SAMPLE 5.1 Equilibrium of a pin. Two rods, AB and BC, are pinned together at point B and to the ground as shown in the figure. A force $F = 100$ N is applied at point B. For $\theta = 45^{\circ}$, find the tension in the two rods.

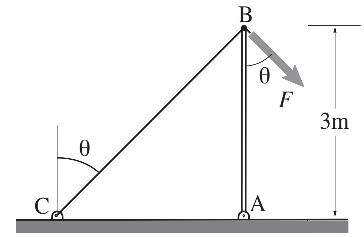

Figure 5.8

Solution The free-body diagram of the pin at B is shown in fig. 5.9 where T_1 and T_2 are the tensions in rods AB and BC respectively. The static equilibrium of the pin at B requires that

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad \vec{F} + \vec{T}_1 + \vec{T}_2 = \vec{0}$$

or

$$F(\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) - T_1\hat{\jmath} + T_2(-\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) = 0. \tag{5.5}$$

This is one vector equation in 2D in two unknowns T_1 and T_2. We can solve for the unknowns in various ways.

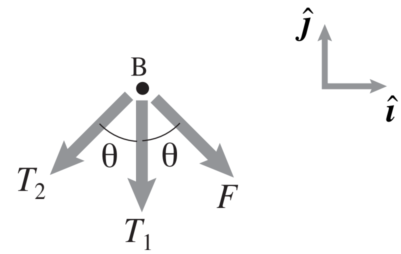

Figure 5.9: The free-body diagram of the pin at B. Although we can see that rod AB is in compression, we use tension as positive and let the solution take care of the proper sign.

Method-1: Separate out scalar equations in x and y directions. The force equilibrium equation, eqn. (5.5), gives us two independent scalar equations in the x and y directions:

$$\sum F_x = 0 \quad \Rightarrow \quad F\sin\theta - T_2\sin\theta = 0$$
$$\sum F_y = 0 \quad \Rightarrow \quad -F\cos\theta - T_1 - T_2\cos\theta = 0.$$

Solving these two equations simultaneously, we get

$$\begin{aligned} T_2 &= F = 100\,\text{N} \\ T_1 &= -(F + T_2)\cos\theta \\ &= -2F\cos\theta = -141.4\,\text{N}. \end{aligned}$$

$$T_1 = -141.4\,\text{N}, \qquad T_2 = 100\,\text{N}$$

①

①Note that the tension in rod AB, T_1 turns out to be negative, that is, rod AB is in *compression*. This is expected since the other two forces at B, F and T_2, are pushing on rod AB. The equality of F and T_2 is also expected from symmetry.

Method-2: Dot the equation with appropriate vectors. The goal here is to dot the vector equation with appropriate vectors that give us one scalar equation in one unknown. Here, $\vec{T}_1$ acts in the $-\hat{\jmath}$ direction; therefore, dotting the equation with $\hat{\imath}$ gets rid of $\vec{T}_1$ and results in a scalar equation involving only T_2:

$$[\text{eqn. (5.5)}] \cdot \hat{\imath} \quad \Rightarrow \quad F\sin\theta - T_2\sin\theta = 0$$
$$\Rightarrow \quad T_2 = F = 100\,\text{N}.$$

Similarly, to get rid of $\vec{T}_2$, dot the equation with a vector $\hat{n}$ normal to $\vec{T}_2$, *i.e.*, with $\hat{n} = \cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$:②

$$[\text{eqn. (5.5)}] \cdot \hat{n} \quad \Rightarrow \quad [-T_1\hat{\jmath} + F(\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}) \cdot (\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})] = 0$$
$$\Rightarrow \quad T_1\sin\theta + F(\cos^2\theta + \sin^2\theta) = 0$$
$$\Rightarrow \quad T_1 = -\frac{F}{\sin\theta} = -\frac{100\,\text{N}}{1/\sqrt{2}} = -141.4\,\text{N}$$

These are the same values of T_1 and T_2, as they must be, obtained by Method-1.

