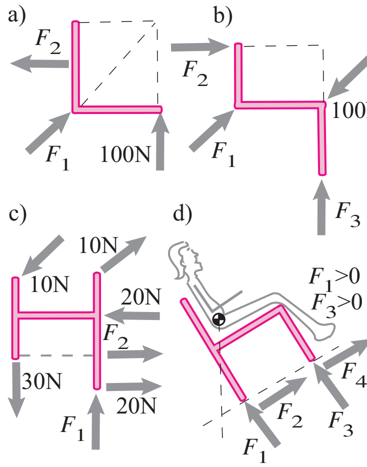
5.2.6 Which of the objects below cannot possibly be in equilibrium and which ones can? (Where the center of mass is indicated, assume non-zero weight acting vertically downwards. Assume dimensions as needed.)

a) Explain in words.

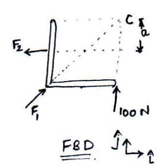
b) Explain using equations.

Note that scalars (e.g., F, F_1, etc.) can be positive or negative unless mentioned otherwise.



Problem 5.6

4.2.6 a)

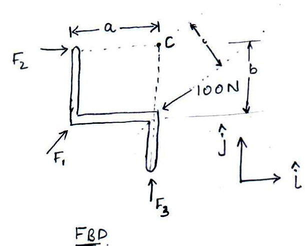


The FBD for this object shows that the three forces acting on the object are not concurrent. Due to this, the moments of 100 N force and F_1 force are zero about an axis passing through point 'c', but that of force F_2 is non-zero. Hence there is always a non-zero moment acting about point 'C'.
Hence, this object can not be in equillibrium, unless $F_2=0$.

Moment balance about point C,

$$\vec{M}_{/c} = -a\hat{j} \times -F_2\hat{\imath} = -aF_2\hat{k} \neq 0 \text{ if } F_2 \neq 0.$$

b



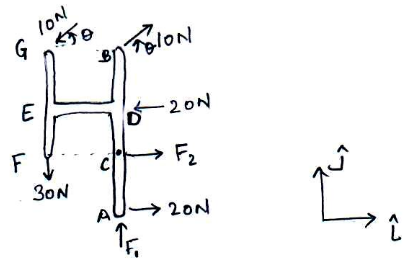
If F_1 is passing through point 'C', moments of all unknown forces, F_1, F_2 and F_3 are zero about point 'C', hence, the overall moment about this point is equal to that caused by 100 N force, and is non-zero.
Hence, the object can not be in static equillibrium.

Moment balance about point C,

$$\vec{M}_{/c} = -100c\,\hat{k}\ [\text{N}\cdot\text{m}]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

4.2.6: C)



FBD.

If we ~~see~~ the moment of all forces about point 'C' we find that,

- the moment of Force F_1 and F_2 are zero as they pass throught point 'c'
- Net moment of the two 20N forces is positive along $\hat{k}$ direction.
- Net moment of the two 10N forces is positive along $\hat{k}$ direction.
- Net moment of the 30N force is positive along $\hat{k}$ direction.

Hence, overall a net positive moment acts on the object along $\hat{k}$ directi

Thus this object cannot be in equillibrium.

Net moment about point C gives:

$$\sum \vec{M}_{/C} = \left[-(AC)\hat{j} \times 20N\,\hat{i}\right] + \left[+DC\,\hat{j} \times (-20N\,\hat{i})\right] + \left[(-FC\,\hat{i}) \times -30N\,\hat{j}\right] \ldots$$

$$+ \left[BC\,\hat{j} \times (10\cos\theta N\,\hat{i} + 10\sin\theta N\,\hat{j})\right] + \left[(BC\,\hat{j} - ED\,\hat{i}) \times (-10\cos\theta N\,\hat{i} - 10\sin\theta N\,\hat{j})\right]$$

$$(FC = ED)$$

$$= (AD)20N\,\hat{k} + (ED)30N\,\hat{k} + (ED\sin\theta)10N\,\hat{k} \quad > \underline{0}$$

because, AD and ED are positive lengths by definition.

Hence, for any value of F_1 and F_2 the object cannot be in static equilibrium.

4.2.6 : b)

Given,

$F_1 > 0,$
$F_3 > 0.$

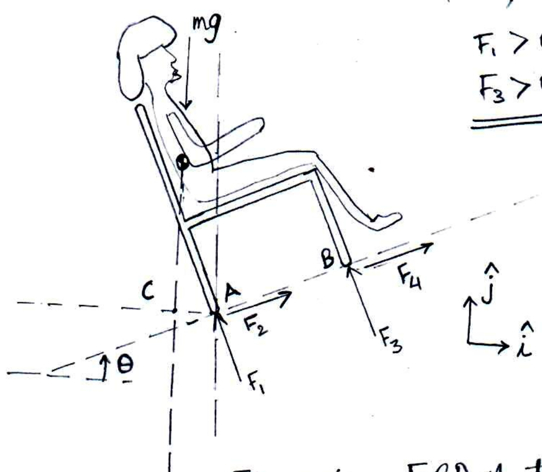


Figure :(i) FBD of the person on chair

Let's say 'AB', 'CA' etc. represent scalar distances between points A and B.

