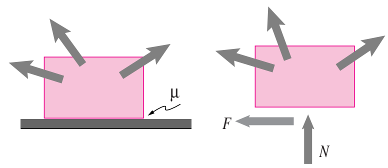
5.3.1 For the block shown in the figure, what do you know about F if

a) the block is sliding to the right

b) the block is sliding to the left

c) the block is not sliding.



Problem 5.1

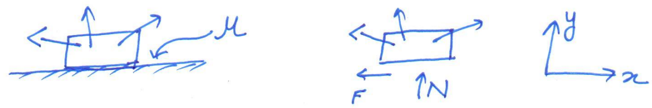
Roberto Padovani

a) $F = \mu N$

b) $F = -\mu N$

c) $|F| \leq \mu N$

4.3.1

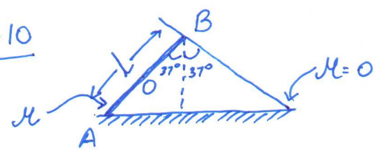


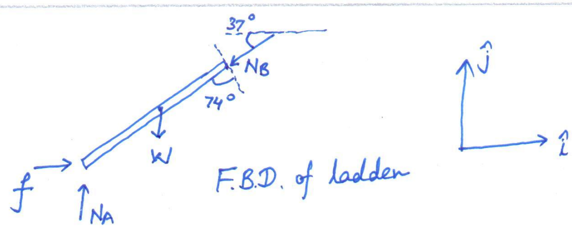
a) $\vec{F} = \mu \vec{N}$ if the block slides to the right.

b) $\vec{F} = -\mu \vec{N}$ if the block slides to the left.

c) $-\mu\vec{N} \leqslant \vec{F} \leqslant \mu\vec{N}$ if the block remains stationary.

4.3.10





F.B.D. of ladder

$$\Sigma \vec{F} = \vec{0}$$

In x-direction, $(f - N_B \cos 37°)\hat{i} = 0\hat{i}$

$\Rightarrow f = N_B \cos 37° \ldots$ (1)

In y-direction, $(N_A - W - N_B \sin 37°)\hat{j} = 0\hat{j}$

$\Rightarrow N_A - W = N_B \sin 37°$

$\Rightarrow \dfrac{N_A - W}{\sin 37°} = N_B \ldots$ (2)

$\Sigma \vec{M} = \vec{0}$ about point B

$\therefore -N_A(l \sin 37°) + W\left(\frac{l}{2}\sin 37°\right) + f(l\cos 37°) = 0$

$\Rightarrow N_A(\cancel{l}\sin 37°) = \dfrac{W\cancel{l}\sin 37°}{2} + f\cancel{l}\cos 37°$

$\Rightarrow N_A(\sin 37°) = \dfrac{W \sin 37°}{2} + f\cos 37° \Rightarrow N_A = \dfrac{W}{2} + f\cot 37° \ldots$ (3)

Substituting (2) in (1), $f = \dfrac{N_A - W}{\sin 37°}\cos 37° \Rightarrow f = (N_A - W)\cot 37° \ldots$ (4)

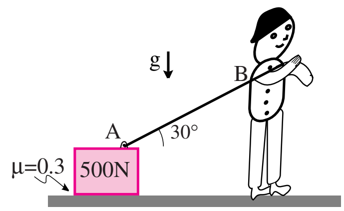
Substituting (3) in (4), we get $f = \left(\frac{-W}{2} + f\cot 37°\right)\cot 37°$. Solving, we get $f = 1.744W \ldots$ (5)

Substituting (5) in (4), we get $\dfrac{1.744W}{\cot 37°} = N_A - W$. Solving, we get $N_A = 2.314W \ldots$ (6)

Now $f \leqslant \mu N_A \Rightarrow 1.744\cancel{W} \leqslant 2.314\cancel{W}\mu \Rightarrow \boxed{\mu \geqslant 0.754}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.2 A block weighing 500 N is dragged slowly on the ground as shown in the figure. Find the tension in the string.



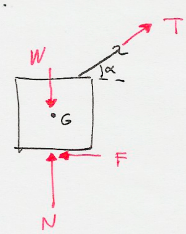
Problem 5.2

Roberto Padovani

$\alpha = 30°$; $\mu = 0.3$

$W = 500\,N$

F.B.D.



$\sum \vec{F}_i = \vec{0}$ (1)

$\{1\} \cdot \hat{i} \Rightarrow -F + T\cos(\alpha) = 0 \Rightarrow$

$\Rightarrow -\mu N + T\cos(\alpha) = 0 \Rightarrow$

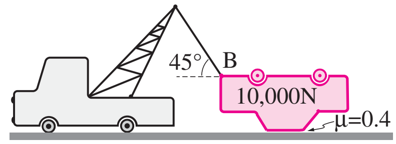
$\Rightarrow N = \frac{1}{\mu} T\cos(\alpha)$

$\{1\} \cdot \hat{j} \Rightarrow N - W + T\sin(\alpha) = 0 \Rightarrow \frac{1}{\mu} T\cos(\alpha) + T\sin(\alpha) = W \Rightarrow$

$\Rightarrow \boxed{T} = W \frac{1}{\frac{\cos(\alpha)}{\mu} + \sin(\alpha)} \simeq \boxed{145\,N}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.3 Find the tension in the cable assuming the car is dragged at constant speed.



