5.1 Static equilibrium of a particle

What is a particle?

The word particle usually means something small. In mechanics a *particle* is an object for which we don't worry about rotation, or the tendency of forces to cause rotation. A particle may or may not be small. Besides, smallness is in the eyes of the beholder. For some purposes a galaxy is well-modeled as a particle and for others a molecule is too big to be thought of as a particle. Big or small, the particle model of a system is defined by the lack of attention paid to the moment-balance equations ①. Either moment balance is trivially satisfied or you can find what you need without worrying about how it is satisfied ②.

For statics of a particle, force-balance tells all: $$\sum_{\substack{\text{All forces}\\ \text{on the FBD}}} \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \qquad (Ic)$$

In two dimensions, this equilibrium equation makes up 2 independent scalar equations (2 components of the net force vector). In 3 dimensions we get 3 independent scalar equations. So we expect to be able to solve for 2 unknown quantities in 2D particle mechanics, and 3 in 3D.

①**Examples of particles.** You might think of a galaxy as an immense thing, not just a dot. But its overall motion through a cluster of galaxies is probably well-described by thinking of it as a particle. The galaxy may rotate and distort in interesting ways, but one can ignore those rotations and distortions when calculating overall translation. That is, one might model an immense rotating and distorting galaxy as a particle.

Similarly for an accelerating car, a block sliding on a ramp or a machine part, one may learn enough about the forces and motion using a particle model. Distortion and rotation might be there, and even large, but might not be important for understanding the overall motion or force balance.

②Equations are 'satisfied' when the right side is equal to the left. Take that as a definition or, if it helps you, think of it as a desire that you would like to accommodate.

The statics-of-a-particle recipe

For particle statics, we work with a simplified form of the general recipe from the inside back cover.

1) **Draw a free-body diagram (FBD) of the part of interest.**
Use knowledge of the contact conditions (see Chapter 3) to draw known and unknown aspects of the forces appropriately (see fig. 2.42 on page 169);
2) **Set the sum of the forces on the FBD to zero: $\sum \vec{\boldsymbol{F}}_i = \vec{\boldsymbol{0}}$.**
('Equilibrium', 'force balance', or 'linear momentum balance in statics');
3) **Solve the equations for unknowns.**
Use vector manipulation skills (Chapter 2) to solve the force balance equation for unknowns of interest.

Scalar mechanics

In scalar mechanics — as opposed to vector mechanics — one takes the dot product of Eqn. (Ic) with unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ and write the three scalar component equations ③.

③Actually, people who think in terms of scalar mechanics do the dot products, but implicitly (or unconsciously), and think of the component equations as fundamental.

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \text{additionally, in 3D,} \qquad \sum F_z = 0.$$

Although you can do most problems plowing through with the ‘component’ or scalar approach, often there are shortcuts or insights that depend on the vectorial view.

1D statics of a particle

Let’s call the one dimension of interest the x direction. The key governing equation is

$$\sum F_x = 0.$$

You could call the special direction y, z, x' or s if you like and then use, say $\sum F_z = 0$. The next two simple examples pretty much cover 1D particle statics.

Figure 5.1: Equilibrium of a particle in 1D.

Example: Balance of two forces
For the particle in fig. 5.1, force balance gives

$$\sum \vec{F}_i = \vec{0} \quad \Rightarrow \quad 10\,\mathrm{N}\hat{\imath} - F\hat{\imath} = \vec{0}.$$

Either by equating x components of both sides, or equivalently, by dotting both sides with $\hat{\imath}$, we get $F = 10\,\mathrm{N}$. Or, we could have just jumped to scalar mechanics,

$$\sum F_x = 0 \quad \Rightarrow \quad 10\,\mathrm{N} - F = 0 \quad \Rightarrow \quad F = 10\,\mathrm{N}.$$

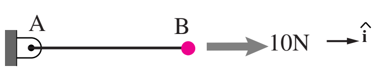


Figure 5.2: Particle held by a string.

Most often we have to contend with forces which don’t show up until we draw a free-body diagram.

Example: Force pulling on a string.
For the particle in fig. 5.2, the quantity of interest, the tension in the cable, doesn’t show in the sketch. We need to draw a free body diagram of the particle which means cutting the string. This FBD is shown in fig. 5.1, where $F = T_{AB}$ represents the tension in cable AB. So force balance gives $T_{AB} = F = 10\,\mathrm{N}$.

2D statics of a particle

The situation is less trivial when we go to 2D.

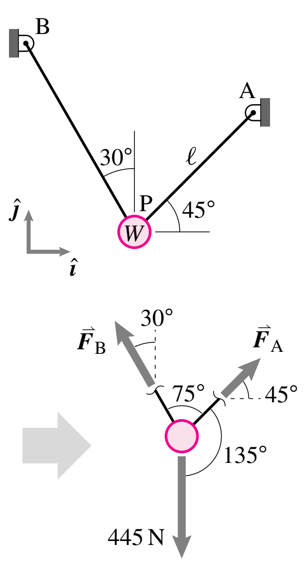


Figure 5.3: Weight hung from two lines with associated free-body diagram.