Moment balance about point 'A' gives

$$\sum \vec{M}_{/A} = \vec{0}$$

$$\Rightarrow \left(-AC\hat{\imath} \times -mg\hat{\jmath}\right) + AB\left(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right) \times F_3\left(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}\right) = \vec{0}\,[\text{N}\cdot\text{m}]$$

$$\Rightarrow (AC)mg\hat{k} + (AB)F_3\left(\cos^2\theta + \sin^2\theta\right)\hat{k} = \vec{0}\ [\text{N}\cdot\text{m}]$$

$$\Rightarrow (AC)mg\,\hat{k} + (AB)F_3\,\hat{k} = \vec{0}\ [\text{N}\cdot\text{m}] \quad \text{—— (i)}$$

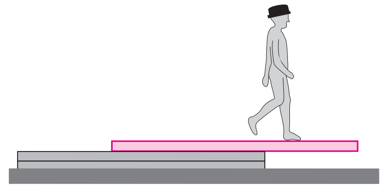
$$\{i\}\cdot\hat{k} \Rightarrow (AC)mg + AB\,F_3 = 0$$

this is possible only if AC is -ve or F_3 is -ve

but as per given figure, that is clearly not the case.

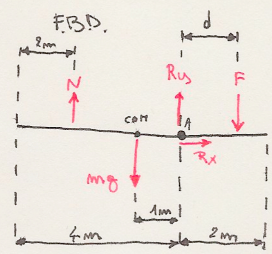
Hence the given system cannot be in equillibrium.

5.2.8 A straight uniform 1000 N beam is 6 m long. It rests on a flat stack of boards with a 2 m overhang. How far out the overhang can an 800 N person walk without the beam tipping over?



Problem 5.8

Roberto Padovani



N is the net support force from the roof

R_x, R_y are the reaction at the "virtual hinge"

F is the person weight at the unknown distance d

moment balance at the virtual hinge A

$$\sum \vec{M}_{/A} = \vec{0} \Rightarrow \vec{r}_N \times \vec{N} + \vec{r}_{COM} \times \vec{W} + \vec{r}_F \times \vec{F} = 0 \Rightarrow$$

$$\Rightarrow (2m)\,N - (1m)\,mg + d\,F = 0 \Rightarrow$$

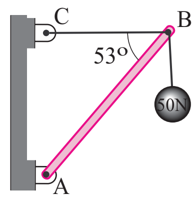
$$\Rightarrow F = \frac{mg - 2N}{d} \cdot 1m$$

tipping begins when $N = 0$ $\Rightarrow$ $d = \frac{mg}{F} \cdot 1m = \frac{2000\,N}{800\,N} \cdot 1m = \boxed{2{,}5\ m}$

5.2.9 The uniform bar AB is 5 m long and weighs 100 N. It is pinned at A and supported by the horizontal cord BC attached at end B. A 50 N weight hangs from end B.

a) Find the tension in cord C.

b) Find the magnitude and direction of the force exerted on the pin at A by the bar.

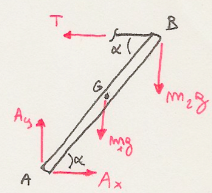


Problem 5.9

Roberto Podavini

$AB = 5m$; $m_1 g = 100\,N$; $m_2 g = 50N$

F.B.D.



$$\sum \vec{M}_{/A} = \vec{0} \Rightarrow \vec{r}_{G/A} \times \vec{W}_1 + \vec{r}_{B/A} \times \vec{W}_2 + \vec{r}_{B/A} \times \vec{T} = 0$$

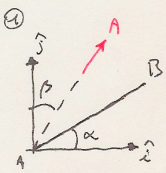
$$\Rightarrow \frac{\cancel{AB}}{2} m_1 g \cos(\alpha) + \cancel{AB}\, m_2 g \cdot \cos(\alpha) - \cancel{AB}\, T \sin(\alpha) = 0 \Rightarrow$$

$$\Rightarrow T = \left(\frac{m_1 g}{2} + m_2 g\right) \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{100\,N}{tg(53^\circ)} \simeq \boxed{75.4\,N}$$

$$\sum \vec{F}_i = 0 \Rightarrow A_x \hat{i} + A_y \hat{j} - m_1 g \hat{j} - m_2 g \hat{j} - T \hat{i} = 0 \quad (1)$$

$$\{1\} \cdot \hat{i} \Rightarrow A_x - T = 0 \Rightarrow A_x = T = 75.4\,N$$

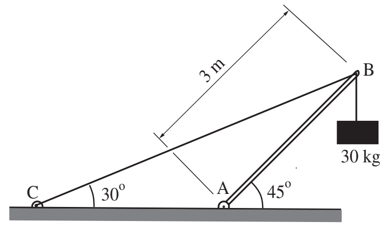
$$\{1\} \cdot \hat{j} \Rightarrow A_y - m_1 g - m_2 g = 0 \Rightarrow A_y = 150\,N$$



$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \simeq \boxed{168\,N} \; ; \; \beta = \arctan\left(\frac{A_x}{A_y}\right) \simeq \boxed{26.7^\circ}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.10 For static equilibrium of the system and the configuration shown in the figure, find the support reaction at end A of the bar.



