# 5.1 Static equilibrium of a particle

## What is a particle?

The word particle usually means something small. In mechanics a *particle* is an object for which we don't worry about rotation, or the tendency of forces to cause rotation. A particle may or may not be small. Besides, smallness is in the eyes of the beholder. For some purposes a galaxy is well-modeled as a particle and for others a molecule is too big to be thought of as a particle. Big or small, the particle model of a system is defined by the lack of attention paid to the moment-balance equations ①. Either moment balance is trivially satisfied or you can find what you need without worrying about how it is satisfied ②.

For statics of a particle, force-balance tells all: $$\sum_{\substack{\text{All forces}\\ \text{on the FBD}}} \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \qquad (Ic)$$

In two dimensions, this equilibrium equation makes up 2 independent scalar equations (2 components of the net force vector). In 3 dimensions we get 3 independent scalar equations. So we expect to be able to solve for 2 unknown quantities in 2D particle mechanics, and 3 in 3D.

## The statics-of-a-particle recipe

For particle statics, we work with a simplified form of the general recipe from the inside back cover.

1) **Draw a free-body diagram (FBD) of the part of interest.**
Use knowledge of the contact conditions (see Chapter 3) to draw known and unknown aspects of the forces appropriately (see fig. 2.42 on page 169);
2) **Set the sum of the forces on the FBD to zero: $\sum \vec{\boldsymbol{F}}_i = \vec{\boldsymbol{0}}$.**
('Equilibrium', 'force balance', or 'linear momentum balance in statics');
3) **Solve the equations for unknowns.**
Use vector manipulation skills (Chapter 2) to solve the force balance equation for unknowns of interest.

①**Examples of particles.** You might think of a galaxy as an immense thing, not just a dot. But its overall motion through a cluster of galaxies is probably well-described by thinking of it as a particle. The galaxy may rotate and distort in interesting ways, but one can ignore those rotations and distortions when calculating overall translation. That is, one might model an immense rotating and distorting galaxy as a particle.

Similarly for an accelerating car, a block sliding on a ramp or a machine part, one may learn enough about the forces and motion using a particle model. Distortion and rotation might be there, and even large, but might not be important for understanding the overall motion or force balance.

②Equations are 'satisfied' when the right side is equal to the left. Take that as a definition or, if it helps you, think of it as a desire that you would like to accommodate.

## Scalar mechanics

In scalar mechanics — as opposed to vector mechanics — one takes the dot product of Eqn. (Ic) with unit vectors $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ and write the three scalar component equations ③.

③Actually, people who think in terms of scalar mechanics do the dot products, but implicitly (or unconsciously), and think of the component equations as fundamental.

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \text{additionally, in 3D,} \qquad \sum F_z = 0.$$

Although you can do most problems plowing through with the 'component' or scalar approach, often there are shortcuts or insights that depend on the vectorial view.

## 1D statics of a particle


Figure 5.1: Equilibrium of a particle in 1D.

Let's call the one dimension of interest the $x$ direction. The key governing equation is

$$\sum F_x = 0.$$

You could call the special direction $y$, $z$, $x'$ or $s$ if you like and then use, say $\sum F_z = 0$. The next two simple examples pretty much cover 1D particle statics.

**Example: Balance of two forces**
For the particle in fig. 5.1, force balance gives

$$\sum \vec{\boldsymbol{F}}_i = \vec{\boldsymbol{0}} \quad \Rightarrow \quad 10\,\mathrm{N}\hat{\boldsymbol{\imath}} - F\hat{\boldsymbol{\imath}} = \vec{\boldsymbol{0}}.$$

Either by equating $x$ components of both sides, or equivalently, by dotting both sides with $\hat{\boldsymbol{\imath}}$, we get $F = 10\,\mathrm{N}$. Or, we could have just jumped to scalar mechanics,

$$\sum F_x = 0 \quad \Rightarrow \quad 10\,\mathrm{N} - F = 0 \quad \Rightarrow \quad F = 10\,\mathrm{N}.$$

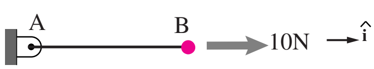



Figure 5.2: Particle held by a string.

Most often we have to contend with forces which don't show up until we draw a free-body diagram.