Example. A 100 pound (445 N) weight hangs from 2 lines in fig. 5.3. We cut the strings, draw a free-body diagram and add the forces to get

$$\sum \vec{F} = \vec{0} \qquad \Rightarrow \qquad 445\,\mathrm{N}(-\hat{\jmath}) + \underbrace{F_A \frac{(\hat{\imath} + \hat{\jmath})}{\sqrt{2}}}_{\vec{F}_A} + \underbrace{F_B(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})}_{\vec{F}_B} = \vec{0}. \quad (5.1)$$

This can be solved various ways (see below) to get $F_A = 230.3\,\mathrm{N}$ and $F_B = 325.8\,\mathrm{N}$.

Although moment balance is technically superfluous in particle mechanics, when the forces are concurrent, moment balance can be used as a shortcut.

How to solve vector statics equations?

Method 1) Pull out both x and y components of the vectors to get 2 equations in 2 unknowns;

$$\begin{aligned}\sum F_x = 0 \quad &\Rightarrow \quad F_A/\sqrt{2} - F_B/2 = 0\\ \sum F_y = 0 \quad &\Rightarrow \quad F_A/\sqrt{2} + F_B\sqrt{3}/2 - 445N = 0.\end{aligned}$$

Method 2) Equivalently, dot both sides of the equation with $\hat{\imath}$ and $\hat{\jmath}$ to get 2 equations in 2 unknowns;

$$\begin{aligned}\{\text{eqn. }(5.1)\}\cdot\hat{\imath} \quad &\Rightarrow \quad 0 = F_A/\sqrt{2} - F_B/2\\ \{\text{eqn. }(5.1)\}\cdot\hat{\imath} \quad &\Rightarrow \quad 0 = F_A/\sqrt{2} + F_B\sqrt{3}/2 - 445N.\end{aligned}$$

Method 3) dot both sides with a vector orthogonal to r_{AP} to get one equation in F_B, similarly dot with a vector orthogonal to $\vec{r}_{\text{BP}}$ to get one equation in F_A.

$$\begin{aligned}\{\text{eqn. }(5.1)\}\cdot(-\hat{\imath}+\hat{\jmath}) \quad &\Rightarrow \quad 0 = -445N + F_B(\frac{1}{2}+\frac{\sqrt{3}}{2})\\ \{\text{eqn. }(5.1)\}\cdot(\sqrt{3}\hat{\imath}+\hat{\jmath}) \quad &\Rightarrow \quad 0 = -445N + F_A\frac{\sqrt{3}+1}{\sqrt{2}}\end{aligned}$$

④

④Note that if $\vec{v} = a\hat{\imath} + b\hat{\jmath}$ then $\vec{w} = -b\hat{\imath} + a\hat{\jmath}$ is orthogonal to it because $\vec{v}\cdot\vec{w} = -ab + ba = 0$. That is, in the plane to get a vector perpendicular to $\vec{v}$ interchange the components and negate one of them (either one).

Method 4) cross both sides with $\vec{r}_{AP}$ to get one equation in F_B, similarly cross with $\vec{r}_{\text{BP}}$ to get F_A.

$$\begin{aligned}\vec{0} &= \left(-\ell\hat{\imath}/\sqrt{2} - \ell\hat{\jmath}/\sqrt{2}\right)\times\left(445\,\text{N}(-\hat{\jmath}) + F_A\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + F_B(-\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})\right)\\ &\Rightarrow \quad 0 = -445N + F_B(\frac{1}{2}+\frac{\sqrt{3}}{2})\\ \vec{0} &= \left(d\hat{\imath}/2 - d\sqrt{3}\hat{\jmath}/2\right)\times\left(445\,\text{N}(-\hat{\jmath}) + F_A\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + F_B(-\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})\right)\\ &\Rightarrow \quad 0 = -445N + F_A\frac{\sqrt{3}+1}{\sqrt{2}}.\end{aligned}$$

See section 1.5 starting on page 106 for more discussion about how to solve vector equations.

Another approach, mathematically equivalent to method 4 above, is to use moment balance.

Example: Moment balance
Consider again fig. 5.3. Moment balance about point A gives

$$\sum \vec{M}_{\text{A}} = \vec{0} \qquad \Rightarrow \qquad \vec{r}_{\text{P/A}}\times(445\,\text{N}(-\hat{\jmath})) + \vec{r}_{\text{P/A}}\times\left(F_B(-\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})\right) + \vec{0} = \vec{0}.$$

Evaluating the cross products one way or another, we again get $F_B = 325.8\,\text{N}$. Similarly moment balance about B gives $F_A = 230.3\,\text{N}$.

Example: A kite.
A kite flying steadily in a breeze is roughly in static equilibrium. The three forces acting on it are from the air, pushing the kite downwind and up; from gravity, pulling the kite down; and from the string pulling the kite upwind and down. The three forces must add to zero.

A funny thing about kites is that they only stay up because you pull them down.