Problem 5.10

4.2.10

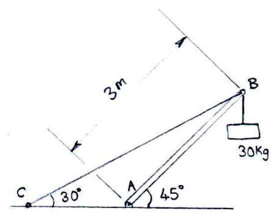


Fig (i)

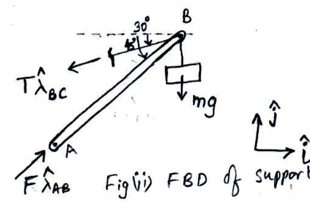


Fig (ii) FBD of supported bar and hanging mass

Since the bar is hinged with a pin joint at point 'A', it can carry force only along its length. Hence the reaction force at end 'A' can be given by $F\hat{\lambda}_{AB}$ where, F is a scalar and $\hat{\lambda}_{AB}$ is unit vector from 'A' to 'B'. Unit vectors, $\hat{\lambda}_{BC}$ and $\hat{\lambda}_{AB}$ can be given as

$$\hat{\lambda}_{BC} = -\cos 30^\circ\,\hat{i} - \sin 30^\circ\,\hat{j} = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j} \qquad \text{(i)}$$

$$\hat{\lambda}_{AB} = \cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \qquad \text{(ii)}$$

By force balance of point B, $\Sigma\vec{F} = \vec{0}$, we get

$$-mg\,\hat{j} + T\hat{\lambda}_{BC} + F\hat{\lambda}_{AB} = \vec{0} \qquad \text{(iii)}$$

Substuting in equ. (iii) we get

$$-mg\,\hat{j} + T\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + F\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right) = \vec{0} \qquad \text{(iv)}$$

separating, x and y components of equation (iv) we get,

$$\frac{F}{\sqrt{2}} = \frac{T\sqrt{3}}{2} \Rightarrow F = T\sqrt{\frac{3}{2}}$$

and

$$\frac{F}{\sqrt{2}} - \frac{T}{2} = mg \Rightarrow \frac{T(\sqrt{3}-1)}{2} = mg \Rightarrow T = \frac{2mg}{\sqrt{3}-1}$$

$$\Rightarrow \boxed{F = \left(\frac{\sqrt{6}}{\sqrt{3}-1}\right) mg}$$

$\Rightarrow$ Hence the support reaction at end A is

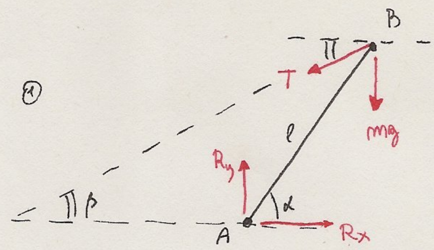
$$F\hat{\lambda}_{AB} = \left[\frac{\sqrt{6}\,mg}{\sqrt{3}-1}\right]\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right) = \left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)(mg\,\hat{i} + mg\,\hat{j})$$

For, m = 30 kg and g = 9.8 m/sec² we get, $\boxed{F\hat{\lambda}_{AB} = 696.3(\hat{i}+\hat{j})\text{ N}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Roberto Padovani

F.B.D.

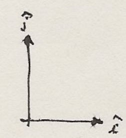


$\alpha = 45°$

$\beta = 30°$

$m = 30\,kg$; $g = 9{,}81\,\frac{N}{s}$

$l = 3\,m$



$\sum \vec{M}_{/A} = 0 \quad \Rightarrow$

$\Rightarrow \bar{r}_{B/A} \times \vec{T} + \bar{r}_{B/A} \times \vec{W} = 0 \quad (1)$

$\vec{T} = T\hat{\lambda}$

$\vec{W} = -mg\hat{j}.$

$(1) \Rightarrow \cancel{l}\, T \sin(\alpha - \beta) - \cancel{l}\, mg \sin(90° - \alpha) = 0 \quad \Rightarrow$

$\Rightarrow T = mg \dfrac{\cos(\alpha)}{\sin(\alpha - \beta)} \simeq 804\,N$

the bar is a two-force body $\Leftrightarrow$ reaction $\bar{R} = -(\vec{T} + \vec{W})$

i.e. $\begin{cases} |\vec{R}| = |\vec{T} + \vec{W}| \\ \vec{R} \,/\!/\, \vec{r}_{AB} \end{cases}$

$\vec{T} = 804\,N\hat{\lambda} = 804\,N\left(-\cos(\beta)\,\hat{i} - \sin(\beta)\,\hat{j}\right)$

$\vec{W} = -294.3\,\hat{j} = 294.3\,N\left(0\,\hat{i} \qquad -\hat{j}\right) \quad \Big| \Rightarrow$

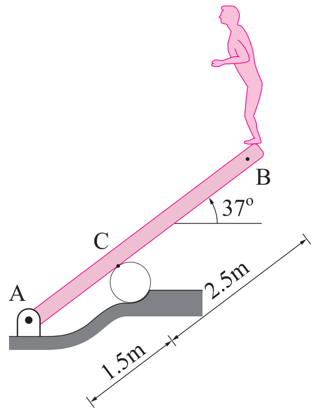
$\vec{R} = -(\vec{T} + \vec{W}) = -\left(-804\,N \cdot \cos(\beta) \cdot \hat{i} - (804\,N \sin(\beta) + 294.3\,N)\,\hat{j}\right) =$

$= \cancel{804\,N}\ \underbrace{696.3\,N}_{R_x}\,\hat{i} + \underbrace{696.3\,N}_{R_y}\,\hat{j} \quad \Rightarrow$

$\Rightarrow |\bar{R}| \simeq \boxed{985\,N}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.11 A 400 N child stands on the end of a uniform 800 N diving plank which is pinned on one end and which also rests on a log (idealized as frictionless). Find the force of the log on the plank and of the pin on the plank.

