5.2 Static equilibrium of one body

Preparatory Problems

5.2.1 For problems below, assume a 2D free-body diagram has been drawn where forces $\vec{F}_1, \vec{F}_2, \ldots, \vec{F}_5$ are applied at positions $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_5$ relative to the origin. Use this information in the answers below.

a) What is the force-balance equation?

b) What is the moment-balance equation about the origin?

c) What are equilibrium conditions?

d) Write equilibrium conditions in as many different ways as you can.

e) How many independent scalar equations can one write using various force and moment-balance equations?

f) If force $\vec{F}_4$ is moved to a new position along its direction, which equilibrium equations are changed and which are not?

g) If force $\vec{F}_4$ is displaced sideways relative to its direction, which equilibrium equations are changed and which are not?

5.2.2 What is the meaning of *the line of action of a force*?

5.2.3 If only two forces, $\vec{F}_1$ and $\vec{F}_2$, act on a body at $\vec{r}_1$ and $\vec{r}_2$, what do the equilibrium conditions tell you about the two forces?

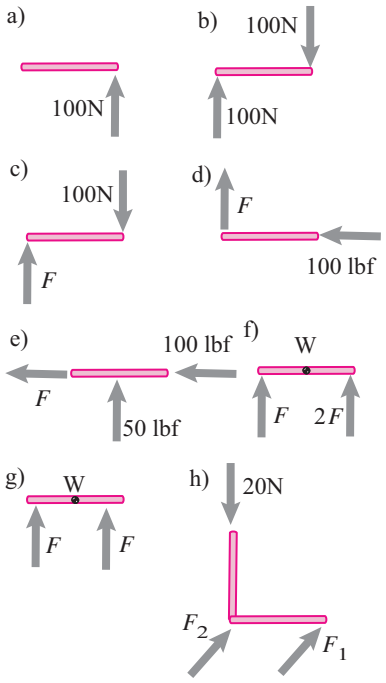
5.2.4 If only three forces, $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$, act on a body at $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$, what do the equilibrium conditions tell you about the three forces?

5.2.5 Which of the bars below cannot possibly be in equilibrium and which ones can? (Where the center of mass is indicated, assume non-zero weight acting vertically downwards. Assume dimensions as needed.)

i) Explain in words.

ii) Explain using equations.

Note that scalars (e.g., F, F_1, etc.) can be positive or negative.



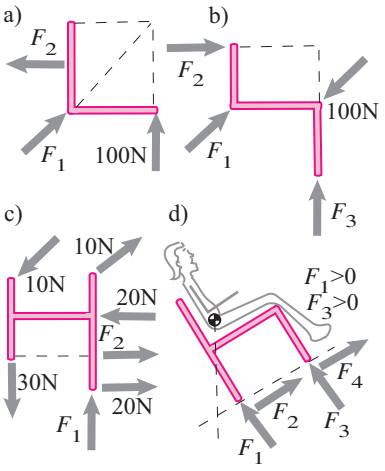
Problem 5.2.5

5.2.6 Which of the objects below cannot possibly be in equilibrium and which ones can? (Where the center of mass is indicated, assume non-zero weight acting vertically downwards. Assume dimensions as needed.)

a) Explain in words.

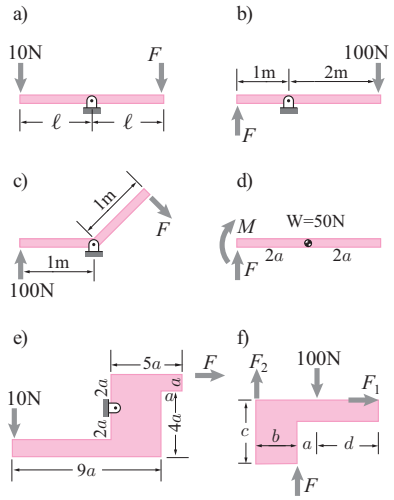
b) Explain using equations.

Note that scalars (e.g., F, F_1, etc.) can be positive or negative unless mentioned otherwise.