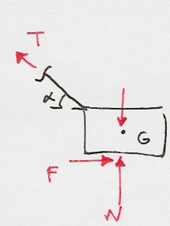
Problem 5.3

Roberto Padovani

$\alpha = 45^\circ \quad ; \quad \mu = 0.4$

$W = 10\,000\,N$

F.B.D.



$\sum \vec{F}_i = \vec{0} \qquad (1)$

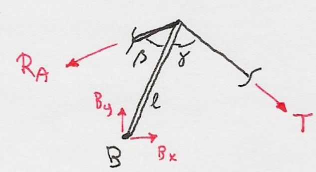
$\{(1)\}\cdot\hat{\imath} \Rightarrow -F + T\cos(\alpha) = 0 \Rightarrow$

$\Rightarrow -\mu N + T\cos(\alpha) = 0 \Rightarrow N = \frac{1}{\mu} T\cos(\alpha)$

$\{(1)\}\cdot\hat{\jmath} \Rightarrow N - W + T\sin(\alpha) = 0 \Rightarrow \frac{1}{\mu} T\cos(\alpha) + T\sin(\alpha) = W \Rightarrow$

$\Rightarrow \boxed{T} = W \dfrac{1}{\dfrac{\cos(\alpha)}{\mu} + \sin(\alpha)} \simeq \boxed{4041\,N}$

F.B.D.



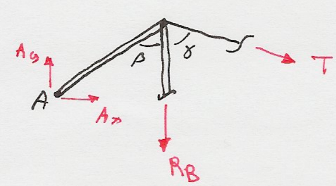
$\left\{\sum \vec{M}_{/B} = \vec{0}\right\}\cdot\hat{k} \Rightarrow$

$\Rightarrow R_A \cancel{\ell} \sin(\beta) = T \cancel{\ell} \sin(\gamma) \Rightarrow$

$\Rightarrow \boxed{R_A} = T \dfrac{\sin(\gamma)}{\sin(\beta)} \quad \boxed{>0}$ because $T > 0$ and $0^\circ < \gamma + \beta < 180^\circ$

↳ trazione!

F.B.D.



$\left\{\sum \vec{M}_{/A} = \vec{0}\right\}\cdot\hat{k} \Rightarrow$

$\Rightarrow R_B \ell \sin(\beta) = -T \ell \sin(\beta + \gamma) \Rightarrow$

$\Rightarrow \boxed{R_B} = -T \dfrac{\sin(\beta+\gamma)}{\sin(\beta)} \quad \boxed{<0}$ because $T > 0$ and $0^\circ < \gamma + \beta < 180^\circ$

↳ compressione!

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.4 Consider the tow truck dragging the car in Problem 5.3.3 again. In order to ensure safety, you would like to minimize the tension in the rope attached to the car. Assume that the angle shown at point B is θ.

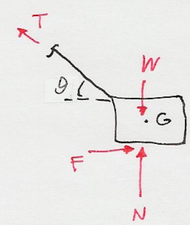
a) What value of θ minimizes the tension in the rope?

b) What is the corresponding value of T?

c) What is the force of the ground on the car?

Roberto Padovani

F.B.D.



$$\sum \vec{F}_i = \vec{0} \qquad (1)$$

$$\{(1)\}\cdot\hat{\imath} \Rightarrow -F + T\cos(\theta) = 0 \Rightarrow$$

$$\Rightarrow -\mu N + T\cos(\theta) = 0 \Rightarrow$$

$$\Rightarrow N = \frac{1}{\mu} T\cos(\theta)$$

$$\{(1)\}\cdot\hat{\jmath} \Rightarrow N - W + T\sin(\theta) = 0 \Rightarrow \frac{1}{\mu} T\cos(\theta) + T\sin(\theta) = W \Rightarrow$$

$$\Rightarrow T = W\left(\frac{\cos(\theta)}{\mu} + \sin(\theta)\right)^{-1}$$

$$\frac{dT}{d\theta} = -W\left(\frac{\cos(\theta)}{\mu} + \sin(\theta)\right)^{-2}\left(-\frac{\sin(\theta)}{\mu} + \cos(\theta)\right)$$

$$\theta^* \text{ maximum or minimum} \Leftrightarrow \left.\frac{dT}{d\theta}\right|_{\theta=\theta^*} = 0$$

$$\Rightarrow -\frac{\sin(\theta^*)}{\mu} + \cos(\theta^*) = 0 \Rightarrow \tan(\theta^*) = \mu \Rightarrow \boxed{\theta^* = \arctan(\mu)}$$

$$\frac{d^2T}{d\theta^2} = 2W\left(\frac{\cos(\theta)}{\mu} + \sin(\theta)\right)^{-3}\left(-\frac{\sin(\theta)}{\mu} + \cos(\theta)\right)^2 - W\left(\frac{\cos(\theta)}{\mu} + \sin(\theta)\right)^{-2}\left(-\frac{\cos(\theta)}{\mu} - \sin(\theta)\right)$$

$$\left.\frac{d^2T}{d\theta^2}\right|_{\theta=\theta^*} = 2W\left(\frac{\cos(\theta^*)}{\mu} + \sin(\theta^*)\right)^{-3}\cancel{\underbrace{\left(-\frac{\sin(\theta^*)}{\mu} + \cos(\theta^*)\right)^2}_{=0}} +$$

