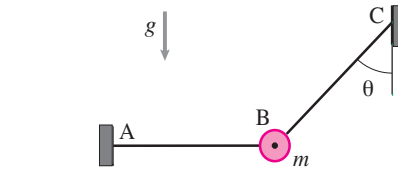


**5.1.5** A particle of mass  $m = 2 \text{ kg}$  hangs from strings AB and AC as shown. AB is horizontal and  $\theta = 45^\circ$ . Find the tension in the two strings.



Problem 5.5

Ans: 4.1.5

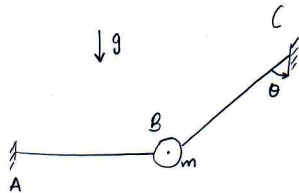
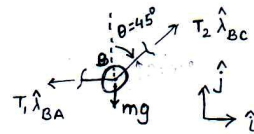


Fig (a)

Fig (b): FBD of the mass  $m$ .

The free body diagram of the mass ' $m$ ' is given in Fig(b).

Force balance,  $\sum \vec{F} = 0$ , gives

$$T_1 \hat{\lambda}_{BA} + T_2 \hat{\lambda}_{BC} + mg(-\hat{j}) = \vec{0} \quad \text{--- (i)}$$

$\hat{\lambda}_{BA}$  and  $\hat{\lambda}_{BC}$  are unit vectors in BA and BC directions, and are given by

$$\hat{\lambda}_{BA} = -\hat{i}$$

$$\hat{\lambda}_{BC} = \sin\theta \hat{i} + \cos\theta \hat{j} = \frac{1}{\sqrt{2}} [\hat{i} + \hat{j}]$$

Substituting in equation (i) we get,

$$-T_1 \hat{i} + \frac{T_2}{\sqrt{2}} (\hat{i} + \hat{j}) - mg \hat{j} = \vec{0} \quad \text{--- (ii)}$$

Separating  $x$  and  $y$  components of equation (ii) we get,

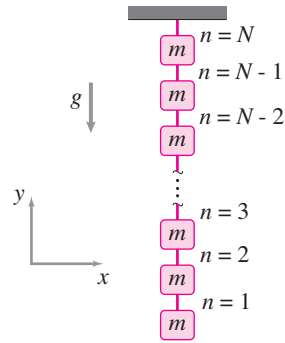
$$T_2 = \sqrt{2} T_1$$

$$\text{and } T_2 = \sqrt{2} mg$$

Substituting,  $m = 2 \text{ kg}$  and  $g = 9.8 \text{ m/sec}^2$  we get

|                                                  |                                                  |
|--------------------------------------------------|--------------------------------------------------|
| $T_2 = 27.7 \text{ N}$<br>$T_1 = 19.6 \text{ N}$ | Tension in string 'AB'<br>Tension in string 'BC' |
|--------------------------------------------------|--------------------------------------------------|

**5.1.7**  $N$  small blocks each of mass  $m$  hang vertically as shown, connected by  $N$  inextensible strings. Find the tension  $T_n$  in string  $n$ .

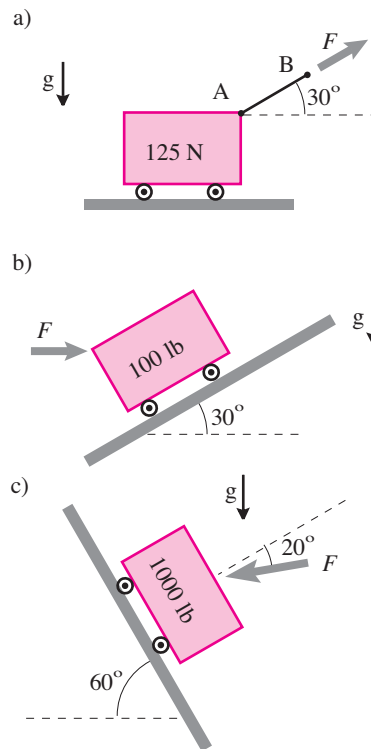


Problem 5.7

**Answer:**

$T_{N-1} = (N - 1)Nmg$ ,  $T_2 = 2mg$ ,  $T_1 = mg$ , and in general  $T_n = nmg$

5.1.13 Assume no sliding friction ( $\mu = 0$ ). Assume equilibrium. Find all reactions, tensions, and forces.

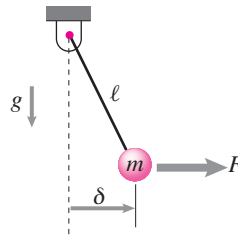


Problem 5.13

**Answer:**

(a) is nonsense; others are fine

5.1.17 For small  $\delta$  what is the relation between  $F$  and  $\delta$  (and  $g$  and  $\ell$ ) for a static pendulum?



