SAMPLE 5.12 **A block on a ramp sliding down or up.** Consider a block of mass $m = 10\,\text{kg}$ pushed up by the force F on the ramp as shown in the figure. The coefficient of friction between the ramp and the block is $\mu = 0.7$.

1. Let $\theta = 60^\circ$ and $\alpha = 0^\circ$. Assuming that the block slides steadily downhill, find the tension in the string.
2. Let $\theta = 30^\circ$ and $\alpha = 30^\circ$. If the applied force $F = 20\,\text{N}$, find the force of friction on the block.
3. Let $\theta = 60^\circ$ and $\alpha = 30^\circ$. If the applied force $F = 10\,\text{N}$, find the force of friction on the block.
4. For $\theta = 60^\circ$ and $\alpha = 30^\circ$, what will be the required tension in the string to make the block just about slide up the slope? Express your answer in terms of the weight of the block.

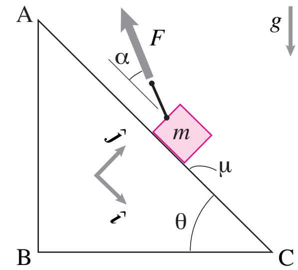

Figure 5.48

Solution The free-body diagram of the block is shown in fig. 5.49. We have assumed that the friction force acts upwards along the inclined plane. The direction of the friction force can be up or down depending on the direction of sliding. We will let the equilibrium equation tell us which way the friction force acts in a particular case. In fig. 5.49, we also use rotated unit vectors $\hat{\imath}$ and $\hat{\jmath}$, parallel and perpendicular to the inclined plane, respectively. This is just to make calculations easier. We can use these basis vectors in any orientation to suit our convenience.

The force balance equation for the static equilibrium of the block gives

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad \vec{F} + \vec{N} + \vec{F}_f + \vec{W} = \vec{0}. \tag{5.22}$$

$$\Rightarrow \quad F(-\cos\alpha\hat{\imath} + \sin\alpha\,\hat{\jmath}) + N\,\hat{\jmath} - F_f\,\hat{\imath} + mg(\sin\theta\hat{\imath} - \cos\theta\,\hat{\jmath}) = \vec{0} \tag{5.23}$$

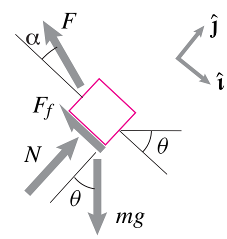

Figure 5.49: Free-body diagram of the block. The friction force $\vec{F}_f$ may be known or unknown depending on whether the block is sliding or not. Under downhill sliding, $F_f = \mu N$, whereas for no sliding $|F_f| \leq \mu N$.

Now depending on what is given and what is unknown, we can manipulate this vector equation to find what we want.

1. **Block sliding down:** If the block slides down steadily or very slowly, we can use the static equilibrium equation written above with $\vec{F}_f = -\mu N\hat{\imath}$ (that is, the friction force is known. This is the case of sliding friction and the friction force is maximum possible). Substituting this value of $\vec{F}_f$ and separating out the $\hat{\imath}$ and $\hat{\jmath}$ components of eqn. (5.23), we get

$$-F\cos\alpha - \mu N + mg\sin\theta = 0 \tag{5.24}$$

$$F\sin\alpha + N - mg\cos\theta = 0. \tag{5.25}$$

Adding μ times eqn. (5.25) to eqn. (5.24) in order to get rid of N, and rearranging terms, we get

$$F(\cos\alpha - \mu\sin\alpha) = mg(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad F = \frac{\sin\theta - \mu\cos\theta}{\cos\alpha - \mu\sin\alpha}mg. \tag{5.26}$$

Substituting $\alpha = 0^\circ$, $\theta = 60^\circ$, and $\mu = 0.7$ in eqn. (5.26), we get

$$F = (\sin 60^\circ - 0.7\cdot\cos 60^\circ)\,mg = 0.52\,mg. = 51\,\text{N}.$$

$$\boxed{\vec{F} = -(51\,\text{N})\hat{\imath}}$$

2. **Block sliding or not sliding – not known:** Now, we are given that $F = 20\,\text{N}$, $\alpha = 30^\circ$, and $\theta = 30^\circ$. We do not know if the block is sliding or not. So, let us assume static equilibrium in the given configuration and solve for the friction force F_f. Then, we will check if it satisfies friction law for static equilibrium ($|F_f| \leq \mu N$).