②How do we find the normal vector $\hat{n}$? Well, we know the direction of $\vec{T}_2$, which is $\hat{\lambda} = -\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$. We can draw a unit vector $\hat{n}$ normal to $\hat{\lambda}$ and find its components from geometry (see figure below), or we can set $\hat{n} = \hat{k} \times \hat{\lambda}$ which guarantees $\hat{n}$ to be normal to $\hat{\lambda}$. In either case we get $\hat{n} = \cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$.

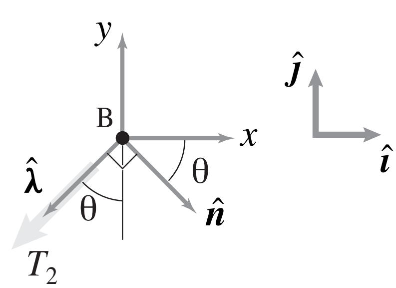

Figure 5.10: Finding a unit vector $\hat{n}$ normal to the direction $\hat{\lambda}$ of T_2.

Method-3: Use matrix equation and solve by hand or on a computer. The two scalar equations obtained from eqn. (5.6) can be written in the matrix form as

$$\begin{bmatrix} 0 & \sin\theta \\ 1 & \cos\theta \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} F\sin\theta \\ -F\cos\theta \end{pmatrix}.$$

Using $F = 100$ N and $\theta = 45° = \pi/4$, and solving the above matrix equation (see Sample 1.28 on page 123 and Sample 1.30 on page 125), we get

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} -141.4 \\ 100 \end{pmatrix} \text{N}$$

which is, of course, the same result as we got above.

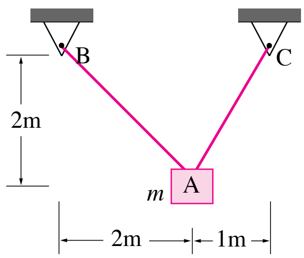

Figure 5.11

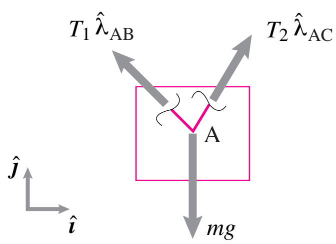

Figure 5.12

SAMPLE 5.2 A mass held in equilibrium by unequal strings in 2D. A 10 kg block m hangs from strings AB and AC in the vertical plane as shown in the figure. Find the tension in the strings.

Solution The free-body diagram of the block is shown in figure 5.12. The equation of force balance, $\sum \vec{F} = \vec{0}$, gives

$$\text{or} \qquad T_1\hat{\boldsymbol{\lambda}}_{AB} + T_2\hat{\boldsymbol{\lambda}}_{AC} - mg\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}, \tag{5.6}$$

where $\hat{\boldsymbol{\lambda}}_{AB}$ and $\hat{\boldsymbol{\lambda}}_{BC}$ are unit vectors in the AB and AC directions:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_{AB} &= \frac{\vec{\boldsymbol{r}}_{AB}}{|\vec{\boldsymbol{r}}_{AB}|} = \frac{-2\,\text{m}\hat{\boldsymbol{\imath}} + 2\,\text{m}\hat{\boldsymbol{\jmath}}}{2\sqrt{2}\,\text{m}} = \frac{1}{\sqrt{2}}(-\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) \\
\hat{\boldsymbol{\lambda}}_{AC} &= \frac{\vec{\boldsymbol{r}}_{AC}}{|\vec{\boldsymbol{r}}_{AC}|} = \frac{1\,\text{m}\hat{\boldsymbol{\imath}} + 2\,\text{m}\hat{\boldsymbol{\jmath}}}{\sqrt{5}\,\text{m}} = \frac{1}{\sqrt{5}}(\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}).
\end{aligned}$$

Substituting in eqn. (5.6) and rearranging terms, we have

$$\left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\boldsymbol{\imath}} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}.$$

Separating x and y components of this equation, we get the scalar equations

$$\begin{aligned}
\sum F_x = 0 \quad &\Rightarrow \quad -\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}} = 0 \\
\sum F_y = 0 \quad &\Rightarrow \quad \frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg = 0.
\end{aligned}$$