Whether force or moment balance is used, for concurrent force systems we only have two independent scalar equilibrium equations in 2D, and three in 3D .

Frictionless contact

As discussed in Chapter 3.1, engineered parts that slide often have bearings or lubrication to reduce the sliding resistance. To simplify analysis, that remaining resistance is often neglected, and we model the contact as 'frictionless' ($\mu = 0$). This makes the interaction force normal (perpendicular) to the contacting surfaces.

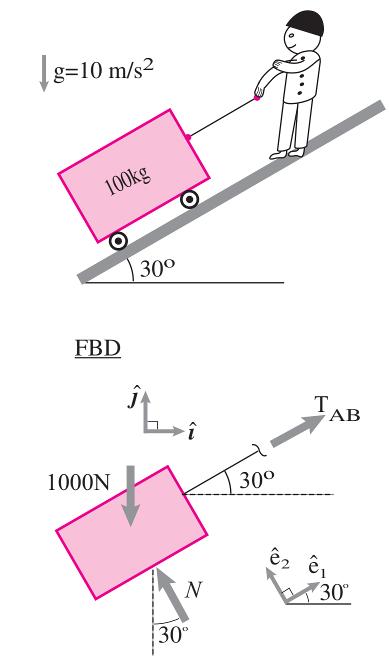


Figure 5.4: A wagon pulled uphill and associated free-body diagram. Because we are doing particle mechanics, we combine the two forces on the wheels into the single resultant N. Further, for particle mechanics we need not worry about where that N is applied.

Example: Pull a wagon uphill
See fig. 5.4. From the free-body diagram we have

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad -1000\,\mathrm{N}\,\hat{\boldsymbol{\jmath}} + N\hat{\boldsymbol{e}}_2 + T_{AB}\hat{\boldsymbol{e}}_1 = \vec{\mathbf{0}}. \tag{5.2}$$

where $\hat{\boldsymbol{e}}_1 = \cos(30^\circ)\hat{\boldsymbol{\imath}} + \sin(30^\circ)\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{e}}_2 = -\sin(30^\circ)\hat{\boldsymbol{\imath}} + \cos(30^\circ)\hat{\boldsymbol{\jmath}}$. N and T_{AB} are unknown forces. Here are two ways to solve for the unknowns.
Method I. Substitute the expressions for $\hat{\boldsymbol{e}}_1$ and $\hat{\boldsymbol{e}}_2$ above into eqn. (5.2), extract x and y components to get 2 equations in two unknowns which you can solve to get $T_{AB} = 500\,\mathrm{N}$ and $N = 500\sqrt{3}\,\mathrm{N}$ (note the font confusion that the force quantity N and unit N have different meanings).
Method II. Using well chosen dot products can simplify the algebra. Take the dot products of both sides of eqn. (5.2) with $\hat{\boldsymbol{e}}_1$ and then separately with $\hat{\boldsymbol{e}}_2$, to get two scalar equations. Dotting eqn. (5.2) with $\hat{\boldsymbol{e}}_1$ eliminates terms orthogonal to $\hat{\boldsymbol{e}}_1$, namely $N\hat{\boldsymbol{e}}_2$. And dotting eqn. (5.2) with $\hat{\boldsymbol{e}}_2$ 'kills' the $T_{AB}\hat{\boldsymbol{e}}_1$ term. So the two equations each have only one unknown. See page 113 for more discussion of this method.

Three-dimensional particle mechanics

The basic idea is the same in 3D as in 2D.

Example: One unknown force.
Assume 3 known forces and one unknown force $\vec{\boldsymbol{F}}$ are acting on a particle (fig. 5.5). Then from force balance

$$\begin{aligned} \vec{\mathbf{0}} &= \sum \vec{\boldsymbol{F}}_i \\ \Rightarrow \quad \vec{\mathbf{0}} &= (36\,\mathrm{lbf}\hat{\boldsymbol{\imath}} - 16\,\mathrm{lbf}\,\hat{\boldsymbol{\jmath}}) + (-52\,\mathrm{lbf}\hat{\boldsymbol{k}} + 5\,\mathrm{lbf}\hat{\boldsymbol{\imath}}) \\ &\quad +(-42\,\mathrm{lbf}\hat{\boldsymbol{k}} + 20\,\mathrm{lbf}\hat{\boldsymbol{\imath}} - 16\,\mathrm{lbf}\,\hat{\boldsymbol{\jmath}}) + \vec{\boldsymbol{F}} \\ \Rightarrow \quad \vec{\boldsymbol{F}} &= \left(-61\hat{\boldsymbol{\imath}} + 32\hat{\boldsymbol{\jmath}} + 94\hat{\boldsymbol{k}}\right)\,\mathrm{lbf}. \end{aligned}$$

.

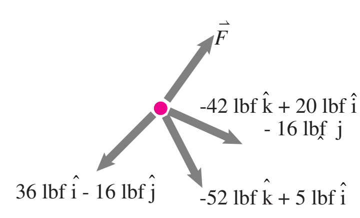


Figure 5.5: One unknown force $\vec{\boldsymbol{F}}$.