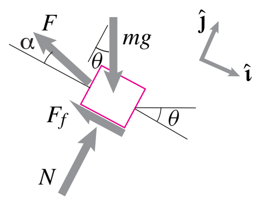

Figure 5.50: Free-body diagram of the block. The friction force $\vec{F}_f$ is not known because we do not know if the block is sliding or not.

Substituting $\vec{F}_f = -F_f\hat{\imath}$ in eqn. (5.23) and separating out the $\hat{\imath}$ and $\hat{\jmath}$ components of the equation, we get

$$\begin{aligned} -F\cos\alpha - F_f + mg\sin\theta &= 0 \\ F\sin\alpha + N - mg\cos\theta &= 0 \end{aligned}$$

which are easily solved for F and N to give

$$\begin{aligned} F_f &= mg\sin\theta - F\cos\alpha \\ N &= mg\cos\theta - F\sin\alpha. \end{aligned}$$

Substituting the given values of F, θ, and α, we get

$$\begin{aligned} F_f &= 10\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2}\cdot\sin 30^\circ - 20\,\mathrm{N}\cdot\cos 30^\circ = 31.73\,\mathrm{N} \\ N &= 10\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2}\cdot\cos 30^\circ - 20\,\mathrm{N}\cdot\sin 30^\circ = 74.96\,\mathrm{N}. \end{aligned}$$

Now, the maximum possible value of friction force is $\mu N = 0.7\cdot 74.96\,\mathrm{N} = 52.47\,\mathrm{N}$. Thus, $|F_f| < \mu N$, and therefore, our assumption of static equilibrium is valid. This equilibrium requires that $F_f = 31.73\,\mathrm{N}$.

$$\boxed{\vec{F}_f = -(31.73\,\mathrm{N})\hat{\imath}}$$

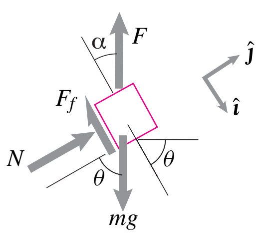

Figure 5.51: Free-body diagram of the block. The friction force $\vec{F}_f$ is not known again because we do not know if the block is sliding or not.

3. **Block sliding or not sliding – not known, again:** In this case, $F = 10\,\mathrm{N}$, $\alpha = 30^\circ$, and $\theta = 60^\circ$. Again, assuming static equilibrium, we do exactly the same calculations as above (in fact, use the same expressions) and substituting the given values, we get

$$\begin{aligned} F_f &= 10\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2}\cdot\sin 60^\circ - 10\,\mathrm{N}\cdot\cos 30^\circ = 76.3\,\mathrm{N} \\ N &= 10\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2}\cdot\cos 60^\circ - 10\,\mathrm{N}\cdot\sin 30^\circ = 44\,\mathrm{N}. \end{aligned}$$

Here, $|F_f| > \mu N$. Clearly, F_f is not less than or equal to μN, and therefore, our assumption of static equilibrium is not valid. In fact, the given parameters of the problem will make the block accelerate downhill — a problem of dynamics. However, the friction force remains constant, at its maximum $\vec{F}_f = \mu N = 30.8\,\mathrm{N}$ once the sliding starts, accelerating or not.

$$\boxed{\vec{F}_f = -30.8\,\mathrm{N}\hat{\imath}}$$

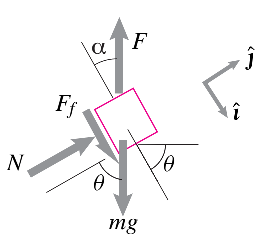

Figure 5.52: Free-body diagram of the block about to slip upwards. The friction force $\vec{F}_f$ is completely known because of impending slip, $\vec{F}_f = \mu N\hat{\imath}$.