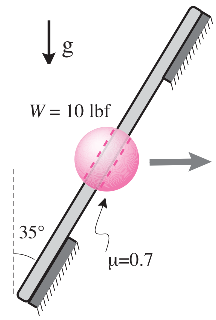
$$+ W\left(\frac{\cos(\theta^*)}{\mu} + \sin(\theta^*)\right)^{-1} = W\left(\frac{\cos(\theta^*)}{\mu} + \sin(\theta^*)\right) > 0$$

$$\Rightarrow \theta^* \text{ is a minimum}$$

$$\boxed{\text{minimum } T^*} = T|_{\theta=\theta^*} \simeq \boxed{3714\,N} \qquad \boxed{N =} \frac{1}{\mu} T^*\cos(\theta^*) \cong \boxed{8621\,N}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.7 A horizontal force F is applied to slide the bead on the rod shown in the figure. Find the value of F that is required to initiate sliding up the rod. Why is F so big or small?



Problem 5.7

4.3.7

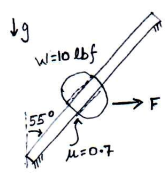


Fig (i)

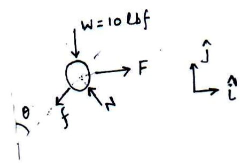


Fig (ii) FBD of the bead

In order to initiate sliding, the component of external force 'F' along the rod has to overcome friction force and the component of weight of the bead, that are acting downward along the rod.
In this scenario the frictional force will be critical friction force given by, $f = \mu N$.

Now by force balance we get;

$$F\hat{i} - w\hat{j} - f\sin\theta\,\hat{i} - f\cos\theta\,\hat{j} - N\cos\theta\,\hat{i} + N\sin\theta\,\hat{j} = \vec{0}$$

Substituting $f = \mu N$, we get,

$$F\hat{i} - w\hat{j} - (\mu N\sin\theta + N\cos\theta)\,\hat{i} - [\mu N\cos\theta - N\sin\theta]\,\hat{j} = \vec{0} \quad \text{—— (i)}$$

$\{equ.(i)\}\cdot\hat{j}$ gives $N\sin\theta - \mu N\cos\theta = w$

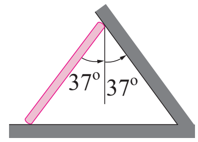
$$\Rightarrow N = \frac{w}{\sin\theta - \mu\cos\theta} \Rightarrow \boxed{N = \frac{10\text{ lbf}}{(\sin 55^\circ - 0.7\cos 55^\circ)} = 23.943\text{ lbf}} \quad \text{—— (ii)}$$

$\{equ.(i)\}\cdot\hat{i}$ gives, $F = \mu N\sin\theta + N\cos\theta$

$$\Rightarrow \boxed{F = \frac{w(\mu\sin\theta + \cos\theta)}{\sin\theta - \mu\cos\theta} = 27.462\text{ lbf}}$$

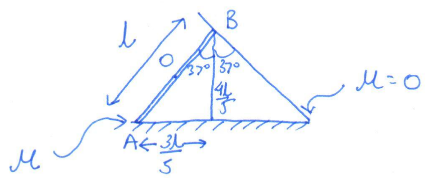
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

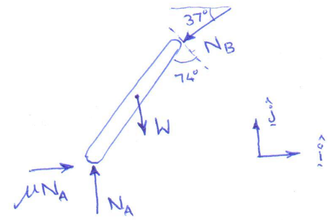
5.3.10 A uniform ladder of length ℓ and weight W rests against a frictionless slanted wall. What is the minimum μ between ladder and ground that is needed to hold the ladder in position?



Problem 5.10

4.3.10





F.B.D

$\sum \vec{F} = \vec{0}$

In x-direction, $\mu N_A - N_B \cos 37° = 0$

$\Rightarrow \mu N_A = N_B \cos 37° \Rightarrow N_B = \dfrac{\mu N_A}{\cos 37°}$ ... (1)

In y-direction, $N_A - W - N_B \sin 37° = 0$

$\Rightarrow N_A = W + N_B \sin 37°$

From (1), $N_A = W + \mu N_A \tan 37°$

$\because \tan 37° = 3/4 \Rightarrow N_A = W + \frac{3}{4}\mu N_A$

$\Rightarrow N_A = \dfrac{4W}{4-3\mu}$ .... (2)

$\sum \vec{M} = \vec{0}$ about point B

$-N_A\left(\frac{3l}{5}\right) + W\left(\frac{3l}{10}\right) + \mu N_A\left(\frac{4l}{5}\right) = 0$ ... (3)

$N_A\left(\dfrac{4\mu l - 3l}{5}\right) + \dfrac{3Wl}{10} = 0$ ... (4)

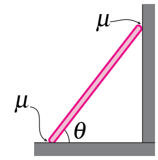
Substituting (2) in (4)

$\dfrac{4\cancel{W}}{4-3\mu}\left(\dfrac{4\mu\cancel{l} - 3\cancel{l}}{5}\right) + \dfrac{3\cancel{W}\cancel{l}}{10} = 0$