Solving these two equations simultaneously we get,

$$T_1 = \frac{\sqrt{2}}{3}mg = 46.24\,\text{N} \quad \text{and} \quad T_2 = \frac{\sqrt{5}}{3}mg = 73.12\,\text{N}.$$

$$\boxed{T_1 = 46.24\,\text{N},\ T_2 = 73.12\,\text{N}}$$

Note:

Solving for T_1 and T_2 If you are comfortable with vector algebra, then solving for T_1 and T_2 from eqn. (5.6) is quite easy. Let us say, we find two unit vectors $\hat{\boldsymbol{n}}_{AB}$ and $\hat{\boldsymbol{n}}_AC$ normal to unit vectors $\hat{\boldsymbol{\lambda}}_{AB}$ and $\hat{\boldsymbol{\lambda}}_{AC}$, respectively. Then dotting eqn. (5.6) with $\hat{\boldsymbol{n}}_{AB}$ and $\hat{\boldsymbol{n}}_AC$, one at a time, we can solve for T_1 and T_2 in one step:

$$T_2 = \frac{mg\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{n}}_{AB}}{\hat{\boldsymbol{\lambda}}_{AC}\cdot\hat{\boldsymbol{n}}_{AB}}, \qquad \text{and} \qquad T_1 = \frac{mg\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{n}}_{AC}}{\hat{\boldsymbol{\lambda}}_{AB}\cdot\hat{\boldsymbol{n}}_{AC}}.$$

For computing the values, we need to carry out the dot products. Noting that $\hat{\boldsymbol{n}}_{AB} = \frac{1}{\sqrt{2}}(i + j)$ and $\hat{\boldsymbol{n}}_{AC} = \frac{1}{\sqrt{5}}(-2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}})$ (you can write these vectors by looking at $\hat{\boldsymbol{\lambda}}_{AB}$ and $\hat{\boldsymbol{\lambda}}_{AC}$), we can carry out the dot product and get the values of T_1 and T_2.

Matrix equation The two scalar equations obtained from eqn. (5.6) can be written in the matrix form as

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ mg \end{pmatrix}.$$

Using $mg = (10\,\text{kg})\cdot(9.81\,\text{m/s}^2) = 98.1\,\text{N}$, and solving the above matrix equation (see Sample 1.28 on page 123 and Sample 1.30 on page 125), we get

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 46.24 \\ 73.12 \end{pmatrix}\text{N}$$

which is, of course, the same result as we got above.

SAMPLE 5.3 **Tensions in a paraglider's ropes.** A paraglider is held by two ropes that in turn connect to the parachute with many ropes. The angles that the two ropes make with the horizontal (or vertical) are not necessarily the same for each rope. Assume that the right rope makes an angle θ_1 with the horizontal and the left one makes an angle θ_2 (in flight, these angles will vary with time). Also assume that the paraglider is descending at some uniform velocity. Find the tensions in the two ropes in terms of the weight of the person and the harness (say, mg), and the angles θ_1 and θ_2.

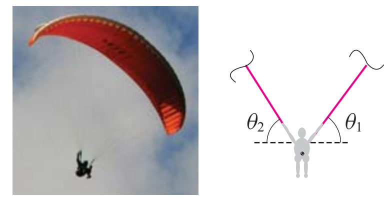

Figure 5.13

Solution The free-body diagram of the glider is shown in fig. 5.14. We assume that the tensions T_1, T_2, and the weight mg are all in one vertical plane. Since the paraglider has a uniform velocity (no acceleration), we can use the force-balance equation for static equilibrium, $\sum \vec{F} = \vec{0}$:

$$T_1\hat{\boldsymbol{\lambda}}_1 + T_2\hat{\boldsymbol{\lambda}}_2 - mg\,\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}} \qquad (5.7)$$

where $\hat{\boldsymbol{\lambda}}_1 = \cos\theta_1\hat{\boldsymbol{\imath}} + \sin\theta_1\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{\lambda}}_2 = -\cos\theta_2\hat{\boldsymbol{\imath}} + \sin\theta_2\hat{\boldsymbol{\jmath}}$ are the unit vectors along the two ropes, respectively. Thus the vector equation of equilibrium is:

$$T_1(\cos\theta_1\hat{\boldsymbol{\imath}} + \sin\theta_1\hat{\boldsymbol{\jmath}}) + T_2(-\cos\theta_2\hat{\boldsymbol{\imath}} + \sin\theta_2\hat{\boldsymbol{\jmath}}) - mg\,\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}. \qquad (5.8)$$

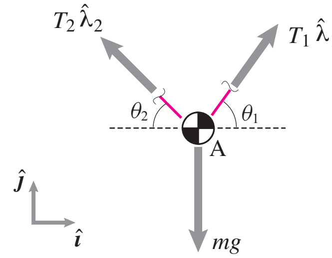

Figure 5.14: The free-body diagram of the paraglider considering the ropes to be at different angles with the horizontal.