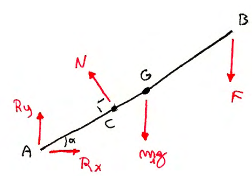


Problem 5.11

Roberto Padovani

$\alpha = 37° \longrightarrow \begin{cases} \sin\alpha \simeq 0.6 \\ \cos\alpha \simeq 0.8 \end{cases}$

F.B.D.



$m_1 g = 800\,N$

$F = 400\,N$

$l_{AG} = l_{GB}$ being uniform

$l_{AC} = 1.5\,m$; $l_{BC} = 2.5\,m$

$$\sum \vec{M}_{/A} = 0 \Rightarrow \vec{r}_{C/A} \times \vec{N} + \vec{r}_{G/A} \times \vec{W} + \vec{r}_{B/A} \times \vec{F} = 0 \Rightarrow$$

$$\Rightarrow l_{AC}\, N - l_{GA}\, m_1 g \sin(90° - \alpha) - l_{AB}\, F \sin(90° - \alpha) = 0 \Rightarrow$$

$$\Rightarrow \boxed{N} = m_1 g \cos(\alpha) \frac{l_{GA}}{l_{AC}} + F \cos(\alpha) \frac{l_{AB}}{l_{AC}} \cong \boxed{1707\,N}$$

$$\sum \vec{F}_i = 0 \Rightarrow (R_x \hat{\imath} + R_y \hat{\jmath}) + \vec{N} + \vec{W} + \vec{F} = 0 \Rightarrow (1)$$

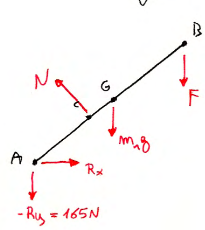
$$\Rightarrow \{(1)\} \cdot \hat{\imath} \Rightarrow R_x - N \sin(\alpha) = 0 \Rightarrow \boxed{R_x} = N \sin(\alpha) = \boxed{1024\,N}$$

$$\{(1)\} \cdot \hat{\jmath} \Rightarrow R_y + N \cos(\alpha) - m_1 g - F = 0 \Rightarrow$$

$$\Rightarrow \boxed{R_y} = m_1 g + F - N \cos(\alpha) = \boxed{-165\,N}$$

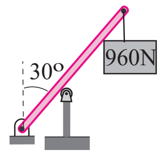
the direction is opposite to the one drawn in the F.B.D. above

physically it is :



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.12 A negligible weight 6 m rod is pinned at one end and leans over a frictionless wall a third of the way up from the bottom. Find the forces of the wall and the pin on the rod.

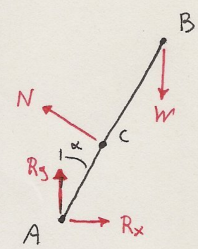


Problem 5.12

Roberto Padovani

$\alpha = 30°$; $W = 960N$; $l_{AC} = \frac{1}{3} l_{AB}$

F.B.D.



$\vec{W} = -W\hat{j}$

$\vec{N} = N\hat{\lambda}$ with $\hat{\lambda} \cdot \vec{r}_{AC} = 0$

$\vec{R} = R_x\hat{i} + R_y\hat{j}$

$\sum \vec{M}_{/A} = \vec{0} \Rightarrow \vec{r}_{AC} \times \vec{N} + \vec{r}_{AB} \times \vec{W} = \vec{0}$

$\Rightarrow \frac{1}{3} \cancel{l_{AB}} N - \cancel{l_{AB}} W \sin(\alpha) = 0 \Rightarrow$

$\Rightarrow \boxed{N} = 3W\sin(\alpha) = \boxed{1440N}$

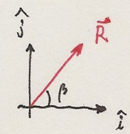
$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{R} + \vec{N} + \vec{W} = 0$ (1)

$\{1\} \cdot \hat{i} \Rightarrow R_x - N\cos(\alpha) = 0 \Rightarrow R_x = N\cos(\alpha) \cong 1247N$

$\{1\} \cdot \hat{j} \Rightarrow R_y + N\sin(\alpha) - W = 0 \Rightarrow R_y = -N\sin(\alpha) + W = 240$

$\Rightarrow |\vec{R}| \cong \boxed{1270N}$

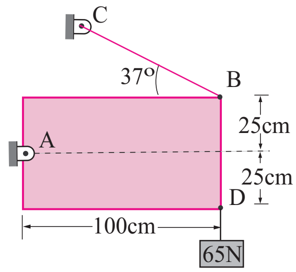
$\boxed{\beta} = \arctan\left(\frac{R_y}{R_x}\right) \cong \boxed{11°}$



5.2.14 The 30 N uniform rectangular plate is supported by a pin at A and cable BC attached at corner B. A 65 N weight hangs from corner D.

a) Find the tension in the cable.

b) Find the force exerted on the plate by the pin at A.



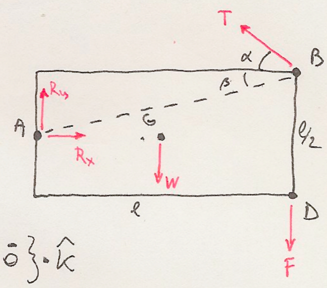
Problem 5.14

$\alpha = 37^\circ$; $\ell = 1\,m$

$W = 30\,N$; $F = 65\,N$

F.B.D.