$\Rightarrow \dfrac{4}{4-3\mu}\left(\dfrac{4\mu - 3}{5}\right) + \dfrac{3}{10} = 0$ .. (5)

Solving (5) we get $\boxed{\mu = 0.521}$

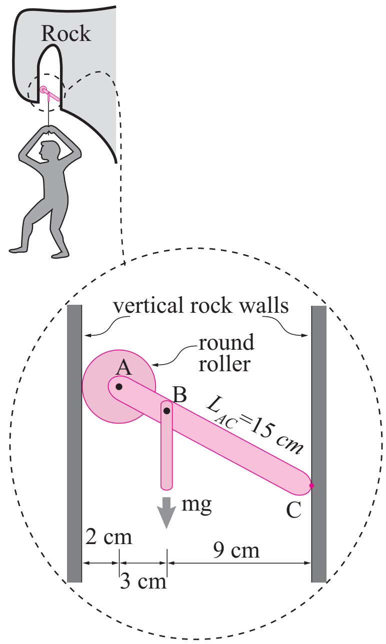
5.3.12 A uniform ladder with weight W and length ℓ leans against a frictional vertical wall and is supported by the frictional ground. The same coefficient of friction μ applies to the wall and to the ground. In terms of the given quantities, find the values of θ between the ladder and ground for which the ladder can be in equilibrium without slipping.



Problem 5.12

Answer:
$\theta \geq \tan^{-1}\left((1-\mu^2)/2\mu\right)$

5.3.15 A candidate rock-climbing device consists of a roller (radius 2 cm) frictionlessly pinned at A to diagonal-member AC. The length of AC from point A to the wall-contact point at C is $L_{AC} = 15cm$. The climber ($m = 60$ kg) hangs from a rope connected to AC by a pin at B. B is on the line AC and located as shown in the figure. If needed, assume $g = 10$N/kg. What is the minimum coefficient of friction μ at C that is needed to hold up the climber?



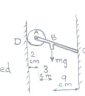
Problem 5.15

Answer:
For this device to hold, $\mu \geq 1$. (Demanding $\mu \geq 1$ is large for a practical device because typical rock friction has $\mu \approx 0.5$. The too-large number follows from the simplified geometry and numbers chosen for a homework problem.)

4.3.15 Rock-climbing device:

Find the minimum coefficient of friction μ needed to hold up the climber.

Let the reaction force vector at 'C' be $\underline{F}_c = F\hat{\underline{j}} - N\hat{\underline{i}}$.



As the diagonal member AC is frictionlessly pinned to the roller center 'A', and the device is in equilibrium, the reaction from wall at D passes through the center 'A' preventing the roller from rolling.

Applying L.M.B. $\Sigma \underline{F} = \underline{0}$

$\Rightarrow \{R\hat{\underline{i}} + F\hat{\underline{j}} - N\hat{\underline{i}} - mg\hat{\underline{j}} = \underline{0}\}$

$\{\}\cdot\hat{\underline{i}} \Rightarrow R - N = 0 \Rightarrow R = N$ — (1)

$\{\}\cdot\hat{\underline{j}} \Rightarrow F - mg = 0 \Rightarrow F = mg$ — (2)

Applying A.M.B, $\Sigma \underline{M}_{/A} = \underline{0}$

$\underline{r}_{C/A} \times (F\hat{\underline{j}} - N\hat{\underline{i}}) + \underline{r}_{B/A} \times mg(-\hat{\underline{j}}) + \underline{r}_{D/A} \times R\hat{\underline{i}} = \underline{0}$

$\Rightarrow L_{AC} F(\hat{\underline{\lambda}} \times \hat{\underline{j}}) - L_{AC} N(\hat{\underline{\lambda}} \times \hat{\underline{i}}) + L_{AB}\, mg(\hat{\underline{\lambda}} \times -\hat{\underline{j}}) + \underbrace{(-2\hat{\underline{i}}(cm)) \times R\hat{\underline{i}}}_{0} = \underline{0}$

$\Rightarrow \{F L_{AC}\left[\frac{12}{15}\right]\hat{\underline{k}} - N L_{AC}\left[\frac{9}{15}\right]\hat{\underline{k}} - mg\, L_{AB}\left[\frac{12}{15}\right]\hat{\underline{k}} = \underline{0}\}$

$\{\}\cdot\hat{\underline{k}} \Rightarrow F L_{AC}\frac{12}{15} - N L_{AC}\frac{9}{15} - mg\, L_{AB}\frac{12}{15} = 0$

From (2)

$N L_{AC} 9 = mg\, L_{AC} 12 - mg\, L_{AB} 12$

$\Rightarrow 9N = 12mg - 12\frac{L_{AB}}{L_{AC}} mg$

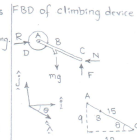
$\Rightarrow 9N = 12mg - 3mg$

$\Rightarrow N = mg$ — (3)

$\therefore$ The minimum coefficient of friction required for equilibrium is, $\mu = \frac{F}{N} = \frac{mg}{mg} = 1$.