The new difficulties in 3D particle mechanics are

5.1 Existence and uniqueness

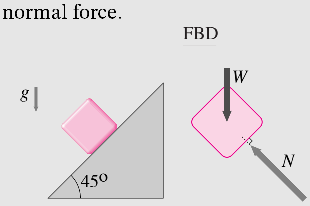
This is a relatively advanced aside.

The words *existence* and *uniqueness* may sound mathematically abstract and irrelevant to the real world. But if you translate 'existence' to 'that's possible' and 'non-existence' to 'no way' you can see the relevance. Likewise 'uniqueness' and 'non-uniqueness' translate to 'there's just one way to do that' and 'there's lots of ways to do that'.

In most homework problems there is an answer, and just one answer: the solution exists and is unique. But in projects and at work problems are often *ill-posed*. Non-existence or non-uniqueness may show up by a structure collapsing or a computer calculation giving erratic answers or error messages.

Existence. Sometimes equations have no solutions, they do not *exist*. For example the set $\begin{aligned} x + 2y &= 7 \\ 2x + 4y &= 15 \end{aligned}$ has no solutions. They don't exist. Why? Because subtracting twice the first equation from the second gives a contradiction: $0 = 1$.

Example: Block on a slippery (frictionless) ramp. Use statics to find the normal force.



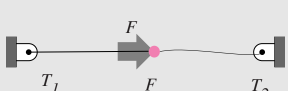
But the block slides and this is not a statics problem. So there is no statics solution. Even without intuition, the statics force-balance equations show that there is no value of N that can make the force vectors add to zero. So no statics solution exists.

Such contradictions can be more subtle. Section 6.5 has some examples where even experts can't intuitively see that there are no solutions.

Uniqueness. Sometimes statics problems have more than one solution, that is a *non-unique* solution: the equation $x+y=1$ has many solutions including $(x,y)=(1,0)$, $(x,y)=(0,1)$, $(x,y)=(10,-9)$ *etc.*

Structural mechanics problems often have non-unique solutions.

Example: Particle held by two strings. Find the tension in the strings to the sides of the point of application of a given load $F = 10\text{N}$.



Force balance along the strings gives us one equation for the two unknown tensions.

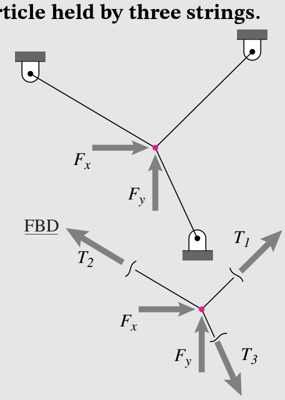
$$\left\{\sum \vec{F}_i = \vec{0}\right\} \cdot \hat{\imath} \qquad \Rightarrow \qquad -T_1 + F + T_2 = 0$$

No other force balance or moment-balance equation gives more information. For any given F this equation has many solutions. The pair (T_1, T_2) could be $(10\,\text{N}, 0)$ or $(0, -10\,\text{N})$ $(20\,\text{N}, 10N)$ $(17\,\text{N}, 7\,\text{N})$, etc.

Of course if you tie strings together like this and apply a force there is some actual tension in each string; reality, at any instant in time, is unique (as far as we know). For example, if you had tied the strings loosely together the right string gets slack and has $T_2 = 0$ and thus $T_1 = 10\,\text{N}$. But it takes an extra assumption of this nature to get a unique solution.

And just because you can make an assumption that leads you to a unique solution doesn't mean that this corresponds to reality. You might assume your friend had tied the strings together loosely and thus calculate $T_2 = 0$ and $T_1 = 10\,\text{N}$. But really she tied them together tightly so $T_2 = 30\,\text{N}$ and $T_1 = 40\,\text{N}$. Here is the same idea in 2D.

Example: Particle held by three strings.



Assume that F_x and F_y are given. What are the three tensions. Planar force balance gives two equations for the 3 unknown tensions. As in the previous example these equations have many solutions.

If you assume a) that one string goes slack and b) that no string can carry compression, then this problem has a unique answer. But you would have to know *a priori* that the strings were initially loose.

The same idea holds for 4 strings holding a particle in 3D. These string examples have a 'one parameter family of solutions'; specifying one number (say that the tension in cable 2 is zero) determines the other tensions. But there can be more non-uniqueness than that by using more strings and then to get a unique solution you have to make more assumptions.

Counting equations and unknowns All of the uniqueness issues above could be detected by counting equations and unknowns. For the block on the ramp we had two equations for the one unknown N. Whereas for the string problems we had more unknowns than equations. In summary,

- If you have more equations than unknowns, existence is likely to be an issue; you probably can't find any solutions.
- If you have more unknowns than equations then uniqueness *is* likely an issue; any solution you find is probably non-unique

But, as for the block on ramp (2 equations with 2 unknowns), there are cases for which equation counting does not tell all about existence and uniqueness: see the lower right corner of the large table in section 6.5. Some simple counter examples: $x = 3, 2x = 6$ has a unique solution; and you can solve the one equation $x^2 + y^2 = 0$ for 2 unknowns.