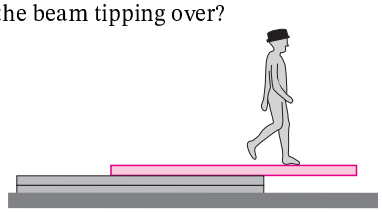
Problem 5.2.6

5.2.7 In the problems shown below, find F for equilibrium.



Problem 5.2.7

5.2.8 A straight uniform 1000 N beam is 6 m long. It rests on a flat stack of boards with a 2 m overhang. How far out the overhang can an 800 N person walk without the beam tipping over?

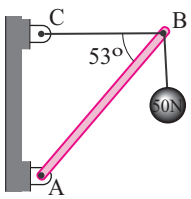


Problem 5.2.8

5.2.9 The uniform bar AB is 5 m long and weighs 100 N. It is pinned at A and supported by the horizontal cord BC attached at end B. A 50 N weight hangs from end B.

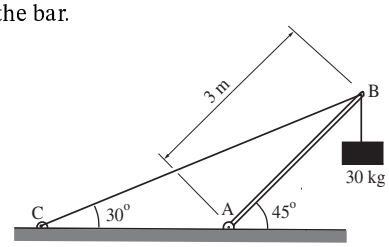
a) Find the tension in cord C.

b) Find the magnitude and direction of the force exerted on the pin at A by the bar.



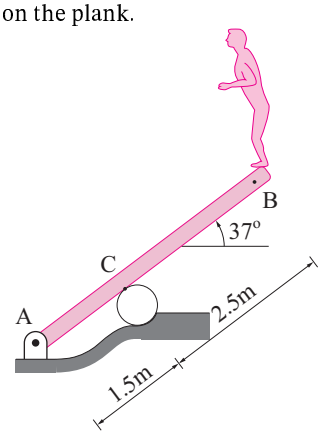
Problem 5.2.9

5.2.10 For static equilibrium of the system and the configuration shown in the figure, find the support reaction at end A of the bar.



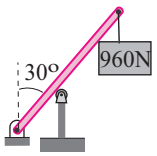
Problem 5.2.10

5.2.11 A 400 N child stands on the end of a uniform 800 N diving plank which is pinned on one end and which also rests on a log (idealized as frictionless). Find the force of the log on the plank and of the pin on the plank.



Problem 5.2.11

5.2.12 A negligible weight 6 m rod is pinned at one end and leans over a frictionless wall a third of the way up from the bottom. Find the forces of the wall and the pin on the rod.



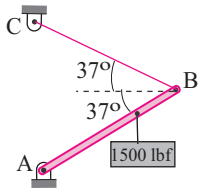
Problem 5.2.12

More-Involved Problems

5.2.13 The uniform boom AB is 20 ft long and weighs 150 lbf. A 1500 lbf weight is suspended from a point 5 ft from end B. The boom is pinned at A and supported by the cable BC attached at end B.

a) Find the tension in the cable.

b) Find the force exerted on the boom by the pin at A.

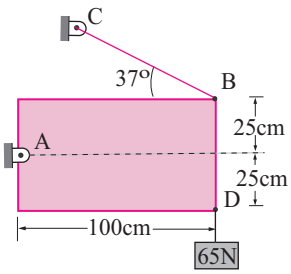


Problem 5.2.13

5.2.14 The 30 N uniform rectangular plate is supported by a pin at A and cable BC attached at corner B. A 65 N weight hangs from corner D.

a) Find the tension in the cable.

b) Find the force exerted on the plate by the pin at A.



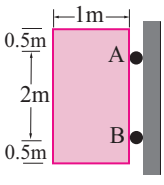
Problem 5.2.14

5.2.15 A uniform door of width 1 m and weight 200 N is supported by two hinges a distance 2 m apart.

a) Find the horizontal component of the force by the door on the upper hinge.

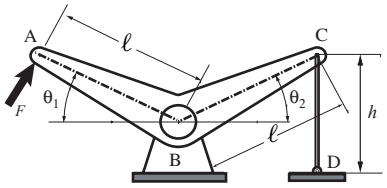
b) Find the horizontal component of the force by the door on the lower hinge.

c) Can you find the vertical force of the door on the upper or lower hinge? If not, what do you know about these forces?