$\therefore$ For the given device to hold mg, $\boxed{\mu \geq 1}$

FBD of climbing device



$L_{AB} = \frac{3}{\cos\theta} = \frac{15}{4}$ cm

$\underline{r}_{B/A} = L_{AB}\hat{\underline{\lambda}}$

$L_{AC} = \frac{12}{\cos\theta} = 15$cm (given)

$\underline{r}_{C/A} = L_{AC}\hat{\underline{\lambda}}$

$\frac{L_{AB}}{L_{AC}} = \frac{1}{4}$

$\hat{\underline{\lambda}} = \cos\theta\,\hat{\underline{i}} - \sin\theta\,\hat{\underline{j}}$

$\hat{\underline{\lambda}} = \frac{1}{15}(12\hat{\underline{i}} - 9\hat{\underline{j}})$

$\hat{\underline{\lambda}} \times \hat{\underline{j}} = \frac{1}{15}(12\hat{\underline{i}} - 9\hat{\underline{j}}) \times \hat{\underline{j}} = \frac{12}{15}\hat{\underline{k}}$

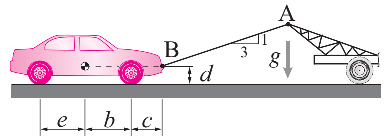
$\hat{\underline{\lambda}} \times \hat{\underline{i}} = \frac{1}{15}(12\hat{\underline{i}} - 9\hat{\underline{j}}) \times \hat{\underline{i}} = \frac{9}{15}\hat{\underline{k}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.19 A car is being towed. Unfortunately all the wheels are locked and skidding with friction coefficient μ. The tow cable AB has a slope of 1/3.

a) In terms of some or all of $e, b, c, d, m,$ g& μ, find the tension in the tow cable AB.

b) Instead of an angle with slope 1/3, what should the cable angle be to minimize the tension.



Problem 5.19

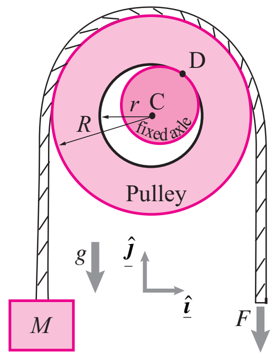
Answer:
$T_{AB} = \sqrt{10}\mu mg/(3+\mu)$
Minimum tension if rope slope is μ (instead of 1/3)

5.3.20 A weight M is steadily raised by pulling with a force F on a rope going over a negligible-mass pulley on an unlubricated journal bearing (no ball bearings). For an ideal frictionless pulley $F = Mg$. Here, however, we have a friction coefficient between the bearing and its axle which is $\mu = \tan\phi$. [Hint: Finding the location of the contact point D is part of the problem.]

a) Find F in terms of M, g, R, r and μ (or ϕ or $\sin\phi$ or $\cos\phi$ — whichever is most convenient. For example $\cos(\tan^{-1}(\mu))$ is more simply expressed as $\cos\phi$), *and*

b) Evaluate F in the special case that $M = 100\,\text{kg}, g = 10\,\text{m/s}^2, r = 1\,\text{cm}, R = 2\,\text{cm}$, and $\mu = \sqrt{3}/3$ (so $\phi = \pi/6, \sin\phi = 1/2, \cos\phi = \sqrt{3}/2$).

c) Referring back to the general case, for fixed r, R, M, and g what happens to F as $\mu \to \infty$ (does it go to ∞)?



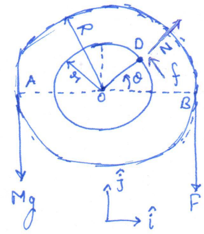
Problem 5.20

4.3.20

There is the friction between pulley and and axle.

We assumed that pulley and axle is in contact at only one point i.e. D

D $(r\cos\theta, r\sin\theta)$



$f = \mu N$

(a)

$$\sum \vec{F} = \vec{0}$$

$$\Rightarrow N(\mu\sin\theta - \cos\theta)\hat{i} - [N(\sin\theta + \mu\cos\theta) + Mg + F]\hat{j} = \vec{0}$$

$$\sum \vec{F} = N(\mu\sin\theta - \cos\theta)\hat{i} - [N(\sin\theta + \mu\cos\theta) + Mg + F]\hat{j}$$

$$\sum F_x \hat{i} + \sum F_y \hat{j} = \sum \vec{F} \Rightarrow \boxed{\begin{array}{l} \sum F_x = \sum \vec{F}\cdot\hat{i} \\ \sum F_y = \sum \vec{F}\cdot\hat{j} \end{array}}$$

$$\therefore \sum F_x = N(\mu\sin\theta - \cos\theta)$$

$$\sum F_y = -[N(\sin\theta + \mu\cos\theta) + Mg + F]$$

Now if $\sum \vec{F} = \vec{0}$ then $\sum F_x$ & $\sum F_y$ need to be zero

① $\sum F_x = 0 \Rightarrow N(\mu\sin\theta - \cos\theta) = 0$

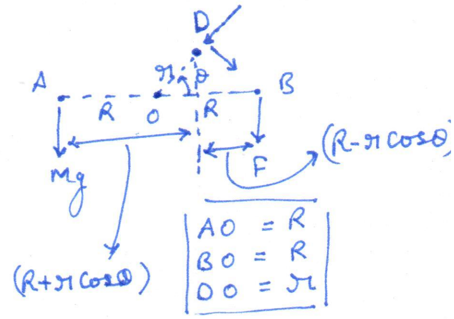
$$\Rightarrow \mu\sin\theta - \cos\theta = 0$$

$$\mu\sin\theta = \cos\theta \quad \text{----} ①$$

$\Rightarrow$ this imply that net Reaction force at point D is vertical only, it doesnot have horizontal component.