- Visualization in 3D. (So practice making and reading 3D drawings.); and
- The vector force-balance equation is 3D and thus equivalent to 3 scalar equations. Solving these is at the upper boundary of what most people can do reliably by hand or even with a non-programmable calculator. So methods that reduce the complexity of the solution are useful, as is the ability to set up the resulting equations on a computer or programmable calculator.

Hint: If the direction of a force is given (possibly implicitly) express the force as a scalar times a unit vector: $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\lambda}}$. (See top row, middle column of fig. 2.42 on page 169.)

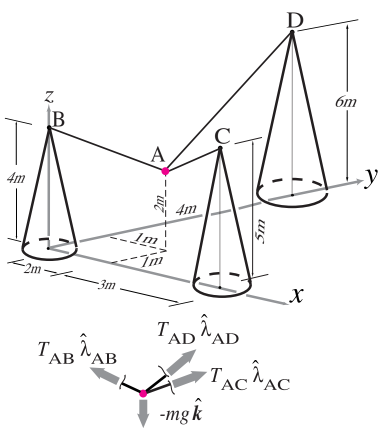


Figure 5.6: Mass A suspended by 3 ropes and a free-body diagram of the mass.

Example: Particle held by 3 ropes.
Say $m = 100\,\text{kg}$ and $g = 10\,\text{N/kg}$ in fig. 5.6. Force balance gives

$$\sum \vec{\boldsymbol{F}}_i = \vec{\boldsymbol{0}} \quad \Rightarrow \quad T_{AB}\hat{\boldsymbol{\lambda}}_{AB} + T_{AC}\hat{\boldsymbol{\lambda}}_{AC} + T_{AD}\hat{\boldsymbol{\lambda}}_{AD} - mg\hat{\boldsymbol{k}} = \vec{\boldsymbol{0}} \tag{5.3}$$

which is a 3D vector equation in 3 unknowns (3 scalar equations and 3 unknowns, good). The $\hat{\boldsymbol{\lambda}}$'s in eqn. (5.6) are known because the position vectors are given in the picture. For example,

$$\hat{\boldsymbol{\lambda}}_{AB} = \frac{\vec{\boldsymbol{r}}_{AB}}{|\vec{\boldsymbol{r}}_{AB}|} = \frac{\vec{\boldsymbol{r}}_{B} - \vec{\boldsymbol{r}}_{A}}{|\vec{\boldsymbol{r}}_{B} - \vec{\boldsymbol{r}}_{A}|}.$$

To get to a numerical answer for the tensions you can use many methods such as (see Sample 5.5 on page 254)

1. Brute force by hand.
2. Systematically set up matrix equations for solution by some means.
3. Set up and solve equations on a computer.
4. Use an appropriate dot product to extract one equation in one unknown.
5. Use moment about an axis to extract one equation in one unknown.

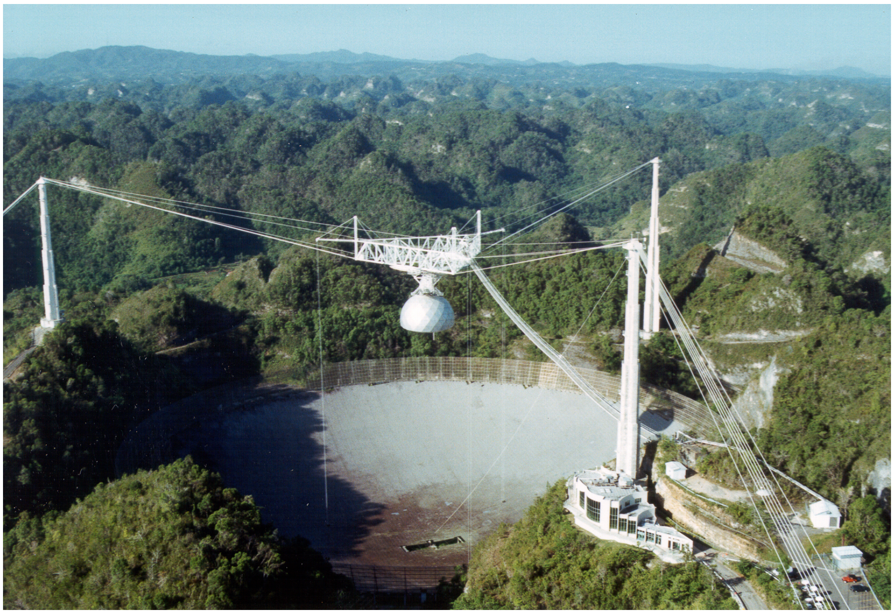


Figure 5.7: The 900 ton platform is suspended 450 feet above the curved reflector surface. The platform hangs on eighteen cables from three towers, each about 300 feet high. *Courtesy of the NSF funded NAIC-Arecibo Observatory in Puerto Rico.*

5.2 The simplification of dynamics to statics

Is statics good enough? The answer below is for your curiosity, not to help you with homework problems.