**Example: Force pulling on a string.**
For the particle in fig. 5.2, the quantity of interest, the tension in the cable, doesn't show in the sketch. We need to draw a free body diagram of the particle which means cutting the string. This FBD is shown in fig. 5.1, where $F = T_{AB}$ represents the tension in cable AB. So force balance gives $T_{AB} = F = 10\,\mathrm{N}$.

## 2D statics of a particle

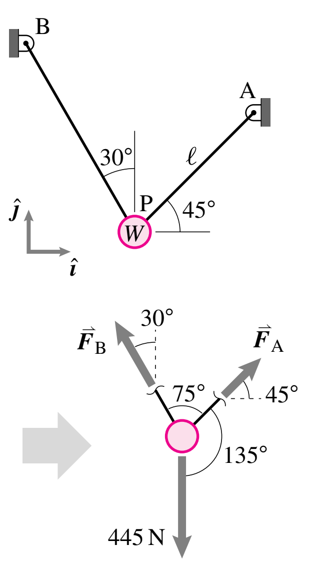



Figure 5.3: Weight hung from two lines with associated free-body diagram.

The situation is less trivial when we go to 2D.

**Example.** A 100 pound (445 N) weight hangs from 2 lines in fig. 5.3. We cut the strings, draw a free-body diagram and add the forces to get

$$\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \quad \Rightarrow \quad 445\,\mathrm{N}(-\hat{\boldsymbol{\jmath}}) + \underbrace{F_A \frac{(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}})}{\sqrt{2}}}_{\vec{F}_A} + \underbrace{F_B(-\tfrac{1}{2}\hat{\boldsymbol{\imath}} + \tfrac{\sqrt{3}}{2}\hat{\boldsymbol{\jmath}})}_{\vec{F}_B} = \vec{\boldsymbol{0}}. \quad (5.1)$$

This can be solved various ways (see below) to get $F_A = 230.3\,\mathrm{N}$ and $F_B = 325.8\,\mathrm{N}$.

Although moment balance is technically superfluous in particle mechanics, when the forces are concurrent, moment balance can be used as a shortcut.

**How to solve vector statics equations?**

**Method 1)** Pull out both $x$ and $y$ components of the vectors to get 2 equations in 2 unknowns;

$$\sum F_x = 0 \quad \Rightarrow \quad F_A/\sqrt{2} - F_B/2 = 0$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_A/\sqrt{2} + F_B\sqrt{3}/2 - 445N = 0.$$

**Method 2)** Equivalently, dot both sides of the equation with $\hat{\imath}$ and $\hat{\jmath}$ to get 2 equations in 2 unknowns;

$$\{\text{eqn. (5.1)}\} \cdot \hat{\imath} \quad \Rightarrow \quad 0 = F_A/\sqrt{2} - F_B/2$$
$$\{\text{eqn. (5.1)}\} \cdot \hat{\imath} \quad \Rightarrow \quad 0 = F_A/\sqrt{2} + F_B\sqrt{3}/2 - 445N.$$

**Method 3)** dot both sides with a vector orthogonal to $r_{\mathrm{AP}}$ to get one equation in $F_B$, similarly dot with a vector orthogonal to $\vec{\boldsymbol{r}}_{\mathrm{BP}}$ to get one equation in $F_A$.

$$\{\text{eqn. (5.1)}\} \cdot (-\hat{\imath} + \hat{\jmath}) \quad \Rightarrow \quad 0 = -445N + F_B(\frac{1}{2} + \frac{\sqrt{3}}{2})$$
$$\{\text{eqn. (5.1)}\} \cdot (\sqrt{3}\hat{\imath} + \hat{\jmath}) \quad \Rightarrow \quad 0 = -445N + F_A\frac{\sqrt{3}+1}{\sqrt{2}}$$

4

4 Note that if $\vec{\boldsymbol{v}} = a\hat{\imath} + b\hat{\jmath}$ then $\vec{\boldsymbol{w}} = -b\hat{\imath} + a\hat{\jmath}$ is orthogonal to it because $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{w}} = -ab + ba = 0$. That is, in the plane to get a vector perpendicular to $\vec{\boldsymbol{v}}$ interchange the components and negate one of them (either one).