From this equation, we get two independent scalar equations by separating the x and y components:

$$\begin{aligned}
\sum F_x = 0 \quad &\Rightarrow \quad T_1\cos\theta_1 - T_2\cos\theta_2 = 0\\
\sum F_y = 0 \quad &\Rightarrow \quad T_1\sin\theta_1 + T_2\sin\theta_2 - mg = 0.
\end{aligned}$$

Solving these two equations simultaneously we get,

$$T_1 = \frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}mg \quad \text{and} \quad T_2 = \frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}mg.$$

$$\boxed{T_1 = \frac{mg\cos\theta_2}{\sin(\theta_1+\theta_2)} \text{ and } T_2 = \frac{mg\cos\theta_1}{\sin(\theta_1+\theta_2)}}$$

Another vector method: We can find T_1 and T_2 directly from eqn. (5.7) by taking cross products with $\hat{\boldsymbol{\lambda}}_2$ and $\hat{\boldsymbol{\lambda}}_1$, respectively:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_2 \times (T_1\hat{\boldsymbol{\lambda}}_1 + T_2\hat{\boldsymbol{\lambda}}_2 - mg\,\hat{\boldsymbol{\jmath}}) &= \vec{\mathbf{0}}\\
\Rightarrow \quad T_1\underbrace{(\hat{\boldsymbol{\lambda}}_2 \times \hat{\boldsymbol{\lambda}}_1)}_{-\sin(\theta_1+\theta_2)\hat{\boldsymbol{k}}} - mg\underbrace{(\hat{\boldsymbol{\lambda}}_2 \times \hat{\boldsymbol{\jmath}})}_{-\cos\theta_2\hat{\boldsymbol{k}}} &= \vec{\mathbf{0}}\\
\Rightarrow \quad T_1 &= \frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}mg
\end{aligned}$$

Similarly, $\lambda_2\times$[eqn. (5.7)] gives us

$$T_2 = \frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}mg.$$

Thus we get the same results as we got above by solving the two scalar equations simultaneously.

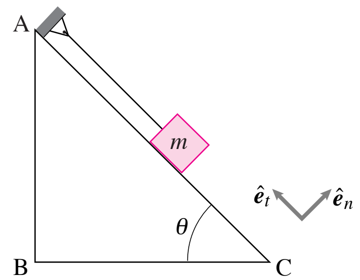

Figure 5.15: A mass-particle on an inclined plane.

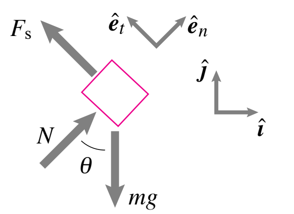

Figure 5.16: Free-body diagram of the mass and the geometry of force vectors.

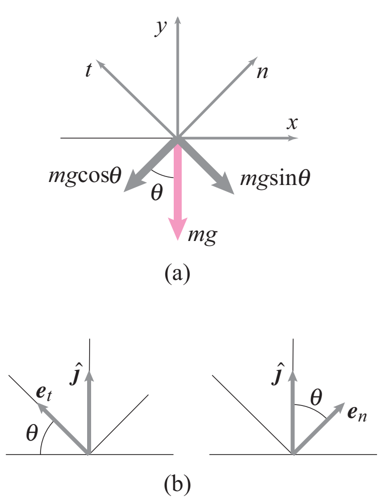

Figure 5.17: (a) Components of mg along t and n directions. (b) The mixed basis dot products: $\hat{\jmath}\cdot\hat{e}_t = \sin\theta$ and $\hat{\jmath}\cdot\hat{e}_n = \cos\theta$

SAMPLE 5.4 A single string holding a mass on a frictionless incline. A block of mass m rests on a frictionless inclined plane with the help of a string that connects the mass to a fixed support at A. Find the force in the string.