Problem 5.2.15

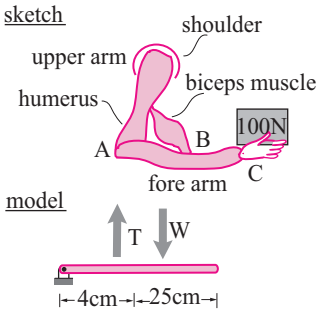
5.2.16 In the mechanism shown, find the maximum force F that can be applied at A normal to the link AB such that the magnitude of the force in rod CD does not exceed 10 kN.



Problem 5.2.16

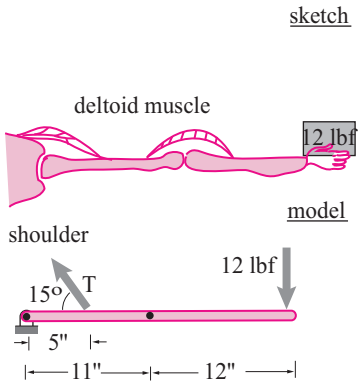
5.2.17 For biomechanics purposes muscles are commonly modeled as massless cables and joints (elbow, shoulder, hip, ankle, etc) as frictionless hinges connecting rigid bones. You will find that the muscle tension and joint reaction forces are large compared to the loads being carried. This is a general feature in biomechanics because muscles usually have short lever-arms relative to the bone lengths.

A human forearm weighs 14 N and supports a 100 N weight. Find the muscle tension and the force of the upper arm on the forearm at the elbow.



Problem 5.2.17

5.2.18 See Problem 5.2.17. An arm weighs 7 pounds and supports a 12 pound weight. Find the tension in the deltoid muscle and the force of the body on the arm at the shoulder joint.

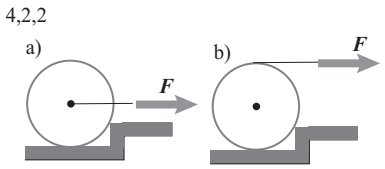


Problem 5.2.18

5.2.19 A 240 N roller is 1 m in diameter. It is being pulled over a 0.1 m curb with a horizontal rope. The roller does not slide on the curb.

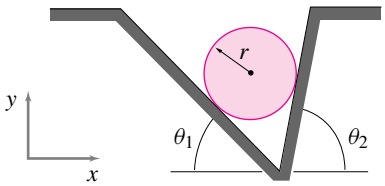
a) What is the force required to lift the roller over the curb with the rope attached at the middle?

b) What is the force required if the rope is instead wrapped around the roller as shown?



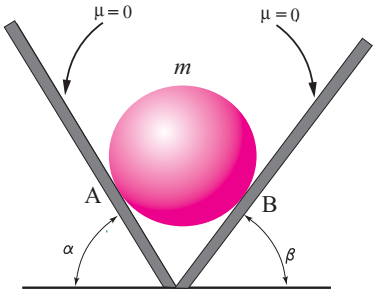
Problem 5.2.19

5.2.20 What are the forces on the disk due to the groove? Define any variables you need.



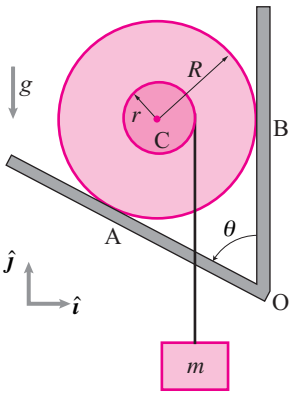
Problem 5.2.20

5.2.21 A solid sphere of mass $m = 5\,\text{kg}$ and radius $R = 250\,\text{mm}$ rests between two frictionless inclined planes. Let $\alpha = 60^\circ$. Find the magnitudes of normal reactions of the plane as functions of β and plot normalized reactions (N_1/mg and N_2/mg for $0 < \beta \leq 90^\circ$). Comment on the plot.



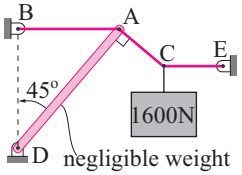
Problem 5.2.21

5.2.22 Assuming the spool is massless and that there is no friction at point A, find the force on the spool at point B in order to maintain equilibrium. Note, there is friction at B. Answer in terms of some or all of r, R, g, θ, and m.



Problem 5.2.22

5.2.23 Find the tension in cord AB.



Problem 5.2.23