Classical mechanics, the equations on the inside cover of this book, is at least 99.99% accurate for at least 99.99% of mechanical engineering problems (see page 33 in the preface). Statics is a subset of classical mechanics. Statics is used for those dynamics situations where the statics approximation is reasonable (remember, no real situation is exactly static). Fortunately, for at least 90% of engineering mechanics calculations statics is at least 90% accurate.

In statics, we set the right hand sides of equations I and II (on inside cover) to zero. We are then thinking that these 'inertial' terms are small enough, compared to other forces, to be neglected. We replace the linear and angular momentum balance equations with their simplified statics forms

$$\sum_{\substack{\text{All external}\\ \text{forces}}} \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}} \qquad \text{and} \qquad \sum_{\substack{\text{All external}\\ \text{torques}}} \vec{\boldsymbol{M}}_C = \vec{\boldsymbol{0}} \quad (Ic, IIc).$$

which are sometimes called the *force balance* and *moment balance* and together are called the *equilibrium* equations. The forces to be summed (added) are the ones you see on a free-body diagram. The torques that are summed are those due to the same forces (by means of $\vec{\boldsymbol{r}}_{i/C} \times \vec{\boldsymbol{F}}_i$) and any applied couples.

In dynamics forces 'balance' mass times acceleration.
In statics, the forces balance each other.

The approximation, the assumption, the 'model', see page 35) in statics is this: Statics assumes that the forces on an object are much larger than the net force which accelerates it. That is, in various ways of thinking:

- The mass times acceleration is small. Small compared to what? Small compared to the individual forces.
- The forces largely cancel. That is, the sum of the forces is much smaller than the individual terms.

Catch 22. Usually dynamics is too complicated for the trouble. So you use statics. But you are insecure and want to know if statics is accurate enough. To know this you have to do dynamics.

Estimating the errors from neglecting dynamics is a dynamics problem.

For each free-body diagram you have to know that

$$\left| m_{\text{tot}} \vec{\boldsymbol{a}}_G \right| = \left| \sum_{\substack{\text{All forces}\\ \text{on the part}}} \vec{\boldsymbol{F}}_i \right| \ll F_{\text{typical}}.$$

To avoid Catch 22 we usually fake it by checking (usually in our heads) this rule of thumb.

Statics is probably accurate enough if

$$m_{\text{part}} a_{\text{part}} \ll \underbrace{m_{\text{load}}(g + |a_{\text{load}}|) + m_{\text{part}} g}_{F_{\text{typical}}}. \qquad (5.4)$$

Given this, the forces are more canceling each other out (balancing each other) than causing acceleration. So you can use statics to figure out just how these forces cancel each other.

Statics equations are often accurate-enough for

- Things that a normal person would call "still" such as a building or bridge on a calm day, and a sleeping person;
- Things that move with little acceleration, such as a tractor plowing a field or most of the parts in a smooth-flying airplane; and
- Parts that mediate the forces needed to accelerate more massive parts, such as gears in a transmission, the rear wheel of an accelerating bicycle, the strut in the landing gear of an airplane, and the individual structural members of a building swaying in an earthquake.

Example: A bicycle wheel. The forces on a bicycle wheel are on the order of the weight of a person plus what is needed to accelerate the person up and down small hills, say about $2m_{\text{person}}g$ in total, at most. As the bike rolls up and down small hills the wheel's acceleration might also be about g (at most) so that $|m_{\text{wheel}}\vec{\boldsymbol{a}}_{\text{wheel}}| \approx m_{\text{wheel}}g$. We ask,

$$\begin{aligned} \left| \mathrm{m}_{\text{wheel}} \vec{\boldsymbol{a}}_G \right| &\ll F_{\text{typical}} \; ? \\ \Rightarrow \qquad m_{\text{wheel}} g &\ll 2m_{\text{person}} g \\ \text{and indeed} \qquad m_{\text{wheel}} &\ll 2m_{\text{person}}. \end{aligned}$$

For a 50 kg rider on a bike with 1 kg wheels, the error from using statics instead of dynamics, for the forces on a bicycle wheel, is about 4%. The load on the wheel might be from acceleration of the rider above, but the analysis of the wheel itself can use statics with pretty good accuracy. Note, a similar argument also often shows that for structures holding other parts we can reasonably neglect, when studying the structure, not just the structure's acceleration, but the weight of the structure too.

What if your statics calculation gives an inaccurate result? If your statics calculations make a bad prediction, one possible source of errors is your neglect of dynamic terms. But that is not the common case for things that seem relatively stationary. Rather,

Most bad statics predictions come from

- Bad estimates of material properties (friction coefficient, failure strength, etc),
- Wrong dimensions (or angles, etc), or
- Math errors.

SAMPLE 5.1 **Equilibrium of a pin.** Two rods, AB and BC, are pinned together at point B and to the ground as shown in the figure. A force $F = 100$ N is applied at point B. For $\theta = 45^\circ$, find the tension in the two rods.