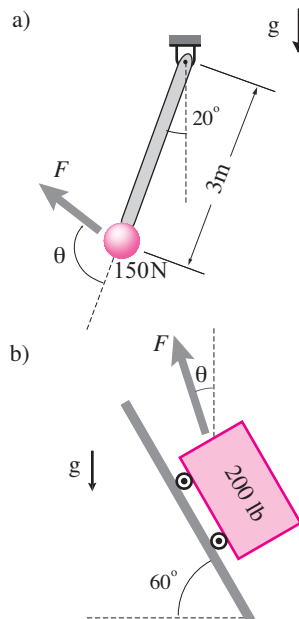
Problem 5.17

Robert Podarani  
Příručka - spring 3

F.B.D.

$\vec{T} = T \hat{\lambda}$   
 $\vec{F} = F \hat{i}$   
 $\vec{W} = -mg \hat{j}$   
 $\sum \vec{F}_i = 0 \Rightarrow T \hat{\lambda} + F \hat{i} - mg \hat{j} = 0 \quad (1)$   
 $\{1\} \cdot \hat{j} \Rightarrow T \hat{\lambda} \cdot \hat{j} = mg \Rightarrow T \cos \theta = mg$   
 $\{1\} \cdot \hat{i} \Rightarrow T \hat{\lambda} \cdot \hat{i} + F = 0 \Rightarrow T \cos(90^\circ + \theta) + F = 0 \Rightarrow$   
 $\Rightarrow -T \sin(\theta) + F = 0 \Rightarrow F = T \sin(\theta) \Rightarrow F = mg \tan(\theta)$   
 $F = mg \tan(\theta) \approx \boxed{mg \cdot \frac{\delta}{\ell}}$  for small  $\delta$

**5.1.18** In the situations shown in the figures, find the value of  $\theta$  that minimizes  $F$ . What is the corresponding value of  $F$  in each case?



Problem 5.18

Problem 4.1.17

a)  $\alpha = 20^\circ$ ;  $l = 3\text{m}$

$\sum \vec{r}_{i/A} \times \vec{F}_i = \vec{0} \Rightarrow$

$\Rightarrow \vec{r}_{G/A} \times \vec{F} + \vec{r}_{G/A} \times (-mg\hat{j}) = \vec{0} \Rightarrow$

$\Rightarrow l \sin \theta \cdot F - l \sin \alpha \cdot mg = 0 \Rightarrow$

$\Rightarrow F \sin \theta = mg \sin \alpha \Rightarrow F = mg \frac{\sin \alpha}{\sin \theta}$

$\frac{dF}{d\theta} = 0 \Rightarrow mg \sin \alpha \cdot \frac{d}{d\theta} [\sin \theta^{-1}] = 0 \Rightarrow (-1) \cdot \frac{1}{\sin^2 \theta} \cos \theta = 0 \Rightarrow$

$\Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} \rightarrow \text{No equilibrium}$

minimum force:  $F = mg \sin \alpha$

b) no friction;  $N$  is the result of the two support wheels

$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{N} + \vec{W} + \vec{F} = \vec{0} \Rightarrow$

$\Rightarrow N(\sin \alpha \hat{i} + \cos \alpha \hat{j}) - mg \hat{j} + F(-\sin \theta \hat{i} + \cos \theta \hat{j}) = \vec{0}$

$\Rightarrow \{ \dots \} \cdot \hat{i} \Rightarrow N \sin \alpha = F \sin \theta \Rightarrow N = F \frac{\sin \theta}{\sin \alpha}$

$\{ \dots \} \cdot \hat{j} \Rightarrow F \sin \theta \frac{\cos \alpha}{\sin \alpha} - mg + F \cos \theta = 0 \Rightarrow$

$\Rightarrow F \left( \frac{\sin \theta}{\tan \alpha} + \cos \theta \right) = mg \Rightarrow F = \frac{mg}{\frac{\sin \theta}{\tan \alpha} + \cos \theta}$

