cut section are tension T, shear force V and the bending moment M. From force balance of part AB in x and y directions, we have

$$T = 0, \quad \text{and} \quad V = F = 500\,\text{N}.$$

From the moment balance about point B, we have

$$M - F\ell/4 = 0 \quad \Rightarrow \quad M = F\ell/4 = 100\,\text{N}\cdot\text{m}.$$

$$\boxed{T = 0, \quad V = 500\,\text{N}, \quad M = 100\,\text{N}\cdot\text{m}}$$

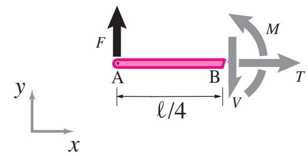

Figure 5.71

For finding the tension in rod CE at the given section, we cut the rod at s-s and draw the free-body diagram of the structure along with the upper part of the rod attached at point C. The tension in bar CE is T and the reaction of the support at pin D is R. We need to find T.

We can write the moment-balance equation about point D, $\sum \vec{\boldsymbol{M}}_{\text{D}} = \vec{\boldsymbol{0}}$, so that the unknown force R (that we are not interested in) disappears from the equation:

$$\vec{\boldsymbol{r}}_{\text{A/D}} \times \vec{\boldsymbol{F}} + \vec{\boldsymbol{r}}_{\text{C/D}} \times \vec{\boldsymbol{T}} \quad = \quad \vec{\boldsymbol{0}}.$$

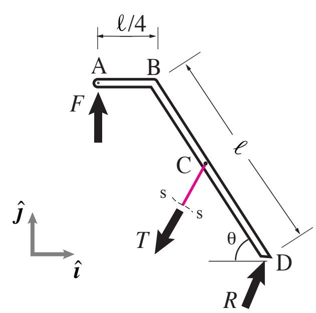

Figure 5.72

The moments of F and T about point D can be easily evaluated using the scalar formula 'force times the lever arm' (see fig. 5.73). Thus, the moment-balance equation in $\hat{\boldsymbol{k}}$ direction is:

$$\begin{aligned} -F\ell(1/4 + \cos\theta) + T\frac{\ell}{2}\sin 2\theta &= 0 \\ \Rightarrow \quad T = \frac{2(1/4+\cos\theta)}{\sin 2\theta}F. \end{aligned}$$

Substituting the given values, $F = 500\,\text{N}$ and $\theta = 60^\circ$, we get

$$T = 866\,\text{N}.$$

$$\boxed{T = 866\,\text{N}}$$

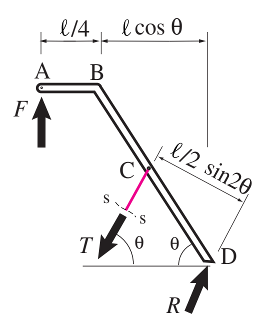

Figure 5.73: Moments of F and T about point D can be evaluated by multiplying the forces with their respective lever arms $\ell/4 + \ell\cos\theta$ and $\ell/2\sin 2\theta$, respectively.

Note: Evaluation of the moment equation about point D using vectors and cross products is as follows. Since $\vec{\boldsymbol{r}}_{\text{A/D}} = \vec{\boldsymbol{r}}_{\text{A/B}} + \vec{\boldsymbol{r}}_{\text{B/D}} = -\frac{\ell}{4}\hat{\boldsymbol{\imath}} + \ell(-\cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\hat{\boldsymbol{\jmath}})$, $\vec{\boldsymbol{r}}_{\text{C/D}} = \frac{\ell}{2}(-\cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\hat{\boldsymbol{\jmath}})$, $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\jmath}}$, and $\vec{\boldsymbol{T}} = T(-\cos\theta\hat{\boldsymbol{\imath}} - \sin\theta\hat{\boldsymbol{\jmath}})$,

$$\vec{\boldsymbol{r}}_{\text{A/D}} \times \vec{\boldsymbol{F}} = -F\left(\frac{\ell}{4} + \ell\cos\theta\right)\hat{\boldsymbol{k}}, \text{ and } \vec{\boldsymbol{r}}_{\text{C/D}} \times \vec{\boldsymbol{T}} = T\ell\cos\theta\sin\theta\hat{\boldsymbol{k}}.$$

Therefore, the moment-balance equation is

$$-F\ell(1/4 + \cos\theta)\hat{\boldsymbol{k}} + T\frac{\ell}{2}\sin 2\theta\hat{\boldsymbol{k}} = \vec{\boldsymbol{0}}.$$

SAMPLE 5.19 A ladder of length $2d = 4\,\mathrm{m}$ rests against a wall as shown. A person of weight $W = 700\,\mathrm{N}$ stands at C. Assume that the ladder does not slip. Neglecting the weight of the ladder, find the internal forces in the ladder at sections a-a and b-b, at mid points of AC and AB, respectively. (See Sample 5.14.)