(2) $F_y = 0$

$$\Rightarrow N(\sin\theta + \mu\cos\theta) + Mg + F = 0 \quad \text{--- (2)}$$



Now

$$\sum \vec{M}_D = \vec{0}$$

$$\left[Mg(R + r\cos\theta) - F(R - r\cos\theta)\right]\hat{k} = \vec{0}$$

$$\Rightarrow F = Mg\left[\frac{R + r\cos\theta}{R - r\cos\theta}\right] \quad \text{---- (3)}$$

Now from eq (1) $\tan\theta = 1/\mu \Rightarrow \boxed{\cos\theta = \frac{\mu}{\sqrt{1+\mu^2}}}$

$$\therefore \boxed{F = Mg\left[\frac{1 + \left(\frac{r}{R}\right)\frac{\mu}{\sqrt{1+\mu^2}}}{1 - \left(\frac{r}{R}\right)\frac{\mu}{\sqrt{1+\mu^2}}}\right]} \quad \text{--- (4)}$$

if $\left(\frac{r}{R}\right) \to 0$, then $F \to Mg$

otherwise $F > Mg$ until $\mu > 0$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(b) Now $M = 100\,kg$, $g = 10\,m/sec^2$, $r = 1\,cm$ & $R = 2\,cm$

$\mu = 1/\sqrt{3} \Rightarrow \mu^2 = 1/3$

using eq (4)

$$F = 1000\left[\frac{5}{3}\right]N > Mg = 1000\,N$$

$$\boxed{F = \frac{5000}{3}N}$$

(c) if $\mu \to \infty$

$$\lim_{\mu\to\infty} F = \lim_{\mu\to\infty}\left[Mg\left[\frac{1 + \left(\frac{r}{R}\right)\frac{1}{\sqrt{1/\mu^2+1}}}{1 - \left(\frac{r}{R}\right)\frac{1}{\sqrt{1/\mu^2+1}}}\right]\right]$$

$$\boxed{F = Mg\left[\frac{1 + r/R}{1 - r/R}\right]}$$

Note if friction is infinite then still Pulley will move.

$\Rightarrow$ If r is small, then F is ~~cots~~ close to Mg even if μ (friction coefficient) is huge.

5.3.21 A reel of mass M and outer radius R is connected by a horizontal string from point P across a pulley to a hanging object of mass m. The inner cylinder of the reel has radius $r = \frac{1}{2}R$. The slope has angle θ. There is no slip between the reel and the slope. There is gravity.

a) Find the ratio of the masses so that the system is at rest.

b) Find the corresponding tension in the string, in terms of M, g, R, and θ.

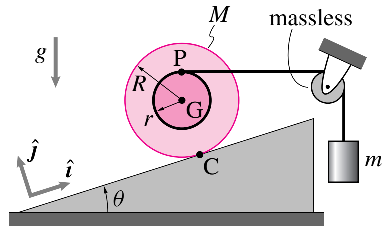
c) Find the corresponding force on the reel at its point of contact with the slope, point C, in terms of M, g, R, and θ.

Harder Draw a careful sketch and find a point where the lines of action of the gravity force and string tension intersect. For the reel to be in static equilibrium, the line of action of the reaction force at C must pass through this point. Using this information, what must the tangent of the angle ϕ of the reaction force at C be, measured with respect to the normal to the slope? Does this answer agree with that you would obtain from your answer in part(c)?

d) What is the relationship between the angle ϕ of the reaction at C, measured with respect to the normal to the ground, and the mass ratio required for static equilibrium of the reel?

e) What is the minimum coefficient of friction μ at C needed to prevent slip.

Check that for $\theta = 0$, your solution gives $\frac{m}{M} = 0$ and $\vec{F}_C = Mg\hat{\jmath}$ and for $\theta = \frac{\pi}{2}$, it gives $\frac{m}{M} = 2$ and $\vec{F}_C = Mg(\hat{\imath} + 2\hat{\jmath})$.

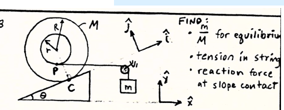


Problem 5.21

Answer:
(a) $\frac{m}{M} = \frac{R\sin\theta}{R\cos\theta + r} = \frac{2\sin\theta}{1+2\cos\theta}$.
(b) $T = mg = 2Mg\frac{\sin\theta}{1+2\cos\theta}$.
(c) $\vec{F}_C = Mg\left[-\frac{2\sin\theta}{2\cos\theta+1}\hat{\imath}' + \hat{\jmath}'\right]$ (where $\hat{\imath}'$ and $\hat{\jmath}'$ are aligned with the horizontal and vertical directions)
(c) $\tan\phi = \frac{\sin\theta}{2+\cos\theta}$. Needs somewhat involved trigonometry, geometry, and algebra.
(d) $\tan\psi = \frac{m}{M} = \frac{2\sin\theta}{1+2\cos\theta}$.