Roberto Radavani



$$\left\{ \sum \vec{M}_{/A} = \vec{0} \right\} \cdot \hat{k} \Rightarrow$$

$$\Rightarrow \left\{ \vec{r}_{AG} \times \vec{W} + \vec{r}_{AB} \times \vec{T} + \vec{r}_{AD} \times \vec{F} = \vec{0} \right\} \cdot \hat{k}$$

$$\Rightarrow \frac{\ell}{2} W - \ell_{AB}\, T \sin(\alpha + \beta) + \ell_{AD}\, F \sin(90^\circ + \beta) = 0 \quad (1)$$

$$\ell_{AB} = \ell_{AD} = \sqrt{\ell^2 + \frac{\ell^2}{4}} = \ell \frac{\sqrt{5}}{2} \quad ; \quad \beta = \arctan\left(\frac{\ell/2}{\ell}\right) = \arctan\left(\frac{1}{2}\right) \simeq 26{,}6^\circ$$

$$(1)\ \frac{\cancel{\ell}}{2} W - \cancel{\ell} \frac{\sqrt{5}}{2} T \sin(\alpha+\beta) + \cancel{\ell} \frac{\sqrt{5}}{2} F \cos(\beta) = 0 \Rightarrow$$

$$\Rightarrow \frac{\sqrt{5}}{\cancel{2}} T \sin(\alpha+\beta) = \frac{W}{\cancel{2}} + \frac{\sqrt{5}}{\cancel{2}} F \cos(\beta) \Rightarrow$$

$$\Rightarrow \boxed{T =} \frac{W/\sqrt{5} + F\cos(\beta)}{\sin(\alpha+\beta)} = \boxed{80\,N}$$

$$\sum \vec{F}_i = \vec{0} \Rightarrow R_x \hat{\imath} + R_y \hat{\jmath} + \vec{W} + \vec{F} + \vec{T} = \vec{0} \quad (2)$$

$$\vec{W} = -W\hat{\jmath} \quad ; \quad \vec{F} = -F\hat{\jmath} \quad ; \quad \vec{T} = T\left(-\cos(\alpha)\hat{\imath} + \sin(\alpha)\hat{\jmath}\right)$$

$$\{(2)\} \cdot \hat{\imath} \Rightarrow R_x - T\cos(\alpha) = 0 \Rightarrow \boxed{R_x} = T\cos(\alpha) = \boxed{64\,N}$$

$$\{(2)\} \cdot \hat{\jmath} \Rightarrow R_y - W - F + T\sin(\alpha) = 0 \Rightarrow$$

$$\Rightarrow \boxed{R_y} = W + F - T\sin(\alpha) = \boxed{47\,N}$$

$$\boxed{|\vec{R}|} = \sqrt{R_x^2 + R_y^2} \cong \boxed{79{,}4\,N}$$

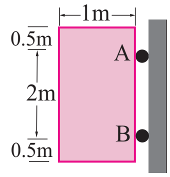
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.15 A uniform door of width 1 m and weight 200 N is supported by two hinges a distance 2 m apart.

a) Find the horizontal component of the force by the door on the upper hinge.

b) Find the horizontal component of the force by the door on the lower hinge.

c) Can you find the vertical force of the door on the upper or lower hinge? If not, what do you know about these forces?

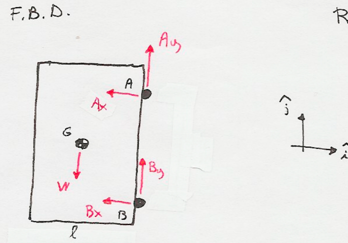


Problem 5.15

$\ell = 1\,m$; $\ell_{AB} = 2\,m$

F.B.D.

Roberto Podavini



$W = 200\,N$

$\vec{W} = -W\hat{j}$

$\{\sum \vec{M}_{/A} = \vec{0}\} \cdot \hat{k} \Rightarrow$

$\Rightarrow \{\vec{r}_{AG} \times \vec{W} + \vec{r}_{AB} \times (-B_x\hat{i}) = \vec{0}\} \cdot \hat{k} \Rightarrow$

$\Rightarrow \frac{\ell}{2}W - \ell_{AB}B_x = 0 \Rightarrow \boxed{B_x} = \frac{1}{2}\frac{\ell}{\ell_{AB}}W = \frac{W}{4} = \boxed{50N}$

$\{\vec{M}_{/B} = \vec{0}\} \cdot \hat{k} \Rightarrow \{\vec{r}_{BG} \times \vec{W} + \vec{r}_{BA} \times (-A_x\hat{i})\} \cdot \hat{k} = 0 \Rightarrow$

$\Rightarrow \frac{\ell}{2}W + \ell_{AB}A_x = 0 \Rightarrow \boxed{A_x} = -\frac{1}{2}\frac{\ell}{\ell_{AB}}W = -\frac{W}{4} = \boxed{-50N}$

$\{\sum \vec{F}_i = \vec{0}\} \cdot \hat{j} \Rightarrow -W + A_y + B_y = 0 \Rightarrow$

$\Rightarrow \boxed{A_y + B_y} = W = \boxed{200N}$

the single components cannot be obtained; only their sum is known

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

and

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

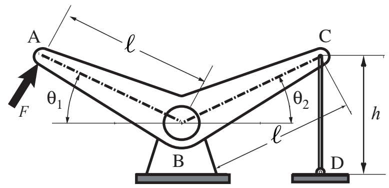
they are the support reactions on the door

$\Downarrow$

the forces by the door on the hinges are $\boxed{-\vec{A}}$ and $\boxed{-\vec{B}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.16 In the mechanism shown, find the maximum force F that can be applied at A normal to the link AB such that the magnitude of the force in rod CD does not exceed 10 kN.