4. **Block just about to slide upwards:** If the block is about to slide upwards, then the friction force must act downwards as shown in fig. 5.52. We also know the magnitude, $F_f = \mu N$ because it is the case of impending slip. Now the force-balance equations in the $\hat{\imath}$ and $\hat{\jmath}$ directions are:

$$\begin{aligned} -F\cos\alpha + \mu N + mg\sin\theta &= 0 \\ F\sin\alpha + N - mg\cos\theta &= 0 \end{aligned}$$

Eliminating N from the two equations, we get F in terms of mg and substituting $\alpha = 30^\circ$, and $\theta = 60^\circ$, we get the desired value:

$$\begin{aligned} F &= \frac{\sin\theta + \mu\cos\theta}{\cos\alpha + \mu\sin\alpha}\,mg \\ &= \frac{1.216}{1.216}\,mg = mg. \end{aligned}$$

$$\boxed{F = mg}$$

Does the answer make sense? Yes, it does. For the given θ and α, the string tension is vertical. If it balances the weight of the block, the normal force goes to zero and so does the friction force. The block is then ready to slide up if the tension increases by any tiny amount.

SAMPLE 5.13 How much friction does the cylinder need? A cylinder of mass m sits between an incline and a vertical wall as shown in the figure. There is no friction between the wall and the cylinder but there is friction between the incline and the cylinder. Take the coefficient of friction to be μ and the angle of incline with the horizontal to be θ. Find the force of friction on the cylinder from the incline.

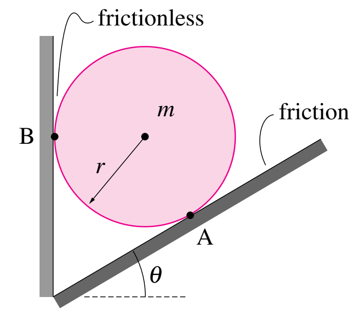

Figure 5.53

Solution The free-body diagram of the cylinder is shown in fig. 5.54. We need to find the force of friction F_s.

Note that the normal reaction of the vertical wall, N, the force of gravity, mg, and the normal reaction of the incline, R, all pass through the center C of the cylinder. So, if we do moment balance about point C, $\sum \vec{\boldsymbol{M}}_C = \vec{\boldsymbol{0}}$, none of these forces will appear in the equation since their moment about C is zero. Therefore, to find F_s, we should use the moment-balance equation about point C. Noting that F_s acts along the inclined plane, its normal distance (*lever arm*) from point C is simply r, the radius of the cylinder, we have,

$$\sum \vec{\boldsymbol{M}}_C = \vec{\boldsymbol{0}} \quad \Rightarrow \quad rF_s(-\hat{\boldsymbol{k}}) = \vec{\boldsymbol{0}}$$
$$\Rightarrow \quad F_s = 0.$$

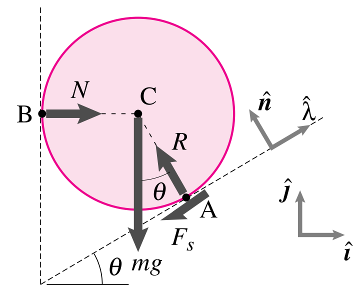

Figure 5.54

Thus the force of friction on the cylinder is zero! Note that F_s is independent of θ, the angle of incline. Thus, irrespective of what the angle of incline is, in the static equilibrium condition, there is no force of friction on the cylinder.

$\boxed{F_s = 0}$

Note: The cylinder here is a three-force body since there are three forces acting on it — two contact forces (at A and B) and one gravity force. Therefore, for equilibrium, all the three forces must intersect at a single point. Now, lines of action of the gravity force and the normal reaction at B intersect at the center C of the cylinder. Therefore, the line of action of the contact force at A also must pass through the center. This is clearly not possible if the contact force is not normal to the incline (see the candidate contact forces marked by the dashed gray arrows in fig. 5.55. If there is any non-zero friction force at A, the contact force (the resultant of the normal reaction and the friction force) at A will be tipped away from the normal, thus making its line of action miss the center of the cylinder and, therefore, violate equilibrium condition.