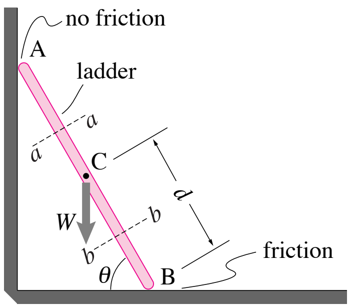

Figure 5.74

Solution To find the internal forces at the indicated sections, we need to cut the ladder at those sections, one at a time, draw the free-body diagram of each part and carry out the force and moment-balance equations. A little anticipation shows that we will need the support reactions at A and B in our calculations. So, let us first determine the support reactions. The free-body diagram of the ladder is shown in fig. 5.75. The moment balance about point B in $\hat{\boldsymbol{k}}$ direction gives

$$-R(2d\sin\theta) + W(d\cos\theta) = 0 \quad \Rightarrow \quad R = \frac{W}{2}\frac{\cos\theta}{\sin\theta}.$$

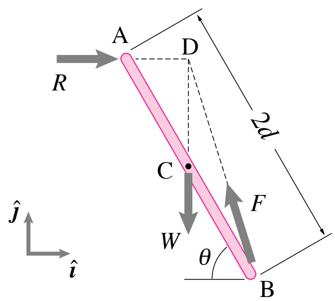

Figure 5.75

The force balance, $\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}$, gives

$$R\hat{\boldsymbol{\imath}} - W\hat{\boldsymbol{\jmath}} + \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \quad \Rightarrow \quad \vec{\boldsymbol{F}} = -R\hat{\boldsymbol{\imath}} + W\hat{\boldsymbol{\jmath}}.$$

Substituting the given values of $\theta(60^{\circ})$ and $W(700\,\mathrm{N})$, we get,

$$\vec{\boldsymbol{R}} = (202\,\mathrm{N})\hat{\boldsymbol{\imath}}, \quad \text{and} \quad \vec{\boldsymbol{F}} = (-202\hat{\boldsymbol{\imath}} + 700\hat{\boldsymbol{\jmath}})\,\mathrm{N}.$$

Section a-a: Now, we cut the ladder at a-a and draw the free-body diagram of the upper part of the ladder as shown in fig. 5.76. The force balance for this part gives

$$\begin{aligned} T\hat{\boldsymbol{\lambda}} - V\hat{\boldsymbol{n}} + R\hat{\boldsymbol{\imath}} &= \vec{\boldsymbol{0}} \\ \Rightarrow \quad T &= -R(\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{\lambda}}) = -R\cos\theta \\ \text{and} \quad V &= R(\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{n}}) = R\sin\theta. \end{aligned}$$

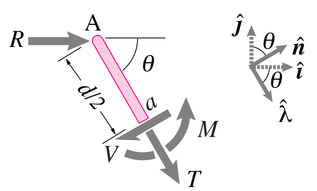

Figure 5.76: Free-body diagram of the upper part of the ladder cut at section a-a. Note that $\hat{\boldsymbol{\lambda}} = \cos\theta\hat{\boldsymbol{\imath}} - \sin\theta\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{n}} = \sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\hat{\boldsymbol{\jmath}}$.

Substituting the numerical values of R and θ, we get $T = -101\,\mathrm{N}$ and $V = 175\,\mathrm{N}$. Now, the moment-balance equation about a (the cut) gives

$$M - R(d/2)\sin\theta = 0 \quad \Rightarrow \quad M = (1/2)Rd\sin\theta$$

which, with numerical values, gives $M = 175\,\mathrm{N{\cdot}m}$.

$$\boxed{T = -101\,\mathrm{N}, \quad V = 175\,\mathrm{N}, \quad M = 175\,\mathrm{N{\cdot}m}}$$

Section b-b: Now we consider the internal forces at section b-b. We cut the ladder at the given section. We can consider the free-body diagram of the upper part or the lower part of the ladder to find the internal forces. Considering the upper part, (see fig. 5.77) we get, from force balance,

$$T\hat{\boldsymbol{\lambda}} - V\hat{\boldsymbol{n}} + R\hat{\boldsymbol{\imath}} - W\hat{\boldsymbol{\jmath}} = \vec{\boldsymbol{0}}$$

which, as the analysis above, gives

$$\begin{aligned} T &= -R(\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{\lambda}}) + W(\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{\lambda}}) = -R\cos\theta + W(-\sin\theta) = -707\,\mathrm{N} \\ V &= R(\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{n}}) - W(\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{n}}) = R\sin\theta - W\cos\theta = -175\,\mathrm{N}. \end{aligned}$$

Similarly, the scalar moment-balance equation about point b gives

$$M - R\frac{3d}{2}\sin\theta + W\frac{d}{2}\cos\theta = 0 \quad \Rightarrow \quad M = 175\,\mathrm{N{\cdot}m}.$$

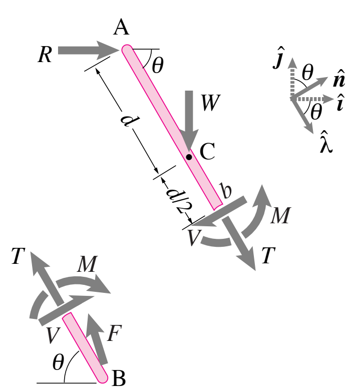

Figure 5.77

$$\boxed{T = -707\,\mathrm{N}, \quad V = -175\,\mathrm{N}, \quad M = 175\,\mathrm{N{\cdot}m}}$$