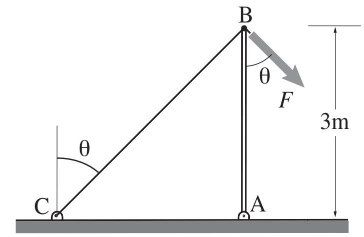


Figure 5.8

Solution The free-body diagram of the pin at B is shown in fig. 5.9 where T_1 and T_2 are the tensions in rods AB and BC respectively. The static equilibrium of the pin at B requires that

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad \vec{F} + \vec{T}_1 + \vec{T}_2 = \vec{0}$$

$$\text{or} \qquad F(\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) - T_1\hat{\jmath} + T_2(-\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}) = 0. \tag{5.5}$$

This is one vector equation in 2D in two unknowns T_1 and T_2. We can solve for the unknowns in various ways.

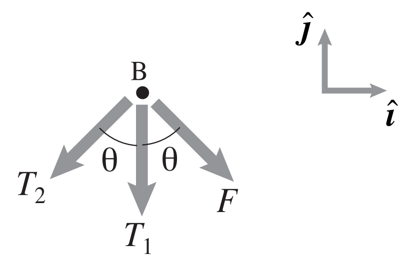


Figure 5.9: The free-body diagram of the pin at B. Although we can see that rod AB is in compression, we use tension as positive and let the solution take care of the proper sign.

Method-1: Separate out scalar equations in x and y directions. The force equilibrium equation, eqn. (5.5), gives us two independent scalar equations in the x and y directions:

$$\sum F_x = 0 \quad \Rightarrow \quad F\sin\theta - T_2\sin\theta = 0$$
$$\sum F_y = 0 \quad \Rightarrow \quad -F\cos\theta - T_1 - T_2\cos\theta = 0.$$

Solving these two equations simultaneously, we get

$$\begin{aligned} T_2 &= F = 100\,\text{N} \\ T_1 &= -(F + T_2)\cos\theta \\ &= -2F\cos\theta = -141.4\,\text{N}. \end{aligned}$$

$$\boxed{T_1 = -141.4\,\text{N}, \qquad T_2 = 100\,\text{N}}$$

①

①Note that the tension in rod AB, T_1 turns out to be negative, that is, rod AB is in *compression*. This is expected since the other two forces at B, F and T_2, are pushing on rod AB. The equality of F and T_2 is also expected from symmetry.

Method-2: Dot the equation with appropriate vectors. The goal here is to dot the vector equation with appropriate vectors that give us one scalar equation in one unknown. Here, $\vec{T}_1$ acts in the $-\hat{\jmath}$ direction; therefore, dotting the equation with $\hat{\imath}$ gets rid of $\vec{T}_1$ and results in a scalar equation involving only T_2:

$$\begin{aligned} [\text{eqn. } (5.5)]\cdot\hat{\imath} \quad &\Rightarrow \quad F\sin\theta - T_2\sin\theta = 0 \\ &\Rightarrow \quad T_2 = F = 100\,\text{N}. \end{aligned}$$

Similarly, to get rid of $\vec{T}_2$, dot the equation with a vector $\hat{n}$ normal to $\vec{T}_2$, *i.e.,* with $\hat{n} = \cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$:②

$$\begin{aligned} [\text{eqn. } (5.5)]\cdot\hat{n} \quad &\Rightarrow \quad [-T_1\hat{\jmath} + F(\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})\cdot(\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})] = 0 \\ &\Rightarrow \quad T_1\sin\theta + F(\cos^2\theta + \sin^2\theta) = 0 \\ &\Rightarrow \quad T_1 = -\frac{F}{\sin\theta} = -\frac{100\,\text{N}}{1/\sqrt{2}} = -141.4\,\text{N} \end{aligned}$$

These are the same values of T_1 and T_2, as they must be, obtained by Method-1.

②How do we find the normal vector $\hat{n}$? Well, we know the direction of $\vec{T}_2$, which is $\hat{\lambda} = -\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$. We can draw a unit vector $\hat{n}$ normal to $\hat{\lambda}$ and find its components from geometry (see figure below), or we can set $\hat{n} = \hat{k}\times\hat{\lambda}$ which guarantees $\hat{n}$ to be normal to $\hat{\lambda}$. In either case we get $\hat{n} = \cos\theta\hat{\imath} - \sin\theta\hat{\jmath}$.

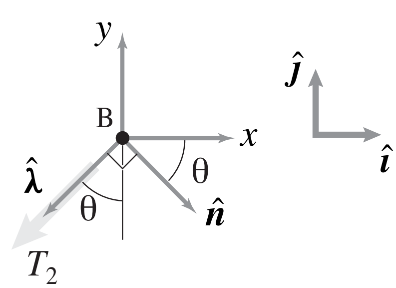


Figure 5.10: Finding a unit vector $\hat{n}$ normal to the direction $\hat{\lambda}$ of T_2.