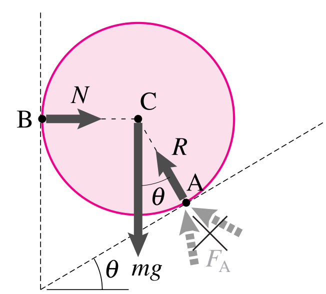

Figure 5.55

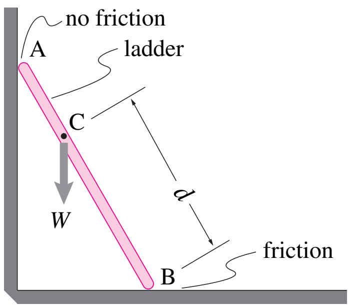

Figure 5.56

SAMPLE 5.14 Will the ladder slip? A ladder of length $\ell = 4\,\mathrm{m}$ rests against a wall at $\theta = 60°$. Assume that there is no friction between the ladder and the vertical wall but there is friction between the ground and the ladder with $\mu = 0.5$. A person weighing 700 N starts to climb up the ladder.

1. Can the person make it to the top safely (without the ladder slipping)? If not, then find the distance d along the ladder that the person can climb safely. Ignore the weight of the ladder in comparison to the weight of the person.
2. Does the "no slip" distance d depend on θ? If yes, then find the angle θ which makes it safe for the person to reach the top.

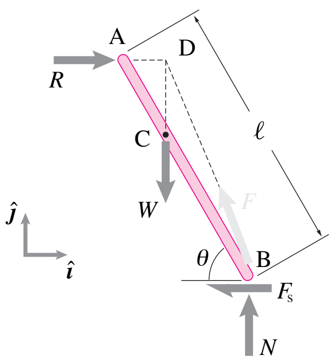

Figure 5.57: The free-body diagram of the ladder indicates that it is a three-force body. Since the direction of the forces acting at points A and C are known (the normal, horizontal reaction at A and the vertical gravity force at C), it is easy to find the direction of the net ground reaction at B — it must pass through point D. The ground reaction F at B (shown by light grey arrow) can be decomposed into a normal reaction N and a horizontal reaction (the force of friction) F_s at B.

Solution

1. The free-body diagram of the ladder is shown in fig. 5.57. There is only a normal reaction $\vec{\boldsymbol{R}} = R\hat{\boldsymbol{\imath}}$ at A since there is no friction between the wall and the ladder. The force of friction at B is $\vec{\boldsymbol{F}}_s = -F_s\hat{\boldsymbol{\imath}}$ where $F_s \leq \mu N$. To determine how far the person can climb the ladder without the ladder slipping, we take the critical case of impending slip. In this case, $F_s = \mu N$. Let the person be at point C, a distance d along the ladder from point B. We need to find d and check if $d < \ell$ (cannot make it to point A).

   From moment balance about point B, $\sum \vec{\boldsymbol{M}}_B = \vec{\boldsymbol{0}}$, we find

$$\begin{aligned} \vec{\boldsymbol{r}}_{A/B} \times \vec{\boldsymbol{R}} + \vec{\boldsymbol{r}}_{C/B} \times \vec{\boldsymbol{W}} &= \vec{\boldsymbol{0}} \\ -R\ell\sin\theta\hat{\boldsymbol{k}} + Wd\cos\theta\hat{\boldsymbol{k}} &= \vec{\boldsymbol{0}} \\ \Rightarrow \quad R &= W\frac{d\cos\theta}{\ell\sin\theta}. \end{aligned} \tag{5.27}$$

   From force equilibrium, we get

$$(R - \mu N)\hat{\boldsymbol{\imath}} + (N - W)\hat{\boldsymbol{\jmath}} = \vec{\boldsymbol{0}}. \tag{5.28}$$