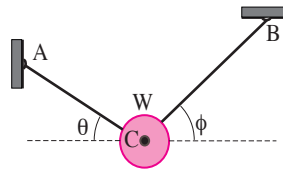
$\frac{dF}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} \left[ (\sin \theta \cot \alpha + \cos \theta)^{-1} \right] = 0 \Rightarrow (-1)(\sin \theta \cot \alpha + \cos \theta)^{-2} (\cos \theta \cot \alpha - \sin \theta) = 0$

$\Rightarrow \cos \theta \frac{1}{\tan \alpha} = \sin \theta \Rightarrow \tan \theta = \frac{1}{\tan \alpha} \Rightarrow \theta_{\min} = 30^\circ$

$F_{\min} = mg \frac{\sqrt{3}}{2}$

**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

**5.1.19** An object of weight  $W = 10 \text{ N}$  is held in equilibrium in the vertical plane by two strings AC and BC. Let  $\theta = 30^\circ$  and  $0 \leq \phi \leq 90^\circ$ . Find and plot the tension in the two strings against  $\phi$  and comment on the variation in the tensions.



Problem 5.19

Problem 4.1.18

Roberto Poobvanni

F.B.D.

$$\vec{T}_A = T_A \cdot \hat{\lambda}_A ; \quad \hat{\lambda}_A = (-\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\vec{T}_B = T_B \cdot \hat{\lambda}_B ; \quad \hat{\lambda}_B = (\cos\phi \hat{i} + \sin\phi \hat{j})$$

$$\vec{W} = -mg \hat{j}$$

$$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{T}_A + \vec{T}_B + \vec{W} = \vec{0} \quad (1)$$

$$\{0\} \cdot \hat{i} \Rightarrow -T_A \cos\theta + T_B \cos\phi = 0 \quad (2)$$

$$\{0\} \cdot \hat{j} \Rightarrow T_A \sin\theta + T_B \sin\phi - mg = 0 \quad (3)$$

$$(2) \Rightarrow T_B = T_A \cdot \frac{\cos\theta}{\cos\phi} \quad (3) \Rightarrow T_A \sin\theta + T_A \cos\theta \cdot \tan\phi = mg \Rightarrow$$

$$\Rightarrow T_A = mg \cdot \frac{1}{\sin\theta + \cos\theta \cdot \tan\phi} ; \quad T_B = mg \cdot \frac{1}{\cos\phi \cdot \tan\theta + \sin\phi}$$

alternative solution:

$$\sum \vec{M}_A = \vec{0} \Rightarrow$$

$$\Rightarrow \vec{r}_{C/A} \times \vec{W} + \vec{r}_{C/A} \times \vec{T}_B = \vec{0}$$

$$\Rightarrow -mg \cdot \ell \cdot \cos\theta + T_B \cdot \ell \cdot \sin(\theta + \phi) = 0$$

$$\Rightarrow T_B = mg \cdot \frac{\cos\theta}{\sin(\theta + \phi)}$$

$$\sum \vec{M}_B = \vec{0} \Rightarrow \vec{r}_{C/B} \times \vec{W} + \vec{r}_{C/B} \times \vec{T}_A = \vec{0} \Rightarrow -mg \ell \cos\phi + T_A \ell \sin(\theta + \phi) = 0$$

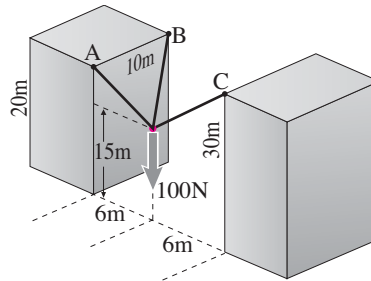
$$\Rightarrow T_A = mg \cdot \frac{\cos\phi}{\sin(\theta + \phi)}$$

they are equivalent to those above, but easier to understand

**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.



5.1.20 Find the tensions in the three strings shown in the figure.



Problem 5.20

4.1.20:

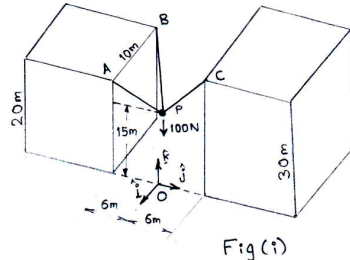


Fig (i)

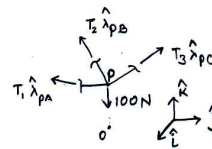


Fig (ii) FBD of point 'P'.