**Method 4)** cross both sides with $\vec{\boldsymbol{r}}_{AP}$ to get one equation in $F_B$, similarly cross with $\vec{\boldsymbol{r}}_{\mathrm{BP}}$ to get $F_A$.

$$\vec{\mathbf{0}} = \left(-\ell\hat{\imath}/\sqrt{2} - \ell\hat{\jmath}/\sqrt{2}\right) \times \left(445\,\mathrm{N}(-\hat{\jmath}) + F_A\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + F_B(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})\right)$$
$$\Rightarrow \quad 0 = -445N + F_B(\frac{1}{2} + \frac{\sqrt{3}}{2})$$
$$\vec{\mathbf{0}} = \left(d\hat{\imath}/2 - d\sqrt{3}\hat{\jmath}/2\right) \times \left(445\,\mathrm{N}(-\hat{\jmath}) + F_A\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + F_B(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})\right)$$
$$\Rightarrow \quad 0 = -445N + F_A\frac{\sqrt{3}+1}{\sqrt{2}}.$$

See section 1.5 starting on page 106 for more discussion about how to solve vector equations.

Another approach, mathematically equivalent to method 4 above, is to use moment balance.

**Example: Moment balance**
Consider again fig. 5.3. Moment balance about point A gives

$$\sum \vec{\boldsymbol{M}}_{\mathrm{A}} = \vec{\mathbf{0}} \quad \Rightarrow \quad \vec{\boldsymbol{r}}_{\mathrm{P/A}} \times (445\,\mathrm{N}(-\hat{\jmath})) + \vec{\boldsymbol{r}}_{\mathrm{P/A}} \times \left(F_B(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})\right) + \vec{\mathbf{0}} = \vec{\mathbf{0}}.$$

Evaluating the cross products one way or another, we again get $F_B = 325.8\,\mathrm{N}$. Similarly moment balance about B gives $F_A = 230.3\,\mathrm{N}$.

**Example: A kite.**
A kite flying steadily in a breeze is roughly in static equilibrium. The three forces acting on it are from the air, pushing the kite downwind and up; from gravity, pulling the kite down; and from the string pulling the kite upwind and down. The three forces must add to zero.

A funny thing about kites is that they only stay up because you pull them down.

Whether force or moment balance is used, for concurrent force systems we only have two independent scalar equilibrium equations in 2D, and three in 3D .

## Frictionless contact

As discussed in Chapter 3.1, engineered parts that slide often have bearings or lubrication to reduce the sliding resistance. To simplify analysis, that remaining resistance is often neglected, and we model the contact as 'frictionless' ($\mu = 0$). This makes the interaction force normal (perpendicular) to the contacting surfaces.

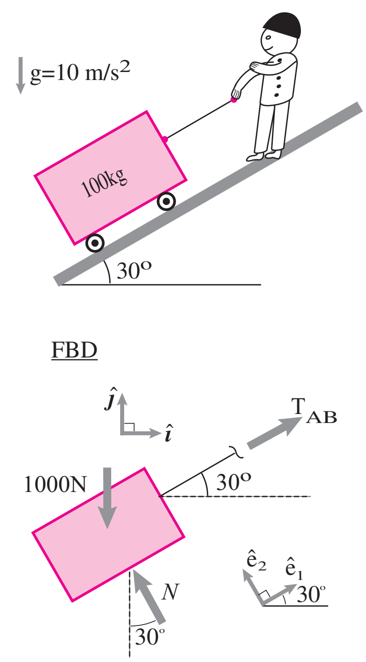



Figure 5.4: A wagon pulled uphill and associated free-body diagram. Because we are doing particle mechanics, we combine the two forces on the wheels into the single resultant $N$. Further, for particle mechanics we need not worry about where that $N$ is applied.

**Example: Pull a wagon uphill**
See fig. 5.4. From the free-body diagram we have

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad -1000\,\mathrm{N}\,\hat{\boldsymbol{\jmath}} + N\hat{\boldsymbol{e}}_2 + T_{AB}\hat{\boldsymbol{e}}_1 = \vec{0}. \tag{5.2}$$