   Dotting eqn. (5.28) with $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{\imath}}$, respectively, we get

$$\begin{aligned} N &= W \\ R &= \mu N = \mu W. \end{aligned}$$

   Substituting this value of R in eqn. (5.27) we get

$$\begin{aligned} \mu W &= W\frac{d\cos\theta}{\ell\sin\theta} \\ \Rightarrow \quad d &= \mu\ell\tan\theta \\ &= 0.5\cdot(4\,\mathrm{m})\cdot\tan 60° = 3.46\,\mathrm{m}. \end{aligned} \tag{5.29}$$

   Thus, the ladder is about to slip when the person is at $d = 3.46\,\mathrm{m}$. But, $d < \ell$, therefore, the person cannot make it to the top of the ladder safely.

   $d = 3.46\,\mathrm{m}$

2. The "no slip" distance d depends on the angle θ via the relationship in eqn. (5.29). The person can climb the ladder safely up to the top if

$$\tan\theta = \frac{1}{\mu} \quad \Rightarrow \quad \theta = \tan^{-1}(\mu^{-1}) = 63.43°.$$

   Thus, any reasonable angle $\theta \geq 64°$ will allow the person to climb up to the top safely.

   $\theta \geq 64°$

SAMPLE 5.15 **Will it tip or will it slide?** Whether or not a box of a given width and height will slide or tip over on an inclined plane depends on the slope of the plane and the coefficient of friction. For a given slope θ, find the relationship between the coefficient of friction μ and the aspect ratio of the box, $\gamma = b/h$ for impending tipping.

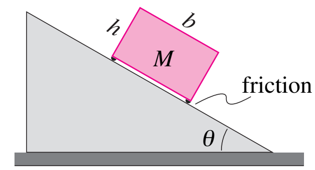

Figure 5.58

Solution Let us imagine that we put the box on a flat surface and then slowly start tilting the surface up with respect to the horizontal. At some slope, the box will either tip over or slide. Just before the instant the box starts to tip over or slide, it is in static equilibrium. The magnitude of the friction force at the contact points is $|F| \leq \mu N$ where N is the magnitude of the normal force at the contact, and the equality holds only in the case of impending slip. That is, if the box is about to slip, then $F = \mu N$ at each contact point.

The free-body diagram of the box is shown in fig. 5.59. Let us first write the equations of static equilibrium assuming there is no impending slip.

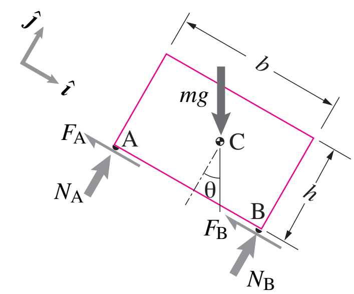

Figure 5.59

The force balance in the $\hat{\imath}$ and $\hat{\jmath}$ directions (see fig. 5.62) gives

$$\begin{aligned} F_A + F_B &= mg\sin\theta & (5.30) \\ N_A + N_B &= mg\cos\theta. & (5.31) \end{aligned}$$

The moment equilibrium about the center-of-mass, $\sum \vec{\boldsymbol{M}}_C = \vec{\boldsymbol{0}}$, in the $\hat{\boldsymbol{k}}$ direction gives

$$N_B \cdot \frac{b}{2} - N_A \frac{b}{2} - (F_A + F_B)\frac{h}{2} = 0. \quad (5.32)$$

Substituting $F_A + F_B = mg\sin\theta$ from eqn. (5.30) in eqn. (5.32), and solving eqns. (5.31) and (5.32) simultaneously, we get

$$N_A = \frac{1}{2}mg\left(\cos\theta - \frac{h}{b}\sin\theta\right), \text{ and } \quad N_B = \frac{1}{2}mg\left(\cos\theta + \frac{h}{b}\sin\theta\right).$$