Let us call the point of application of the 100 N external force as 'P'.

If we assume origin of the coordinate system as shown in Fig (i) and (ii) by the symbol 'O', the coordinates of points P, A, B, and C can be given as,

$$P: (0, 0, 15) \text{ m}$$

$$A: (0, -6, 20) \text{ m}$$

$$B: (-10, -6, 20) \text{ m}$$

$$C: (0, 6, 30) \text{ m}$$

The unit vectors of direction PA, PB and PC can be given as

$$\hat{\lambda}_{PA} = \frac{\vec{r}_{P/O} - \vec{r}_{A/O}}{|\vec{r}_{P/O} - \vec{r}_{A/O}|} = \frac{0\hat{i} - 6\hat{j} + 5\hat{k}}{\sqrt{6^2 + 5^2}} = \frac{1}{\sqrt{61}}(-6\hat{j} + 5\hat{k})$$

$$\hat{\lambda}_{PB} = \frac{\vec{r}_{P/O} - \vec{r}_{B/O}}{|\vec{r}_{P/O} - \vec{r}_{B/O}|} = \frac{-10\hat{i} - 6\hat{j} + 5\hat{k}}{\sqrt{161}}$$

$$\text{and } \hat{\lambda}_{PC} = \frac{\vec{r}_{P/O} - \vec{r}_{C/O}}{|\vec{r}_{P/O} - \vec{r}_{C/O}|} = \frac{6\hat{j} + 15\hat{k}}{\sqrt{261}}$$

By force balance,  $\sum \vec{F} = 0$ , we get

$$T_1 \hat{\lambda}_{PA} + T_2 \hat{\lambda}_{PB} + T_3 \hat{\lambda}_{PC} - 100 \hat{k} \text{ N} = \vec{0} \quad \text{--- (i)}$$

By doing dot product of equ (i) with  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  we get,

$$(i) \cdot \hat{i} \Rightarrow T_2 \left( \frac{-10}{\sqrt{161}} \right) = 0 \text{ N} \Rightarrow \boxed{T_2 = 0 \text{ N}} \quad \text{--- (ii.a)}$$

$$(ii) \cdot \hat{j} \Rightarrow T_1 \left( \frac{-6}{\sqrt{61}} \right) + T_2 \left( \frac{-6}{\sqrt{161}} \right) + T_3 \left( \frac{6}{\sqrt{261}} \right) = 0 \text{ N} \Rightarrow T_1 = T_3 \frac{\sqrt{61}}{\sqrt{261}} \quad \text{--- (ii.b)}$$

$$(iii) \cdot \hat{k} \Rightarrow T_1 \left( \frac{5}{\sqrt{61}} \right) + T_2 \left( \frac{5}{\sqrt{161}} \right) + T_3 \left( \frac{15}{\sqrt{261}} \right) - 100 \text{ N} = 0$$

$$\Rightarrow T_3 \left( \frac{5}{\sqrt{261}} \right) + T_3 \left( \frac{15}{\sqrt{261}} \right) = 100 \text{ N} \Rightarrow \boxed{T_3 = 5\sqrt{261} \text{ N}} \quad \text{--- (ii.c)}$$

From (ii.b)  $T_1 = 5\sqrt{61} \text{ N}$

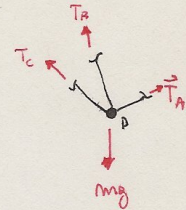
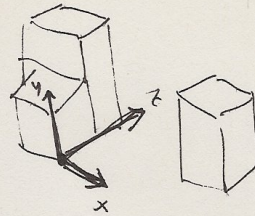
**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

positions:  $\vec{r}_A = \begin{pmatrix} x & y & z \\ 14 & 15 & 5 \end{pmatrix}$

$$\vec{r}_B = \begin{pmatrix} 0 & 20 & 10 \end{pmatrix}$$

$$\vec{r}_C = \begin{pmatrix} 0 & 10 & 0 \end{pmatrix}$$

$$\vec{r}_D = \begin{pmatrix} 8 & 10 & 0 \end{pmatrix}$$



$$\vec{W} = -mg \hat{j} = 100 \text{ N} \begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$$