where $\hat{\boldsymbol{e}}_1 = \cos(30^\circ)\hat{\boldsymbol{\imath}} + \sin(30^\circ)\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{e}}_2 = -\sin(30^\circ)\hat{\boldsymbol{\imath}} + \cos(30^\circ)\hat{\boldsymbol{\jmath}}$. $N$ and $T_{AB}$ are unknown forces. Here are two ways to solve for the unknowns.
**Method I.** Substitute the expressions for $\hat{\boldsymbol{e}}_1$ and $\hat{\boldsymbol{e}}_2$ above into eqn. (5.2), extract $x$ and $y$ components to get 2 equations in two unknowns which you can solve to get $T_{AB} = 500\,\mathrm{N}$ and $N = 500\sqrt{3}\,\mathrm{N}$ (note the font confusion that the force quantity $N$ and unit N have different meanings).
**Method II.** Using well chosen dot products can simplify the algebra. Take the dot products of both sides of eqn. (5.2) with $\hat{\boldsymbol{e}}_1$ and then separately with $\hat{\boldsymbol{e}}_2$, to get two scalar equations. Dotting eqn. (5.2) with $\hat{\boldsymbol{e}}_1$ eliminates terms orthogonal to $\hat{\boldsymbol{e}}_1$, namely $N\hat{\boldsymbol{e}}_2$. And dotting eqn. (5.2) with $\hat{\boldsymbol{e}}_2$ 'kills' the $T_{AB}\hat{\boldsymbol{e}}_1$ term. So the two equations each have only one unknown. See page 113 for more discussion of this method.

## Three-dimensional particle mechanics

The basic idea is the same in 3D as in 2D.

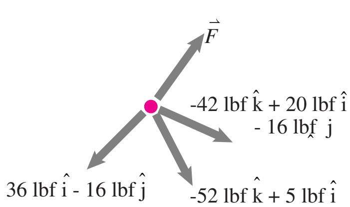



Figure 5.5: One unknown force $\vec{F}$.

**Example: One unknown force.**
Assume 3 known forces and one unknown force $\vec{F}$ are acting on a particle (fig. 5.5). Then from force balance

$$
\begin{aligned}
\vec{0} &= \sum \vec{F}_i \\
\Rightarrow \quad \vec{0} &= (36\,\mathrm{lbf}\,\hat{\boldsymbol{\imath}} - 16\,\mathrm{lbf}\,\hat{\boldsymbol{\jmath}}) + (-52\,\mathrm{lbf}\,\hat{\boldsymbol{k}} + 5\,\mathrm{lbf}\,\hat{\boldsymbol{\imath}}) \\
&\quad + (-42\,\mathrm{lbf}\,\hat{\boldsymbol{k}} + 20\,\mathrm{lbf}\,\hat{\boldsymbol{\imath}} - 16\,\mathrm{lbf}\,\hat{\boldsymbol{\jmath}}) + \vec{F} \\
\Rightarrow \quad \vec{F} &= \left(-61\hat{\boldsymbol{\imath}} + 32\hat{\boldsymbol{\jmath}} + 94\hat{\boldsymbol{k}}\right)\,\mathrm{lbf}.
\end{aligned}
$$

.

The new difficulties in 3D particle mechanics are

# 5.1 Existence and uniqueness

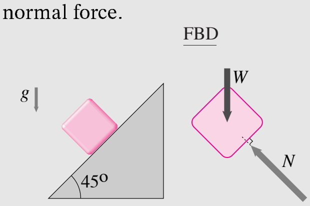
This is a relatively advanced aside.

The words *existence* and *uniqueness* may sound mathematically abstract and irrelevant to the real world. But if you translate 'existence' to 'that's possible' and 'non-existence' to 'no way' you can see the relevance. Likewise 'uniqueness' and 'non-uniqueness' translate to 'there's just one way to do that' and 'there's lots of ways to do that'.

In most homework problems there is an answer, and just one answer: the solution exists and is unique. But in projects and at work problems are often *ill-posed*. Non-existence or non-uniqueness may show up by a structure collapsing or a computer calculation giving erratic answers or error messages.

**Existence.** Sometimes equations have no solutions, they do not *exist*. For example the set $\begin{array}{ll} x+2y & =7 \\ 2x+4y & =15 \end{array}$ has no solutions. They don't exist. Why? Because subtracting twice the first equation from the second gives a contradiction: $0 = 1$.

**Example: Block on a slippery (frictionless) ramp.** Use statics to find the normal force.



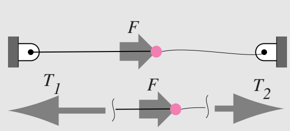

But the block slides and this is not a statics problem. So there is no statics solution. Even without intuition, the statics force-balance equations show that there is no value of $N$ that can make the force vectors add to zero. So no statics solution exists.

Such contradictions can be more subtle. Section 6.5 has some examples where even experts can't intuitively see that there are no solutions.