If the box were to tip over (about point B), the support forces at A will go to zero (because of loss of contact). Thus, for impending tipping,

$$N_A = 0 \quad \Rightarrow \quad \cos\theta - \frac{h}{b}\sin\theta = 0 \quad \Rightarrow \quad \tan\theta = \frac{b}{h} = \gamma.$$

Thus, the condition for impending tipping is

$$\tan\theta = \gamma. \quad (5.33)$$

This condition, however, does not guarantee that the box *will* tip over. In fact, it may start sliding before it tips over. We need to check if sliding condition is met before eqn. (5.33) is satisfied. In other words, we need to check the value of friction forces and make sure that $|F_A + F_B| \leq \mu(N_A + N_B)$. Thus, for no slipping,

$$F_A + F_B \leq \mu(N_A + N_B) \quad \Rightarrow \quad mg\sin\theta \leq \mu mg\cos\theta \quad \Rightarrow \quad \tan\theta \leq \mu.$$

Using this condition (with equality) in eqn. (5.33), we get the *critical* condition for tipping:

$$\gamma = \mu.$$

$$\gamma = \tan\theta \leq \mu$$

You may know this condition geometrically as the line of action of the weight of the box must pass through B and beyond for tipping over (see fig. 5.60).

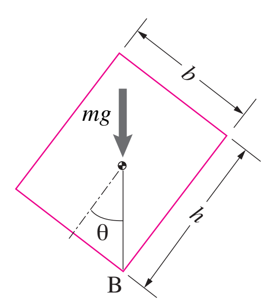

Figure 5.60: The impending tipping condition $\tan\theta = \gamma \equiv b/h$, provided $\tan\theta \leq \mu$.

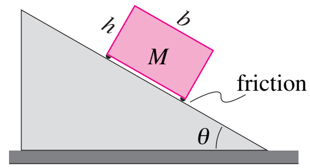

Figure 5.61

SAMPLE 5.16 How big does the friction force get? Consider the box on the inclined plane of Sample 5.15 again. The box has aspect ratio $\gamma = b/h$. The coefficient of friction is μ. Imagine that the angle θ of the inclined plane can be varied. How does the force of friction on the box vary with θ? How does the maximum value of this force depend on μ?

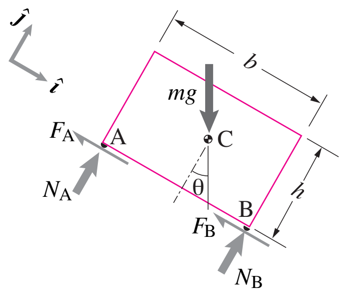

Figure 5.62

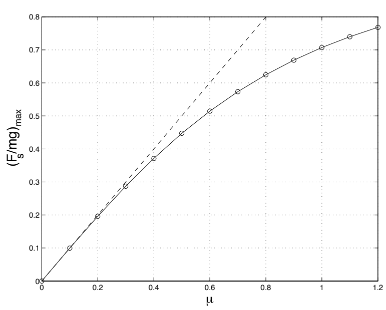

Figure 5.63: The maximum net friction force on the box as a function of the friction coefficient μ. Note that the relationship is almost linear for most practical values of μ.

Solution If we imagine the inclined plane to be not inclined ($\theta = 0$) but horizontal and the box to be just sitting there, the force of friction on the box has to be zero. As we tilt the plane up ($\theta > 0$), the friction force starts increasing. It increases up to the point of impending slip unless the box tips over before that. Assuming that the aspect ratio of the box prevents it from tipping (see Sample 5.15), we can determine the maximum value up to which the friction force rises before the box starts slipping.