$$\vec{T}_A = T_A \hat{\lambda}_A ; \quad \hat{\lambda}_A = \frac{\vec{r}_A - \vec{r}_D}{|\vec{r}_A - \vec{r}_D|} = \frac{1}{\sqrt{86}} \begin{pmatrix} 6 & 5 & 5 \end{pmatrix}$$

$$\vec{T}_B = T_B \hat{\lambda}_B ; \quad \hat{\lambda}_B = \frac{\vec{r}_B - \vec{r}_D}{|\vec{r}_B - \vec{r}_D|} = \frac{1}{\sqrt{264}} \begin{pmatrix} -8 & 10 & 10 \end{pmatrix}$$

$$\vec{T}_C = T_C \hat{\lambda}_C ; \quad \hat{\lambda}_C = \frac{\vec{r}_C - \vec{r}_D}{|\vec{r}_C - \vec{r}_D|} = \frac{1}{8} \begin{pmatrix} -8 & 0 & 0 \end{pmatrix}$$

$$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{W} + \vec{T}_A + \vec{T}_B + \vec{T}_C = \vec{0} \Rightarrow$$

$$\Rightarrow \hat{\lambda}_A T_A + \hat{\lambda}_B T_B + \hat{\lambda}_C T_C = mg \hat{j}$$

$$\Rightarrow \begin{bmatrix} \frac{6}{\sqrt{86}} & -\frac{8}{\sqrt{264}} & -1 \\ \frac{5}{\sqrt{86}} & \frac{10}{\sqrt{264}} & 0 \\ \frac{5}{\sqrt{86}} & \frac{10}{\sqrt{264}} & 0 \end{bmatrix} \begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 0 \\ 100 \text{ N} \\ 0 \end{pmatrix} \Rightarrow$$

$\vec{A} \quad \cdot \quad \vec{T} \quad = \quad \vec{k}$

$$\Rightarrow \vec{A}^{-1} \vec{A} \vec{T} = \vec{A}^{-1} \vec{k} \Rightarrow \vec{T} = \vec{A}^{-1} \vec{k}$$

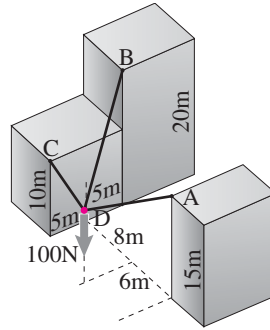
$$\det(\vec{A}) = -1 \frac{5}{\sqrt{86}} \frac{10}{\sqrt{264}} - (-1) \frac{5}{\sqrt{86}} \frac{10}{\sqrt{264}} = 0$$

$$\vec{A}^{-1} \text{ does not exist} \Rightarrow \text{the system has no solution} \Rightarrow \sum \vec{F} \neq \vec{0}$$

$\Rightarrow$  there is no equilibrium!



**5.1.21** Find the tensions in the three strings shown in the figure. String CD is horizontal and the force at D is 100 N straight down. [Hint: this problem has a trick to it.]



Problem 5.21

**Answer:**

The cables happen to be co-planar and the force is not in that plane. So there is no solution.

posibilities:  $\vec{r}_B = 6\hat{i} + 15\hat{j}$  (6; 15; 0)

$\vec{r}_A = 20\hat{j}$  (0; 20; 0)

$\vec{r}_C = 20\hat{j} + 10\hat{k}$  (0; 20; 10)

$\vec{r}_D = 12\hat{i} + 30\hat{j}$  (12; 30; 0)

F.B.D.

$\vec{T}_A = T_A \hat{\lambda}_A$ ;  $\hat{\lambda}_A = \vec{r}_A - \vec{r}_D = \left(-\frac{6}{\sqrt{61}}; \frac{5}{\sqrt{61}}; 0\right)$

$\vec{T}_B = T_B \hat{\lambda}_B$ ;  $\hat{\lambda}_B = \vec{r}_B - \vec{r}_D = \left(-\frac{6}{\sqrt{161}}; \frac{5}{\sqrt{161}}; \frac{10}{\sqrt{161}}\right)$