Solution The free-body diagram of the mass is shown in Fig. 5.16. The string force F_s and the normal reaction of the plane N are unknown forces. The force-balance equation, $\sum \vec{F} = \vec{0}$, is

$$\vec{F}_s + \vec{N} + m\vec{g} = \vec{0}.$$

We can express the forces in terms of their components in various ways and then dot the vector equation with appropriate unit vectors to get two independent scalar equations. For example, we write the force-balance equation using mixed basis vectors $\hat{e}_t$ and $\hat{e}_n$, and $\hat{\imath}$ and $\hat{\jmath}$:

$$F_s\hat{e}_t + N\hat{e}_n - mg\hat{\jmath} = \vec{0}. \qquad (5.9)$$

We can now find F_s directly by taking the dot product of the above equation with $\hat{e}_t$ since the other unknown N is in the $\hat{e}_n$ direction and $\hat{e}_n \cdot \hat{e}_t = 0$:

$$\begin{aligned} \{\text{eqn. (5.9)}\}\cdot\hat{e}_t \quad &\Rightarrow \quad F_s - mg\overbrace{(\hat{\jmath}\cdot\hat{e}_t)}^{\sin\theta} = 0 \\ &\Rightarrow \quad F_s = mg\sin\theta. \end{aligned}$$

$$\boxed{F_s = mg\sin\theta}$$

. . .

Note: We can also find N from a single equation by taking the dot product of eqn. (5.9) with $\hat{n}$:

$$\begin{aligned} \{\text{eqn. (5.9)}\}\cdot\hat{e}_n \quad &\Rightarrow \quad N - mg\overbrace{(\hat{\jmath}\cdot\hat{e}_n)}^{\cos\theta} = 0 \\ &\Rightarrow \quad N = mg\cos\theta. \end{aligned}$$

Scalar approach: We resolve all forces into their $\hat{e}_t$ and $\hat{e}_n$ components and then sum the forces. Here, F_s is along the plane and therefore, has no component perpendicular to the plane. Force N is perpendicular to the plane and therefore, has no component along the plane. We resolve the weight mg into two components: (1) $mg\cos\theta$ perpendicular to the plane (along $\hat{e}_n$) and (2) $mg\sin\theta$ along the plane (along $\hat{e}_t$). Now we can sum the forces:

$$\begin{aligned} \sum F_t = 0 \quad &\Rightarrow \quad F_s - mg\sin\theta = 0; \\ \text{and} \quad \sum F_n = 0 \quad &\Rightarrow \quad N - mg\cos\theta = 0 \end{aligned}$$

which, of course, is essentially the same as the equations obtained above.

SAMPLE 5.5 **A particle in 3D.** A particle of mass 1 kg is attached to two strings tied at points C and D shown in the figure. Another string, AB, attached to the particle, passes over a pulley and is used to hold the particle in equilibrium under gravity such that it loses contact with the ground at point A. Find the tension in string AB.

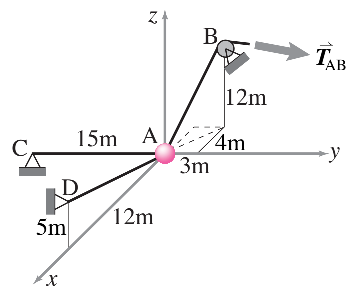

Figure 5.18

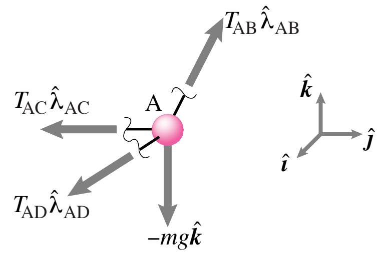

Figure 5.19

Solution The free-body diagram of the particle is shown in fig. 5.19. Assuming the tensions in strings AB, AC, and AD to be T_{AB}, T_{AC}, and T_{AD} respectively, we can represent the string forces acting on the particle as $T_{AB}\hat{\boldsymbol{\lambda}}_{AB}$, $T_{AC}\hat{\boldsymbol{\lambda}}_{AC}$, and $T_{AD}\hat{\boldsymbol{\lambda}}_{AD}$, where the $\hat{\boldsymbol{\lambda}}$'s are the unit vectors along the strings.