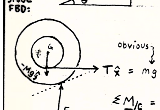
\# 2.78

$r = \frac{R}{2}$



FIND:
- $\frac{m}{M}$ for equilibrium
- tension in string
- reaction force at slope contact

SPOOL FBD:



obvious

$T\hat{x} = mg\hat{x}$

note: different unit vectors than used in class example

a) AMB about C:

$\sum \underline{M}_{/C} = \dot{\underline{H}}_{/C} = \underline{0}$ (statics)

$(\underline{r}_{P/C} \times T\hat{x}) - (r_{G/C} \times Mg\hat{y}) = \underline{0}$

$(R\hat{\jmath} - \frac{R}{2}\hat{y})$ $(R\hat{\jmath})$

$= (R\hat{\jmath} - \frac{R}{2}\hat{y}) \times (mg\hat{x}) - (R\hat{\jmath} \times Mg\hat{y}) = \underline{0}$

$= Rmg(\hat{\jmath}\times\hat{x}) - \frac{R}{2}mg(\hat{y}\times\hat{x}) - RMg(\hat{\jmath}\times\hat{y}) = \underline{0}$

aside on cross products:
$\hat{y}\times\hat{x} = -\hat{k}$
$\hat{\jmath}\times\hat{y} = -\sin\theta\,\hat{k}$
$\hat{\jmath}\times\hat{x} = -\cos\theta\,\hat{k}$

substitute and cancel $R, g, \hat{k}$

$-m\cos\theta + \frac{m}{2} + M\sin\theta = 0$

$2M\sin\theta = m(2\cos\theta - 1)$

$\boxed{\frac{m}{M} = \frac{2\sin\theta}{2\cos\theta - 1}}$

b) $\boxed{T = mg = \frac{2Mg\sin\theta}{2\cos\theta - 1}}$ (Using result from (a))

c) LMB: $\sum \underline{F} = \underline{0}$, $\underline{F}_c = Mg\hat{y} - mg\hat{x}$

$\boxed{\underline{F}_c = -\frac{2Mg\sin\theta}{2\cos\theta - 1}\hat{x} + Mg\hat{y}}$

can dot with $\hat{\imath}, \hat{\jmath}$ to get components in those directions

CONTINUED

2.78 CONT'D

Check solution: at $\theta = 0$ $\frac{m}{M} = \frac{2\sin(0)}{2\cos(0) - 1} = 0$

no mass needed to keep spool at rest.

at $\theta = \frac{\pi}{2}$, $F_c = -2Mg\hat{x} + Mg\hat{y}$

and since at $\theta = \frac{\pi}{2}$ $\hat{\imath} = \hat{y}$, $\hat{\jmath} = -\hat{x}$ answer agrees.

and $\frac{m}{M} = -2$ ("suction")

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.3.22 This problem is similar to problem 5.3.21. A reel of mass M and outer radius R is connected by an inextensible string from point P across a pulley to a hanging object of mass m. The inner cylinder of the reel has radius $r = \frac{1}{2}R$. The slope has angle θ. There is no slip between the reel and the slope. There is gravity. In terms of M, g, R, and θ, find:

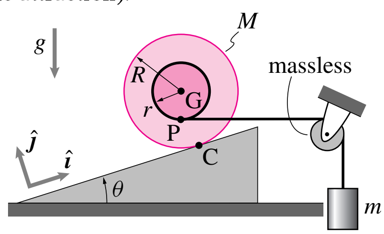
a) the ratio of the masses so that the system is at rest,

b) the corresponding tension in the string, and

c) the corresponding force on the reel at its point of contact with the slope, point C.

d) What is the minimum coefficient of friction μ at C needed to prevent slip.

Check that for $\theta = 0$, your solution gives $\frac{m}{M} = 0$ and $\vec{\boldsymbol{F}}_C = Mg\hat{\boldsymbol{j}}$ and for $\theta = \frac{\pi}{2}$, it gives $\frac{m}{M} = -2$ and $\vec{\boldsymbol{F}}_C = Mg(\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{j}})$.The negative mass ratio is impossible since mass cannot be negative and the negative normal force is impossible unless the wall or the reel or both can 'suck' or they can 'stick' to each other (that is, provide some sort of suction, adhesion, or magnetic attraction).



Problem 5.22

Answer:

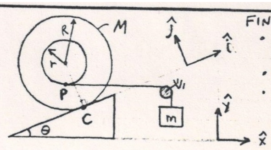
(a) $\frac{m}{M} = \frac{R\sin\theta}{R\cos\theta r} = \frac{2\sin\theta}{2\cos\theta-1}$.

(b) $T = mg = 2Mg\frac{\sin\theta}{2\cos\theta-1}$.

(c) $\vec{\boldsymbol{F}}_C = \frac{Mg}{1-2\cos\theta}[\sin\theta\hat{\boldsymbol{\imath}} + (\cos\theta - 2)\hat{\boldsymbol{j}}]$.