$\vec{T}_C = T_C \hat{\lambda}_C$ ;  $\hat{\lambda}_C = \vec{r}_C - \vec{r}_D = \left(\frac{6}{\sqrt{261}}; \frac{15}{\sqrt{261}}; 0\right)$

$\vec{W} = -mg \hat{j}$

$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{W} = \vec{0} \Rightarrow$

$\Rightarrow \hat{\lambda}_A T_A + \hat{\lambda}_B T_B + \hat{\lambda}_C T_C = mg \hat{j} \Rightarrow$

$\Rightarrow \begin{bmatrix} -\frac{6}{\sqrt{61}} & -\frac{6}{\sqrt{161}} & \frac{6}{\sqrt{261}} \\ \frac{5}{\sqrt{61}} & \frac{5}{\sqrt{161}} & \frac{15}{\sqrt{261}} \\ 0 & \frac{10}{\sqrt{161}} & 0 \end{bmatrix} \cdot \begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 0 \\ mg \\ 0 \end{pmatrix}$

$\vec{A} \cdot \vec{T} = \vec{k}$

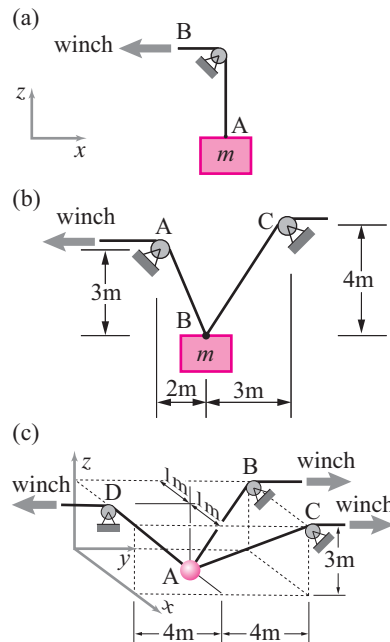
$\Rightarrow \vec{A}^{-1} \vec{A} \cdot \vec{T} = \vec{A}^{-1} \vec{k} \Rightarrow \vec{T} = \vec{A}^{-1} \vec{k}$

$\det(\vec{A}) = \frac{6}{\sqrt{261}} - \frac{5}{\sqrt{61}} - \frac{10}{\sqrt{161}} = \frac{-6}{\sqrt{61}} - \frac{10}{\sqrt{161}} - \frac{15}{\sqrt{261}} \neq 0$

A can be inverted  $\Rightarrow \dots$  computation  $\dots \Rightarrow \begin{cases} T_A = 5\sqrt{61} \\ T_B = 0 \\ T_C = 15\sqrt{29} \end{cases}$

**Disclaimer:** We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

**5.1.24** For the three cases (a), (b), and (c), below, find the tension in the string AB. In all cases the strings hold up the mass  $m = 3$  kg. You may assume the local gravitational constant is  $g = 10 \text{ m/s}^2$ . In all cases the winches are pulling in the string so that the velocity of the mass is a constant  $4 \text{ m/s}$  upwards (in the  $\hat{k}$  direction). [Note that in problems (b) and (c), in order to pull the mass up at constant rate the winches must pull in the strings at an unsteady speed.]



Problem 5.24

**Answer:**

$$(a) T_{AB} = 30 \text{ N}, \quad (b) T_{AB} = \frac{300}{17} \text{ N},$$

$$(c) T_{AB} = \frac{5\sqrt{26}}{2} \text{ N}$$

3.28 NOTE: In all three parts of this problem the acceleration of the mass is zero.  $\therefore$  Use  $(\sum \vec{F} = 0)$

To Find: Tension in string AB for each case.

(a)

$m = 3 \text{ kg},$   
 $g = 10 \text{ m/s}^2$

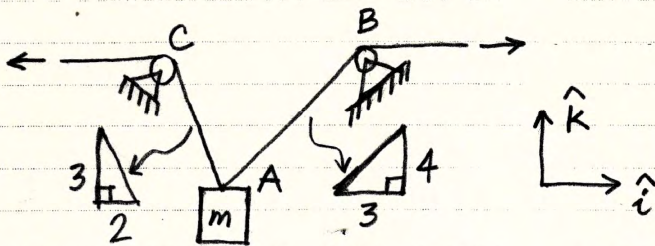
FBD

$\therefore T_{AB} = 30 \text{ N}$

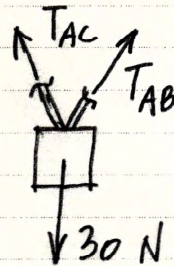
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(b) Ignore dimensions of pulleys