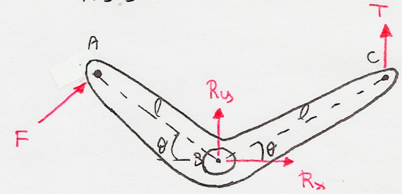


Problem 5.16

$T_{max} = 10\,kN$

F.B.D.

Roberto Padovani



$\sum \vec{M}_{/B} = \vec{0} \Rightarrow$

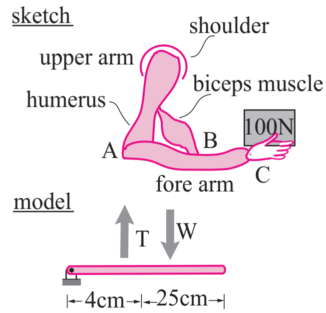
$\Rightarrow \vec{r}_{BA} \times \vec{F} + \vec{r}_{BC} \times \vec{T} = \vec{0}$ (1)

$\{(1)\}\cdot\hat{k} \Rightarrow \ell F - \ell T \cos\theta = 0 \Rightarrow F = T\cos\theta$

$\Rightarrow \boxed{F_{max} = T_{max}\cos\theta}$

5.2.17 For biomechanics purposes muscles are commonly modeled as massless cables and joints (elbow, shoulder, hip, ankle, etc) as frictionless hinges connecting rigid bones. You will find that the muscle tension and joint reaction forces are large compared to the loads being carried. This is a general feature in biomechanics because muscles usually have short lever-arms relative to the bone lengths.

A human forearm weighs 14 N and supports a 100 N weight. Find the muscle tension and the force of the upper arm on the forearm at the elbow.



Problem 5.17

Roberto Padovani

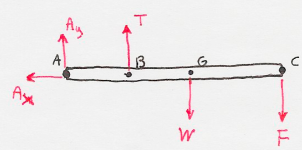
F.B.D.

$\ell_{AB} = 4$ cm

$\ell_{BC} = 25$ cm

$W = 14$ N

$F = 100$ N

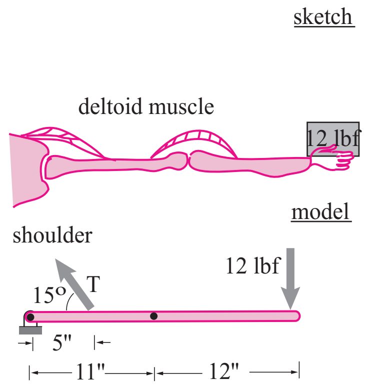


$$\left\{ \sum \vec{M}_{/A} = \vec{0} \right\} \cdot \hat{k} \Rightarrow \ell_{AB} T - \ell_{AG} W - \ell_{AC} F = 0 \Rightarrow$$

$$\Rightarrow T = \frac{\ell_{AG}}{\ell_{AB}} W + \frac{\ell_{AC}}{\ell_{AB}} F = \frac{29\text{ cm}}{2 \cdot 4\text{ cm}} W + \frac{29\text{ cm}}{4\text{ cm}} F \Rightarrow$$

$$\Rightarrow \boxed{T} = \frac{29}{4}\left(\frac{W}{2} + F\right) = \boxed{775{,}75\ N}$$

5.2.18 See Problem 5.2.17. An arm weighs 7 pounds and supports a 12 pound weight. Find the tension in the deltoid muscle and the force of the body on the arm at the shoulder joint.



Problem 5.18

4.2.18

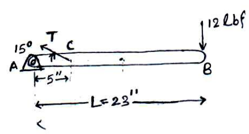
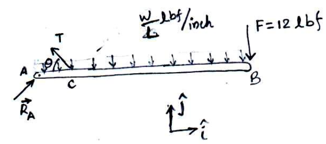


fig (i)



fig(ii) FBD

Let us use following representations

Complete length of the arm $= 12'' + 11'' = 23'' = L$.

Length AC $= 5'' = l_1$

'w', weight of arm is uniformly distributed along the length and shown by weight per unit length, $\frac{w}{L} = \frac{7}{23}$ lbf/in.

Now, doing moment balance of the system [FBD in fig (ii)] about point 'A', we get:

$$\vec{r}_{B/A} \times (-F\hat{j}) + \vec{r}_{C/A} \times (-T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j}) + \int_0^L r\hat{i} \times \frac{w}{L} dr(-\hat{j}) = \vec{0}$$

$$\Rightarrow \quad -FL\hat{k} + l_1 T\sin\theta\,\hat{k} - \frac{wL}{2}\hat{k} = \vec{0} \qquad \text{(i)}$$

$$\{\text{eqn (i)}\} \cdot \hat{k} \Rightarrow \quad T = \left(F + \frac{w}{2}\right)\frac{L}{l_1 \sin\theta} \Rightarrow T = \frac{(12 + 3.5)\,23}{5\,\sin(15^\circ)}$$

$$\Rightarrow \boxed{T = 275.48 \text{ lbf}}$$

Force balance of the body gives:

$$\vec{R}_A - T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j} - F\hat{j} - w\hat{j} = \vec{0}$$

$$\Rightarrow \quad \vec{R}_A = T\cos\theta\,\hat{i} + (F + w - T\sin\theta)\,\hat{j}$$

$$\Rightarrow \boxed{\vec{R}_A = (266.1\,\hat{i} - 52.3\,\hat{j}) \text{ lbf}}$$

The magnitude of force of the body on the arm at the joint is $|\vec{R}_A| = 271.2$ lbf.