**Uniqueness.** Sometimes statics problems have more than one solution, that is a *non-unique* solution: the equation $x+y=1$ has many solutions including $(x,y)=(1,0)$, $(x,y)=(0,1)$, $(x,y)=(10,-9)$ *etc.*

Structural mechanics problems often have non-unique solutions.

**Example: Particle held by two strings.** Find the tension in the strings to the sides of the point of application of a given load $F = 10\text{N}$.




Force balance along the strings gives us one equation for the two unknown tensions.

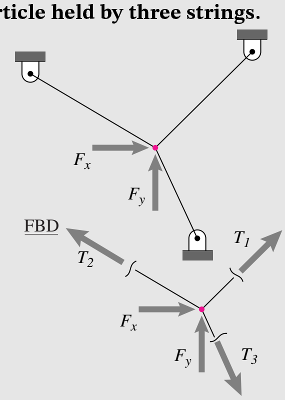
$$\left\{\sum \vec{F}_i = \vec{0}\right\}\cdot\hat{\imath} \qquad \Rightarrow \qquad -T_1 + F + T_2 = 0$$

No other force balance or moment-balance equation gives more information. For any given $F$ this equation has many solutions. The pair $(T_1, T_2)$ could be $(10\,\text{N}, 0)$ or $(0, -10\,\text{N})$ $(20\,\text{N}, 10N)$ $(17\,\text{N}, 7\,\text{N})$, etc.

Of course if you tie strings together like this and apply a force there is some actual tension in each string; reality, at any instant in time, is unique (as far as we know). For example, if you had tied the strings loosely together the right string gets slack and has $T_2 = 0$ and thus $T_1 = 10\,\text{N}$. But it takes an extra assumption of this nature to get a unique solution.

And just because you can make an assumption that leads you to a unique solution doesn't mean that this corresponds to reality. You might assume your friend had tied the strings together loosely and thus calculate $T_2 = 0$ and $T_1 = 10\,\text{N}$. But really she tied them together tightly so $T_2 = 30\,\text{N}$ and $T_1 = 40\,\text{N}$. Here is the same idea in 2D.

**Example: Particle held by three strings.**




Assume that $F_x$ and $F_y$ are given. What are the three tensions. Planar force balance gives two equations for the 3 unknown tensions. As in the previous example these equations have many solutions.

If you assume a) that one string goes slack and b) that no string can carry compression, then this problem has a unique answer. But you would have to know *a priori* that the strings were initially loose.

The same idea holds for 4 strings holding a particle in 3D. These string examples have a 'one parameter family of solutions'; specifying one number (say that the tension in cable 2 is zero) determines the other tensions. But there can be more non-uniqueness than that by using more strings and then to get a unique solution you have to make more assumptions.

**Counting equations and unknowns** All of the uniqueness issues above could be detected by counting equations and unknowns. For the block on the ramp we had two equations for the one unknown $N$. Whereas for the string problems we had more unknowns than equations. In summary,

- If you have more equations than unknowns, existence is likely to be an issue; you probably can't find any solutions.
- If you have more unknowns than equations then uniqueness *is* likely an issue; any solution you find is probably non-unique

But, as for the block on ramp (2 equations with 2 unknowns), there are cases for which equation counting does not tell all about existence and uniqueness: see the lower right corner of the large table in section 6.5. Some simple counter examples: $x = 3, 2x = 6$ has a unique solution; and you can solve the one equation $x^2 + y^2 = 0$ for 2 unknowns.

- Visualization in 3D. (So practice making and reading 3D drawings.); and
- The vector force-balance equation is 3D and thus equivalent to 3 scalar equations. Solving these is at the upper boundary of what most people can do reliably by hand or even with a non-programmable calculator. So methods that reduce the complexity of the solution are useful, as is the ability to set up the resulting equations on a computer or programmable calculator.

Hint: If the direction of a force is given (possibly implicitly) express the force as a scalar times a unit vector: $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\lambda}}$. (See top row, middle column of fig. 2.42 on page 169.)

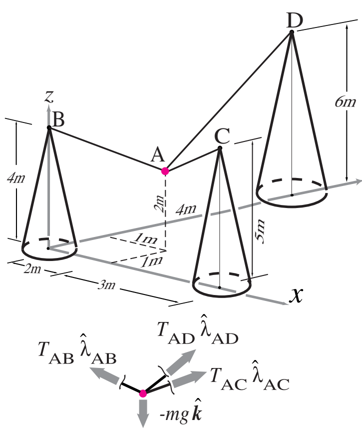



Figure 5.6: Mass A suspended by 3 ropes and a free-body diagram of the mass.