From Sample 5.15, we know that the total friction force $F_s = F_A + F_B = mg\sin\theta$. Thus the normalized friction force (as a fraction of the weight of the block), F_s/mg is

$$\frac{F_s}{mg} = \sin\theta.$$

Thus the total friction force varies as sine of the ramp angle. However, this variation is valid only upto the maximum value of the friction force (μN) when the block starts sliding. The critical angle at which this maximum is attained is $\theta_{\text{slip}} = \tan^{-1}\mu \equiv \phi$ (friction angle). Thus,

$$\left.\frac{F_s}{mg}\right|_{\max} = \sin\phi.$$

Figure 5.63 shows how the maximum normalized friction force varies with μ. Note that for lower values of μ (which covers most practical values of μ), the relationship is almost linear. Thus, $|F_s/mg| \approx \mu$ for $\mu \leq 0.5$.

$$\boxed{\frac{F_s}{mg} = \sin\theta, \quad \left.\frac{F_s}{mg}\right|_{\max} = \sin\phi}$$

What happens to the friction force after it attains the maximum value $F_s = mg\sin\phi$? For a given ramp angle, the friction force remains constant and the box slides.

SAMPLE 5.17 A spool of mass $m = 2\,\text{kg}$ rests on an incline as shown in the figure. The inner radius of the spool is $r = 200\,\text{mm}$ and the outer radius is $R = 500\,\text{mm}$. The coefficient of friction between the spool and the incline is $\mu = 0.4$, and the angle of incline $\theta = 60°$.

1. Which way does the force of friction act, up or down the incline?
2. What is the required horizontal pull T to balance the spool on the incline?
3. Is the spool about to slip?

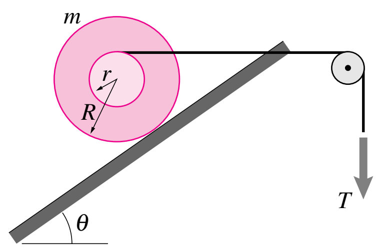

Figure 5.64

Solution

1. The free-body diagram of the spool is shown in fig. 5.65. Note that the spool is a 3-force body. Therefore, in static equilibrium all the three forces — the force of gravity mg, the horizontal pull T, and the incline reaction F — must intersect at a point. Since T and mg intersect at the top of the inner drum (point B), the reaction force $\vec{\boldsymbol{F}}$ of the incline must be along the direction AB. Now the incline reaction $\vec{\boldsymbol{F}}$ is the vector sum of two forces — the normal (to the incline) reaction N and the friction force F_s (along the incline). The normal reaction force N passes though the center C of the spool. Therefore, the force of friction F_s must point up along the incline to make the resultant $\vec{\boldsymbol{F}}$ point along AB.

Up the incline

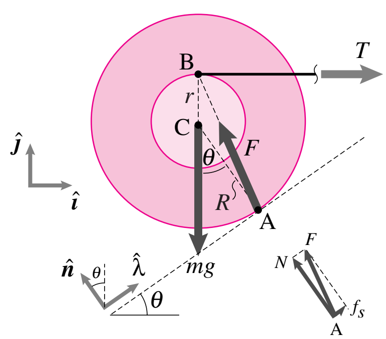

Figure 5.65

2. We need to find the tension T in the string. From the free-body diagram, we see that the force equilibrium will involve $\vec{\boldsymbol{T}}$ along with another unknown force $\vec{\boldsymbol{F}}$, the reaction of the incline. On the other hand, if we do moment balance about point A, we can get rid of $\vec{\boldsymbol{F}}$ and get one scalar equation involving T and mg, giving T in terms of mg. So, writing the moment equilibrium equation about point A,

$$\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\boldsymbol{0}},$$

we get

$$\vec{\boldsymbol{r}}_{\text{C/A}} \times (-mg\,\hat{\boldsymbol{\jmath}}) + \vec{\boldsymbol{r}}_{\text{B/A}} \times (T\hat{\boldsymbol{\imath}}) = \vec{\boldsymbol{0}}. \tag{5.34}$$

These cross products can be easily evaluated by using the scalar form of the moment of a force—the product of force and the lever arm. Thus the moment of mg is $mg \cdot R\sin\theta$ and the moment of T is $-T \cdot (r + R\cos\theta)$ about point A in the $\hat{\boldsymbol{k}}$ direction. Thus the scalar form of the moment-balance equation gives

$$\begin{aligned} mgR\sin\theta &= T(R\cos\theta + r) \\ \Rightarrow \quad T &= mg\,\frac{\sin\theta}{\cos\theta + r/R} \\ &= 2\,\text{kg} \cdot 9.81\,\text{m/s}^2 \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{.2\,\text{m}}{.5\,\text{m}}} \\ &= 18.88\,\text{N}. \end{aligned}$$