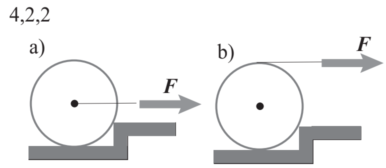
Notice that, the muscle tension 'T' is much greater than external force because of smaller lever arm and smaller angle to the arm, when the arm is horizontally extended.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.19 A 240 N roller is 1 m in diameter. It is being pulled over a 0.1 m curb with a horizontal rope. The roller does not slide on the curb.

a) What is the force required to lift the roller over the curb with the rope attached at the middle?

b) What is the force required if the rope is instead wrapped around the roller as shown?



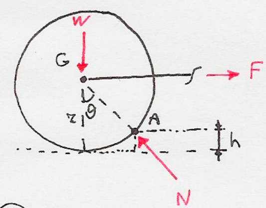
Problem 5.19

Roberto Padovani

a) $r = 1\,m$; $h = 0.1\,m$

$W = 240\,N$

F.B.D.



roller lifting $\Leftrightarrow$ no support reaction from the ground

$$\sum \vec{M}_{/A} = \vec{0} \Rightarrow \vec{r}_{AG} \times \vec{W} + \vec{r}_{AG} \times \vec{F} = \vec{0} \quad (1)$$

$$\{(1)\} \cdot \hat{k} \Rightarrow rW\sin(\theta) - rF\sin(90^\circ - \theta) = 0 \Rightarrow$$

$$\Rightarrow \cancel{r}W\sin(\theta) - \cancel{r}F\cos(\theta) = 0 \Rightarrow$$

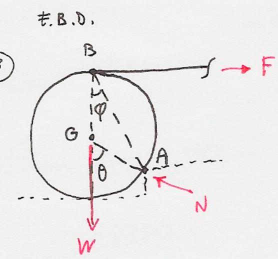
$$\Rightarrow F = W\tan(\theta) \quad (2)$$

$$\cos\theta = \frac{r-h}{r} = 1 - \frac{h}{r} = 0.9 \Rightarrow \theta = \arccos(0.9) \simeq 25.8^\circ$$

$$(2) \Rightarrow \boxed{F} = W\tan(\theta) \simeq \boxed{116\,N}$$

b)

F.B.D.



$$\sum \vec{M}_{/A} = \vec{0} \Rightarrow \vec{r}_{AG} \times \vec{W} + \vec{r}_{AB} \times \vec{F} = \vec{0} \quad (3)$$

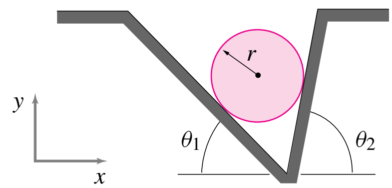
$$\{(3)\} \cdot \hat{k} \Rightarrow W\cancel{r}\sin\theta - F\cancel{r}\sin(90^\circ - \varphi) = 0 \Rightarrow$$

$$\Rightarrow \boxed{F} = W\frac{\sin\theta}{\cos\varphi} = W\frac{\sin\theta}{\cos(\theta/2)} =$$

$$= W\frac{\sqrt{1-\cos^2\theta}}{\sqrt{\frac{1+\cos\theta}{2}}} \cong \boxed{107\,N}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.2.20 What are the forces on the disk due to the groove? Define any variables you need.



Problem 5.20

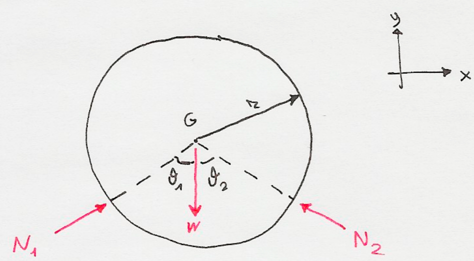
Roberto Padovani

$$\vec{W} = -W\hat{j}$$

$$\vec{N}_1 = N_1\left(\sin(\theta_1)\hat{i} + \cos(\theta_1)\hat{j}\right)$$

$$\vec{N}_2 = N_2\left(-\sin(\theta_2)\hat{i} + \cos(\theta_2)\hat{j}\right)$$

F.B.D.



$$\sum \vec{F} = \vec{0} \qquad (1)$$

$$\{1\}\cdot\hat{i} \Rightarrow N_1\sin(\theta_1) - N_2\sin(\theta_2) = 0 \Rightarrow N_1 = N_2\frac{\sin(\theta_2)}{\sin(\theta_1)}$$