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FBD



$$\vec{AB} = 3\hat{i} + 4\hat{k}$$

$$\hat{\lambda}_{AB} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}$$

$$\vec{AC} = -2\hat{i} + 3\hat{k}$$

$$\hat{\lambda}_{AC} = \frac{-2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{k}$$

$$\sum \vec{F} = 0$$

$$T_{AB} \hat{\lambda}_{AB} + T_{AC} \hat{\lambda}_{AC} - 30\text{ N } \hat{k} = 0$$

$$\Rightarrow \sum \vec{F} \cdot \hat{i} = T_{AB} \cdot \frac{3}{5} - T_{AC} \cdot \frac{2}{\sqrt{13}} = 0 \quad \text{--- ①}$$

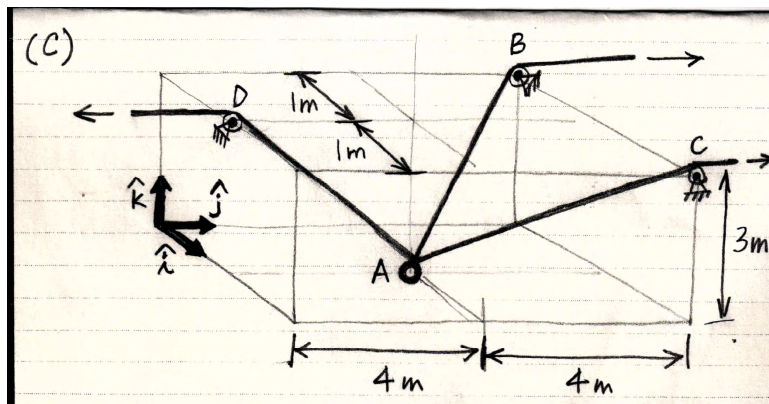
$$\sum \vec{F} \cdot \hat{k} = T_{AB} \cdot \frac{4}{5} + T_{AC} \cdot \frac{3}{\sqrt{13}} - 30\text{ N} = 0 \quad \text{--- ②}$$

$$\text{From ①, } T_{AC} \cdot \frac{3}{\sqrt{13}} = \frac{3}{2} \cdot T_{AB} \cdot \frac{3}{5} = \frac{9}{10} T_{AB}$$

$$\therefore \left( \frac{4}{5} + \frac{9}{10} \right) T_{AB} = 30\text{ N}$$

$$\boxed{T_{AB} = \frac{300}{17}\text{ N}}$$





Relevant unit vectors:

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$$\vec{AD} = -4\hat{j} + 3\hat{k} \text{ (m)}$$

$$\therefore \hat{\lambda}_{AD} = -\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$$

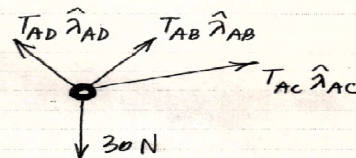
$$\vec{AB} = -1\hat{i} + 4\hat{j} + 3\hat{k} \text{ (m)}$$

$$\hat{\lambda}_{AB} = \frac{-1}{\sqrt{26}}\hat{i} + \frac{4}{\sqrt{26}}\hat{j} + \frac{3}{\sqrt{26}}\hat{k}$$

Similarly (or by symmetry)

$$\hat{\lambda}_{AC} = \frac{1}{\sqrt{26}}\hat{i} + \frac{4}{\sqrt{26}}\hat{j} + \frac{3}{\sqrt{26}}\hat{k}$$

FBD



$$\sum \vec{F} = 0$$

$$\text{or } T_{AD} \left( -\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k} \right)$$

$$+ \frac{T_{AB}}{\sqrt{26}} \left( -1\hat{i} + 4\hat{j} + 3\hat{k} \right)$$

$$+ \frac{T_{AC}}{\sqrt{26}} \left( 1\hat{i} + 4\hat{j} + 3\hat{k} \right) - 30\text{ N } \hat{k} = 0$$

Solving by equating coefficients,  
(omitted)

We obtain

$$T_{AB} = \frac{5\sqrt{26}}{2}$$

$$T_{AC} = \frac{5\sqrt{26}}{2}$$

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