$T = 18.88\,\text{N}$

Alternatively,
We can also evaluate the net moment on the spool, given by eqn. (5.34), using direct cross products of vectors in the equation. We can use mixed basis vectors ($\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, $\hat{\boldsymbol{\lambda}}$, and $\hat{\boldsymbol{n}}$) as shown in fig. 5.65. Since,

$$\vec{\boldsymbol{r}}_{\text{C/A}} = R\hat{\boldsymbol{n}} \quad \text{and} \quad \vec{\boldsymbol{r}}_{\text{B/A}} = R\hat{\boldsymbol{n}} + r\,\hat{\boldsymbol{\jmath}},$$

we have,

$$\begin{aligned}\vec{r}_{C/A} \times (-mg\hat{\jmath}) &= -mgR(\hat{n} \times \hat{\jmath}) \\ \vec{r}_{B/A} \times T\hat{\imath} &= T[R(\hat{n} \times \hat{\imath}) + r(\hat{\jmath} \times \hat{\imath})].\end{aligned}$$

Now, from the geometry of the basis vectors (see fig. 5.65), we have,

$$\hat{n} \times \hat{\jmath} = -\sin\theta\hat{k} \quad \text{and} \quad \hat{n} \times \hat{\imath} = -\cos\theta\hat{k}.$$

Therefore,

$$\begin{aligned}\vec{r}_{C/A} \times (-mg\hat{\jmath}) &= mgR\sin\theta\hat{k}, \\ \text{and} \qquad \vec{r}_{B/A} \times T\hat{\imath} &= -TR\cos\theta\hat{k} - r\hat{k}.\end{aligned}$$

Hence, eqn. (5.34) becomes

$$mgR\sin\theta\hat{k} - TR\cos\theta\hat{k} - r\hat{k} = \vec{0}.$$

Dotting both sides of this equation with $\hat{k}$, we get the scalar equation

$$mgR\sin\theta = T(R\cos\theta + r)$$

which is the same equation as obtained above using moment lever arms.

3. To find if the spool is about to slip, we need to find the force of friction $|F_s|$ and see if it satisfies the condition of impending slip: $F_s = \mu N$. The force balance on the spool, $\sum \vec{F} = \vec{0}$ gives

$$T\hat{\imath} - mg\hat{\jmath} + F_s\hat{\lambda} + N\hat{n} = \vec{0} \tag{5.35}$$

where $\hat{\lambda}$ and $\hat{n}$ are unit vectors along the incline and normal to the incline, respectively. Dotting eqn. (5.35) with $\hat{\lambda}$ we get

$$\begin{aligned}F_s &= -T(\underbrace{\hat{\imath} \cdot \hat{\lambda}}_{\cos\theta}) + mg(\underbrace{\hat{\jmath} \cdot \hat{\lambda}}_{\sin\theta}) \\ &= -T\cos\theta + mg\sin\theta \\ &= -18.88\,\text{N}(1/2) + 19.62\,\text{N}(\sqrt{3}/2) \\ &= 7.55\,\text{N}.\end{aligned}$$

Similarly, we compute the normal force N by dotting eqn. (5.35) with $\hat{n}$:

$$\begin{aligned}N &= -T(\hat{\imath} \cdot \hat{n}) + mg(\hat{\jmath} \cdot \hat{n}) \\ &= T\sin\theta + mg\cos\theta \\ &= 18.88\,\text{N}(\sqrt{3}/2) + 19.62\,\text{N}(1/2) \\ &= 26.16\,\text{N}.\end{aligned}$$

Now we find that $\mu N = 0.4(26.16\,\text{N}) = 10.46\,\text{N}$ which is greater than $F_s = 7.55\,\text{N}$. Thus $F_s < \mu N$, and therefore, the spool is not about to slip.

Not about to slip