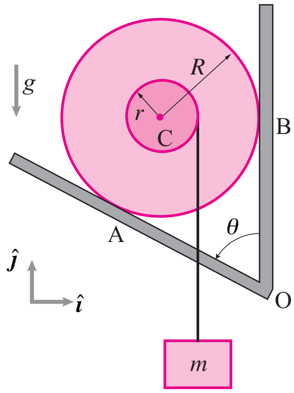
$$\{1\}\cdot\hat{j} \Rightarrow N_1\cos(\theta_1) + N_2\cos(\theta_2) = W \Rightarrow$$

$$\Rightarrow N_2\frac{\sin(\theta_2)}{\sin(\theta_1)}\cos(\theta_1) + N_2\cos(\theta_2) = W \Rightarrow$$

$$\Rightarrow \boxed{N_2 = \frac{W}{\dfrac{\sin(\theta_2)}{\tan(\theta_1)} + \cos(\theta_2)}}$$

$$\Rightarrow \boxed{N_1 = \frac{W}{\cos(\theta_1) + \dfrac{\sin(\theta_1)}{\tan(\theta_2)}}}$$

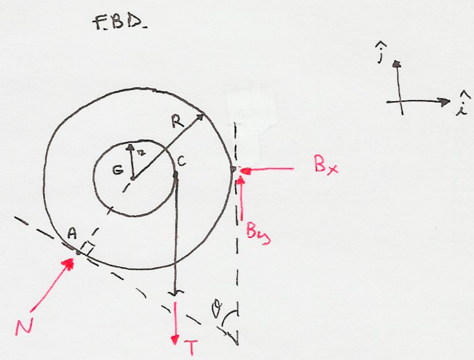
5.2.22 Assuming the spool is massless and that there is no friction at point A, find the force on the spool at point B in order to maintain equilibrium. Note, there is friction at B. Answer in terms of some or all of r, R, g, θ, and m.



Problem 5.22

Roberto Radovani

from the F.B.D of the load :

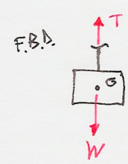


$\sum \vec F = \vec 0 \Rightarrow \vec W + \vec T = \vec 0 \Rightarrow$

$\Rightarrow -W\hat j + T\hat j = \vec 0 \Rightarrow$

$\Rightarrow T = W = mg$

from the F.B.D. of the wheel :



$\sum \vec M_{/G} = \vec 0 \Rightarrow$

$\Rightarrow \vec r_{GC} \times \vec T + \underbrace{\cancel{\vec r_{GA} \times} \vec N}_{=\vec 0} + \vec r_{GB} \times \vec B = \vec 0$

$\Rightarrow \vec r_{GC} \times (-T\hat j) + \vec r_{GB} \times (\underbrace{\cancel{-B_x \hat i}}_{=\vec 0} + B_y \hat j) = \vec 0 \Rightarrow$

$\Rightarrow r\hat i \times (-T\hat j) + R\hat i \times (B_y \hat i) = \vec 0 \Rightarrow -rT\hat k + RB_y \hat k = \vec 0$

$\Rightarrow \boxed{B_y = mg\frac{r}{R}}$

$\sum \vec F = \vec 0 \Rightarrow \vec N + \vec T + \vec B = 0 \quad (1)$

$\{1\}\cdot\hat j \Rightarrow N\sin(\theta) - mg + mg\frac{r}{R} = 0 \Rightarrow N = mg\frac{(1-\frac{r}{R})}{\sin(\theta)}$

$\{2\}\cdot\hat i \Rightarrow N\cos(\theta) - B_x = 0 \Rightarrow B_x = N\cos\theta \Rightarrow$

$\Rightarrow \boxed{B_x = mg\frac{(1-\frac{r}{R})}{\tan(\theta)}}$

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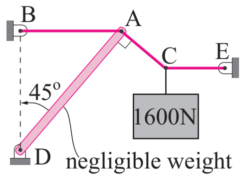
5.2.23 Find the tension in cord AB.

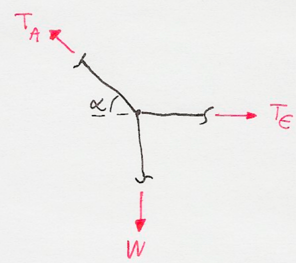


Problem 5.23

Roberto Padovani

$\alpha = 45^\circ \quad ; \quad W = 1600\,N$

F.B.D.



$\sum \vec{F} = \vec{0} \Rightarrow \vec{T}_A + \vec{T}_E + \vec{W} = \vec{0}$ ①

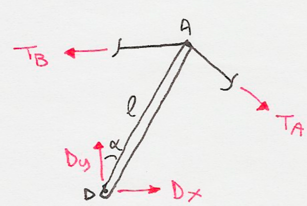
$\vec{T}_A = T_A \left(-\cos(\alpha)\hat{\imath} + \sin(\alpha)\hat{\jmath} \right)$

$\vec{T}_E = T_E\,\hat{\imath}$

$\vec{W} = -W\,\hat{\jmath}$

$\{①\}\cdot\hat{\jmath} \Rightarrow T_A \sin(\alpha) - W = 0 \Rightarrow T_A = \dfrac{W}{\sin(\alpha)} \simeq 2263\,N$

F.B.D.



$\sum \vec{M}_{/D} = \vec{0} \Rightarrow$

$\Rightarrow \vec{r}_{DA} \times \vec{T}_A + \vec{r}_{DA} \times \vec{T}_B = \vec{0} \Rightarrow$

$\Rightarrow \cancel{\ell}\, T_A - \cancel{\ell}\, T_B \sin(90^\circ - \alpha) = 0 \Rightarrow$

$\Rightarrow \boxed{T_B} = \dfrac{T_A}{\cos(\alpha)} = \dfrac{W}{\sin(\alpha)\cos(\alpha)} \boxed{= 3200\,N}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*