The force balance on the particle gives us

$$T_{AB}\hat{\boldsymbol{\lambda}}_{AB} + T_{AC}\hat{\boldsymbol{\lambda}}_{AC} + T_{AD}\hat{\boldsymbol{\lambda}}_{AD} - mg\hat{\boldsymbol{k}} = 0. \tag{5.10}$$

This is the equation we need to solve to find T_{AB}. We show various methods below that you can use to get T_{AB}.

1. **Brute force (by hand).**
   From the given figure, the unit vectors are:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_{AB} &= \frac{-4\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} + 12\hat{\boldsymbol{k}}}{\sqrt{4^2+3^2+12^2}} = -\frac{4}{13}\hat{\boldsymbol{\imath}} + \frac{3}{13}\hat{\boldsymbol{\jmath}} + \frac{12}{13}\hat{\boldsymbol{k}} \\
\hat{\boldsymbol{\lambda}}_{AC} &= -\hat{\boldsymbol{\jmath}} \\
\hat{\boldsymbol{\lambda}}_{AD} &= \frac{12\hat{\boldsymbol{\imath}} + 5\hat{\boldsymbol{k}}}{\sqrt{12^2+5^2}} = \frac{12}{13}\hat{\boldsymbol{\imath}} + \frac{5}{13}\hat{\boldsymbol{k}}.
\end{aligned}$$

   Substituting these vectors in eqn. (5.10) and equating the x, y and z components of the equation to zero separately, we get

$$\begin{aligned}
-\frac{4}{13}T_{AB} + \frac{12}{13}T_{AD} &= 0 \\
\frac{3}{13}T_{AB} - T_{AC} &= 0 \\
\frac{12}{13}T_{AB} + \frac{5}{13}T_{AD} &= mg.
\end{aligned} \tag{5.11}$$

   We can solve the three equations simultaneously to get

$$T_{AB} = \frac{39}{41}mg, \quad T_{AC} = \frac{9}{41}mg, \quad \text{and} \quad T_{AD} = \frac{13}{41}mg.$$

   Substituting $m = 1\,\text{kg}$ and $g = 9.81\,\text{m/s}^2$, we get the required values.

$$T_{AB} = 9.33\,\text{N}, \quad T_{AC} = 2.15\,\text{N}, \quad T_{AD} = 3.11\,\text{N}$$

2. **Systematically set up matrix equations.** Eqn. 5.10 can be written in matrix form as

$$\underbrace{\begin{bmatrix} [\hat{\boldsymbol{\lambda}}_{AB}]'_{xyz} & [\hat{\boldsymbol{\lambda}}_{AC}]'_{xyz} & [\hat{\boldsymbol{\lambda}}_{AD}]'_{xyz} \end{bmatrix}}_{3\times 3\text{ matrix}} \begin{bmatrix} T_{AB} \\ T_{AB} \\ T_{AB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

   where $[\hat{\boldsymbol{\lambda}}_{AB}]'_{xyz}$ is a column of 3 numbers, namely the x, y, and z components of $\hat{\boldsymbol{\lambda}}_{AB}$; similarly for the other two columns of the 3×3 matrix. This matrix equation is then ready to hand to a calculator or computer for a matrix solution. Thus, eqn. (5.11) can be written as,

$$\begin{bmatrix} -4/13 & 0 & 12/13 \\ 3/13 & -1 & 0 \\ 12/13 & 0 & 5/13 \end{bmatrix} \begin{pmatrix} T_{AB} \\ T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix}.$$

3 Pseudo-code:

```
Let m=1, 9=9.81
A = [ -4/13 0 12/13
       3/13 -1 0
       12/13 0 5/13 ]
b = [ 0 0 m*g]'
solve A*T = b for T
```

Using the pseudo code shown on the side 3 we solve the equations on a computer and get,

```
T = [9.33   2.15   3.11]
```

which is the solution that we obtained above by hand calculation.