**Example: Particle held by 3 ropes.**
Say $m = 100\,\text{kg}$ and $g = 10\,\text{N}/\text{kg}$ in fig. 5.6. Force balance gives

$$\sum \vec{\boldsymbol{F}}_i = \vec{\boldsymbol{0}} \quad \Rightarrow \quad T_{AB}\hat{\boldsymbol{\lambda}}_{AB} + T_{AC}\hat{\boldsymbol{\lambda}}_{AC} + T_{AD}\hat{\boldsymbol{\lambda}}_{AD} - mg\hat{\boldsymbol{k}} = \vec{\boldsymbol{0}} \tag{5.3}$$

which is a 3D vector equation in 3 unknowns (3 scalar equations and 3 unknowns, good). The $\hat{\boldsymbol{\lambda}}$'s in eqn. (5.6) are known because the position vectors are given in the picture. For example,

$$\hat{\boldsymbol{\lambda}}_{AB} = \frac{\vec{\boldsymbol{r}}_{AB}}{|\vec{\boldsymbol{r}}_{AB}|} = \frac{\vec{\boldsymbol{r}}_{B} - \vec{\boldsymbol{r}}_{A}}{|\vec{\boldsymbol{r}}_{B} - \vec{\boldsymbol{r}}_{A}|}.$$

To get to a numerical answer for the tensions you can use many methods such as (see Sample 5.5 on page 254)

1. Brute force by hand.
2. Systematically set up matrix equations for solution by some means.
3. Set up and solve equations on a computer.
4. Use an appropriate dot product to extract one equation in one unknown.
5. Use moment about an axis to extract one equation in one unknown.

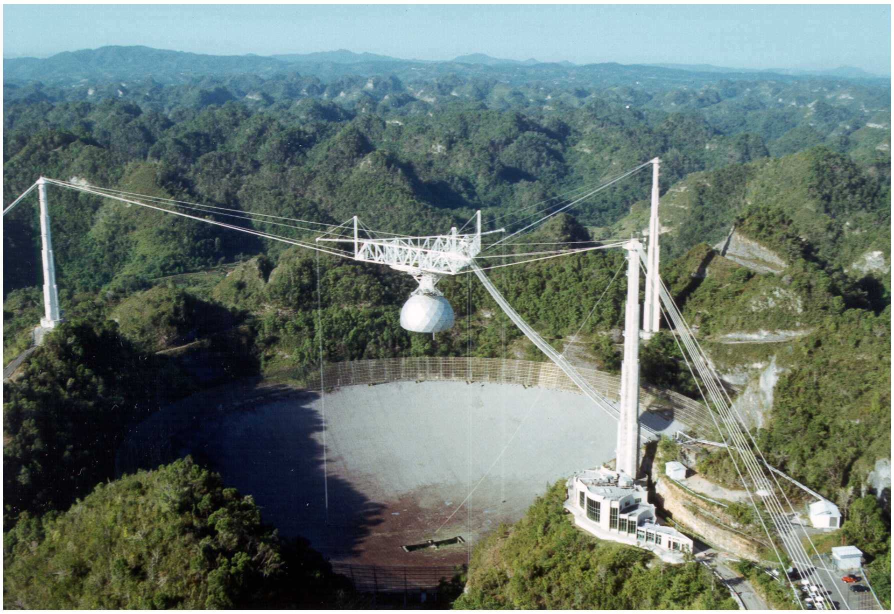


Figure 5.7: The 900 ton platform is suspended 450 feet above the curved reflector surface. The platform hangs on eighteen cables from three towers, each about 300 feet high. *Courtesy of the NSF funded NAIC-Arecibo Observatory in Puerto Rico.*

# 5.2 The simplification of dynamics to statics

**Is statics good enough?** The answer below is for your curiosity, not to help you with homework problems.

Classical mechanics, the equations on the inside cover of this book, is at least 99.99% accurate for at least 99.99% of mechanical engineering problems (see page 33 in the preface). Statics is a subset of classical mechanics. Statics is used for those dynamics situations where the statics approximation is reasonable (remember, no real situation is exactly static). Fortunately, for at least 90% of engineering mechanics calculations statics is at least 90% accurate.