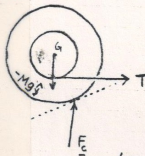
2.78

$r = \frac{R}{2}$



FIND:
- $\frac{m}{M}$ for equilibriu[m]
- tension in string
- reaction force at slope contact

SPOOL FBD:



note: different unit vectors than used in class example

obvious

$T\hat{x} = mg\hat{x}$

a) AMB about C:

$\sum \underline{M}_{/C} = \dot{\underline{H}}_{/C} = \underline{0}$ (statics)

$(\underline{r}_{P/C} \times T\hat{x}) - (r_{G/C} \times Mg\hat{y}) = \underline{0}$

$(R\hat{j} - \frac{R}{2}\hat{y})$ $(R\hat{j})$

$= (R\hat{j} - \frac{R}{2}\hat{y}) \times (mg\hat{x}) - (R\hat{j} \times Mg\hat{y}) = \underline{0}$

$= Rmg(\hat{j} \times \hat{x}) - \frac{R}{2}mg(\hat{y} \times \hat{x}) - RMg(\hat{j} \times \hat{y}) = \underline{0}$

aside on cross products:
$\hat{y} \times \hat{x} = -\hat{k}$
$\hat{j} \times \hat{y} = -\sin\theta\,\hat{k}$
$\hat{j} \times \hat{x} = -\cos\theta\,\hat{k}$

substitute and cancel $R, g, \hat{k}$

$-m\cos\theta + \frac{m}{2} + M\sin\theta = 0$

$2M\sin\theta = m(2\cos\theta - 1)$

$\boxed{\frac{m}{M} = \frac{2\sin\theta}{2\cos\theta - 1}}$

b) $\boxed{T = mg = \frac{2Mg\sin\theta}{2\cos\theta - 1}}$ (Using result from (a))

c) LMB: $\sum \underline{F} = \underline{0}$, $\underline{F}_c = Mg\hat{y} - mg\hat{x}$

$\boxed{\underline{F}_c = -\frac{2Mg\sin\theta}{2\cos\theta - 1}\hat{x} + Mg\hat{y}}$

can dot with $\hat{i}, \hat{j}$ to get components in those directions

CONTINUED

2.78 CONT'D

Check solution: at $\theta = 0$ $\frac{m}{M} = \frac{2\sin(0)}{2\cos(0) - 1} = 0$

no mass needed to keep spool at rest.

at $\theta = \frac{\pi}{2}$, $F_c = -2Mg\hat{x} + Mg\hat{y}$

and since at $\theta = \frac{\pi}{2}$ $\hat{i} = \hat{y}$, $\hat{j} = -\hat{x}$ answer agrees.

and $\frac{m}{M} = -2$ ("suction")

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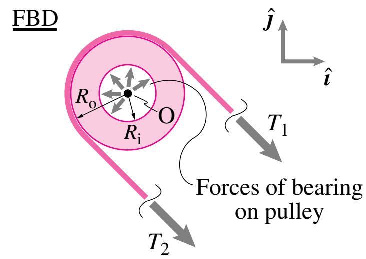
5.3.23 Assume a massless pulley is round and has outer radius R_2. It slides on a shaft that has radius R_i. Assume there is friction between the shaft and the pulley with coefficient of friction μ, and friction angle ϕ defined by $\mu = \tan(\phi)$. Assume the two ends of the line that are wrapped around the pulley are parallel.

a) What is the relation between the two tensions when the pulley is turning? You may assume that the bearing shaft touches the hole in the pulley at only one point. .

b) Plug in some reasonable numbers for R_i, R_o and μ (or ϕ) to see one reason why wheels (say pulleys) are such a good idea even when the bearings are not all that well lubricated.



Problem 5.23

Answer:
(a) $\frac{F_1}{F_2} = \frac{R_o + R_i \sin\phi}{R_o - R_i \sin\phi}$
(b) For $R_o = 3R_i$ and $\mu = 0.2$, $\frac{F_1}{F_2} \approx 1.14$.

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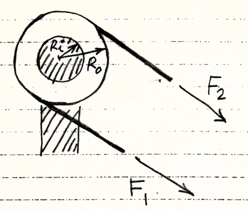
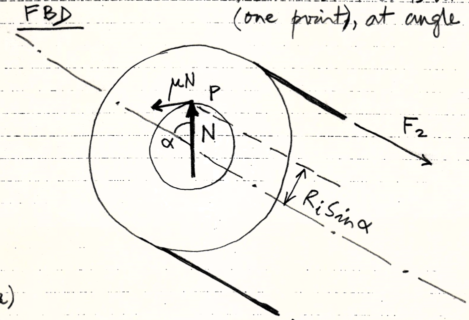


NOTE
The contact point is not known.

Assume $F_2 > F_1$

Assume contact is at P (one point), at angle α

FBD



(a)
Pulley is massless,
∴ use statics equations

$$\sum \vec{F} \cdot \hat{\lambda} = 0$$
$$\Rightarrow N\sin\alpha - \mu N\cos\alpha = 0 \Rightarrow \tan\alpha = \mu$$
$$\therefore \boxed{\alpha = \phi}$$

$$\sum M_{/P} = 0 \Rightarrow \boxed{\frac{F_1}{F_2} = \frac{R_o + R_i\sin\phi}{R_o - R_i\sin\phi}}$$

$$\therefore \frac{F_1}{F_2} \to \frac{R_o + R_i}{R_o - R_i} \text{ as } \phi \to \pi/2 \text{ or } \mu \to \infty$$
(AS PER THIS MODEL)

(b) $\boxed{\text{For } R_o = 3R_i,\ \mu = 0.2 \Rightarrow \frac{F_1}{F_2} \cong 1.14}$

(c) Using result from B&J, $\frac{F_1}{F_2} = e^{\mu\theta} = e^{0.2\pi} \cong 1.87$

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