3. **Computer solution.** All the math can be handed to a computer by a sequence of commands like this, working from the knowns to the unknowns (see page 22), all in consistent units:

```
% Get all the knowns into the computer
  rA = [0 0 0]' ;   rB = [-4 3 12]'
  rC = [0 -15 0]' ;   rD = [12 0 5]'
  m = 1 ;  g = 9.81 ;
% Make relative position vectors
  rAB = rB- rA;   rAC = rC - rA;  rAD = rD - rA
% Make unit vectors
  lamdaAB = rAB/magnitude(rAB);
  lamdaAC = rAC/magnitude(rAC);
  lamdaAD = rAD/magnitude(rAD);
% Set up and solve the matrix equation
  M = [lamdaAB  lamdaAC  lamdaAD]  %3x3 matrix
  F = [0 0 mg]'   %column vector of known force
  solve {M T = F} for T
```

The column of numbers `T` will be the tensions in the 3 cables. Using this pseudo-code on a computer, we get `T = [9.33   2.15   3.11]` again.

4. **Be tricky to get one equation in one unknown.** Since we are interested only in T_{AB}, we can get rid of the terms we don't know or care about. 4 The vector $\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}$ is orthogonal to both $\vec{\boldsymbol{r}}_{AC}$ and $\vec{\boldsymbol{r}}_{AD}$, so it is orthogonal to $\hat{\boldsymbol{\lambda}}_{AC}$ and $\hat{\boldsymbol{\lambda}}_{AD}$. So taking the dot product of both sides of eqn. (5.10) with $\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}$, we get

$$\left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \{T_{AB}\hat{\boldsymbol{\lambda}}_{AB} + T_{AC}\hat{\boldsymbol{\lambda}}_{AC} + T_{AD}\hat{\boldsymbol{\lambda}}_{AD} - mg\hat{\boldsymbol{k}}\} = \left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \vec{\boldsymbol{0}}$$
$$\left(\left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \hat{\boldsymbol{\lambda}}_{AB}\right) T_{AB} = mg\left(\left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \hat{\boldsymbol{k}}\right)$$
$$T_{AB} = \frac{mg\left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \hat{\boldsymbol{k}}}{\left(\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD}\right) \cdot \hat{\boldsymbol{\lambda}}_{AB}}.$$

Since, $\vec{\boldsymbol{r}}_{AC} \times \vec{\boldsymbol{r}}_{AD} = (-15\hat{\boldsymbol{\jmath}})\,\mathrm{m} \times (12\hat{\boldsymbol{\imath}} + 5\hat{\boldsymbol{k}})\,\mathrm{m} = (180\hat{\boldsymbol{k}} - 75\hat{\boldsymbol{\imath}})\,\mathrm{m}^2$, substituting this cross product and other known quantities, we get

$$\begin{aligned} T_{AB} &= \frac{mg \cdot 180}{180 \cdot 12/13 + 75 \cdot 4/13} \\ &= 0.95\,mg \\ &= 9.33\,\mathrm{N}. \end{aligned}$$

$$\boxed{T_{AB} = 9.33\,\mathrm{N}}$$

4 Another way of doing this is by taking **Moment about an axis.** This approach is similar in spirit to the previous approach. Instead of the equilibrium eqn. (5.10) we could have used moments about axis CD to 'kill off' the tensions in ropes AC and AD (they have no moment about that axis), like this,

$$\sum M_{\text{axis CD}} = 0$$
$$\vec{\boldsymbol{r}}_{CD} \cdot \{\vec{\boldsymbol{r}}_{DA} \times \{T_{AB}\hat{\boldsymbol{\lambda}}_{AB} - mg\hat{\boldsymbol{k}}\}\} = 0$$
$$T_{AB} = \frac{mg\vec{\boldsymbol{r}}_{CD} \cdot (\vec{\boldsymbol{r}}_{DA} \times \hat{\boldsymbol{k}})}{\vec{\boldsymbol{r}}_{CD} \cdot (\vec{\boldsymbol{r}}_{DA} \times \hat{\boldsymbol{\lambda}}_{AB})}.$$

Again we have found one equation for one unknown, T_{AB}. All the quantities on the right can be evaluated to give T_{AB}.