In statics, we set the right hand sides of equations I and II (on inside cover) to zero. We are then thinking that these 'inertial' terms are small enough, compared to other forces, to be neglected. We replace the linear and angular momentum balance equations with their simplified statics forms

$$\sum_{\substack{\text{All external}\\ \text{forces}}} \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \qquad \text{and} \qquad \sum_{\substack{\text{All external}\\ \text{torques}}} \vec{\boldsymbol{M}}_C = \vec{\boldsymbol{0}} \quad (Ic, IIc).$$

which are sometimes called the *force balance* and *moment balance* and together are called the *equilibrium* equations. The forces to be summed (added) are the ones you see on a free-body diagram. The torques that are summed are those due to the same forces (by means of $\vec{\boldsymbol{r}}_{i/C} \times \vec{\boldsymbol{F}}_i$) and any applied couples.

> In dynamics forces 'balance' mass times acceleration.
> In statics, the forces balance each other.

The approximation, the assumption, the 'model', see page 35) in statics is this: Statics assumes that the forces on an object are much larger than the net force which accelerates it. That is, in various ways of thinking:

- The mass times acceleration is small. Small compared to what? Small compared to the individual forces.
- The forces largely cancel. That is, the sum of the forces is much smaller than the individual terms.

**Catch 22.** Usually dynamics is too complicated for the trouble. So you use statics. But you are insecure and want to know if statics is accurate enough. To know this you have to do dynamics.

> Estimating the errors from neglecting dynamics is a dynamics problem.

For each free-body diagram you have to know that

$$\left| m_{\text{tot}} \vec{\boldsymbol{a}}_G \right| = \left| \sum_{\substack{\text{All forces}\\ \text{on the part}}} \vec{\boldsymbol{F}}_i \right| \ll F_{\text{typical}}.$$

To avoid Catch 22 we usually fake it by checking (usually in our heads) this rule of thumb.

> Statics is probably accurate enough if
>
> $$m_{\text{part}} a_{\text{part}} \ll \underbrace{m_{\text{load}}(g + |a_{\text{load}}|) + m_{\text{part}} g}_{F_{\text{typical}}}. \qquad (5.4)$$

Given this, the forces are more canceling each other out (balancing each other) than causing acceleration. So you can use statics to figure out just how these forces cancel each other.

Statics equations are often accurate-enough for

- Things that a normal person would call "still" such as a building or bridge on a calm day, and a sleeping person;
- Things that move with little acceleration, such as a tractor plowing a field or most of the parts in a smooth-flying airplane; and
- Parts that mediate the forces needed to accelerate more massive parts, such as gears in a transmission, the rear wheel of an accelerating bicycle, the strut in the landing gear of an airplane, and the individual structural members of a building swaying in an earthquake.

**Example: A bicycle wheel.** The forces on a bicycle wheel are on the order of the weight of a person plus what is needed to accelerate the person up and down small hills, say about $2m_{\text{person}}g$ in total, at most. As the bike rolls up and down small hills the wheel's acceleration might also be about $g$ (at most) so that $|m_{\text{wheel}}\vec{\boldsymbol{a}}_{\text{wheel}}| \approx m_{\text{wheel}}g$. We ask,

$$\begin{aligned} \left| \mathrm{m}_{\text{wheel}} \vec{\boldsymbol{a}}_G \right| &\ll F_{\text{typical}} \; ? \\ \Rightarrow \qquad m_{\text{wheel}} g &\ll 2m_{\text{person}} g \\ \text{and indeed} \qquad m_{\text{wheel}} &\ll 2m_{\text{person}}. \end{aligned}$$

For a 50 kg rider on a bike with 1 kg wheels, the error from using statics instead of dynamics, for the forces on a bicycle wheel, is about 4%. The load on the wheel might be from acceleration of the rider above, but the analysis of the wheel itself can use statics with pretty good accuracy. Note, a similar argument also often shows that for structures holding other parts we can reasonably neglect, when studying the structure, not just the structure's acceleration, but the weight of the structure too.

**What if your statics calculation gives an inaccurate result?** If your statics calculations make a bad prediction, one possible source of errors is your neglect of dynamic terms. But that is not the common case for things that seem relatively stationary. Rather,

> Most bad statics predictions come from
>
> - Bad estimates of material properties (friction coefficient, failure strength, etc),
> - Wrong dimensions (or angles, etc), or
> - Math errors.