Method-3: Use matrix equation and solve by hand or on a computer. The two scalar equations obtained from eqn. (5.6) can be written in the matrix form as

$$\begin{bmatrix} 0 & \sin\theta \\ 1 & \cos\theta \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} F\sin\theta \\ -F\cos\theta \end{pmatrix}.$$

Using $F = 100$ N and $\theta = 45^\circ = \pi/4$, and solving the above matrix equation (see Sample 1.28 on page 123 and Sample 1.30 on page 125), we get

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} -141.4 \\ 100 \end{pmatrix} \text{N}$$

which is, of course, the same result as we got above.

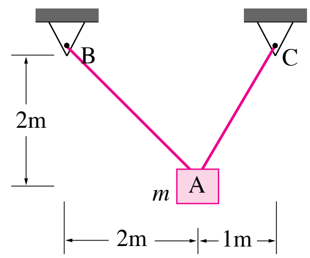


Figure 5.11

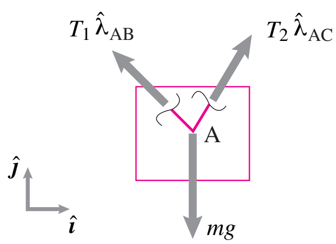


Figure 5.12

SAMPLE 5.2 A mass held in equilibrium by unequal strings in 2D. A 10 kg block m hangs from strings AB and AC in the vertical plane as shown in the figure. Find the tension in the strings.

Solution The free-body diagram of the block is shown in figure 5.12. The equation of force balance, $\sum \vec{F} = \vec{0}$, gives

$$\text{or} \qquad T_1\hat{\boldsymbol{\lambda}}_{\text{AB}} + T_2\hat{\boldsymbol{\lambda}}_{\text{AC}} - mg\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}, \tag{5.6}$$

where $\hat{\boldsymbol{\lambda}}_{\text{AB}}$ and $\hat{\boldsymbol{\lambda}}_{BC}$ are unit vectors in the AB and AC directions:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_{AB} &= \frac{\vec{\boldsymbol{r}}_{AB}}{|\vec{\boldsymbol{r}}_{AB}|} = \frac{-2\,\text{m}\hat{\boldsymbol{\imath}} + 2\,\text{m}\hat{\boldsymbol{\jmath}}}{2\sqrt{2}\,\text{m}} = \frac{1}{\sqrt{2}}(-\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) \\
\hat{\boldsymbol{\lambda}}_{\text{AC}} &= \frac{\vec{\boldsymbol{r}}_{AC}}{|\vec{\boldsymbol{r}}_{AC}|} = \frac{1\,\text{m}\hat{\boldsymbol{\imath}} + 2\,\text{m}\hat{\boldsymbol{\jmath}}}{\sqrt{5}\,\text{m}} = \frac{1}{\sqrt{5}}(\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}).
\end{aligned}$$

Substituting in eqn. (5.6) and rearranging terms, we have

$$\left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\boldsymbol{\imath}} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}.$$

Separating x and y components of this equation, we get the scalar equations

$$\begin{aligned}
\sum F_x = 0 \quad &\Rightarrow \quad -\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}} = 0 \\
\sum F_y = 0 \quad &\Rightarrow \quad \frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg = 0.
\end{aligned}$$

Solving these two equations simultaneously we get,

$$T_1 = \frac{\sqrt{2}}{3}mg = 46.24\,\text{N} \quad \text{and} \quad T_2 = \frac{\sqrt{5}}{3}mg = 73.12\,\text{N}.$$

$$\boxed{T_1 = 46.24\,\text{N},\ T_2 = 73.12\,\text{N}}$$

Note:

Solving for T_1 and T_2 If you are comfortable with vector algebra, then solving for T_1 and T_2 from eqn. (5.6) is quite easy. Let us say, we find two unit vectors $\hat{\boldsymbol{n}}_{AB}$ and $\hat{\boldsymbol{n}}_A C$ normal to unit vectors $\hat{\boldsymbol{\lambda}}_{AB}$ and $\hat{\boldsymbol{\lambda}}_{AC}$, respectively. Then dotting eqn. (5.6) with $\hat{\boldsymbol{n}}_{AB}$ and $\hat{\boldsymbol{n}}_A C$, one at a time, we can solve for T_1 and T_2 in one step:

$$T_2 = \frac{mg\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{n}}_{AB}}{\hat{\boldsymbol{\lambda}}_{AC}\cdot\hat{\boldsymbol{n}}_{AB}}, \qquad \text{and} \qquad T_1 = \frac{mg\hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{n}}_{AC}}{\hat{\boldsymbol{\lambda}}_{AB}\cdot\hat{\boldsymbol{n}}_{AC}}.$$

For computing the values, we need to carry out the dot products. Noting that $\hat{\boldsymbol{n}}_{AB} = \frac{1}{\sqrt{2}}(i + j)$ and $\hat{\boldsymbol{n}}_{AC} = \frac{1}{\sqrt{5}}(-2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}})$ (you can write these vectors by looking at $\hat{\boldsymbol{\lambda}}_{AB}$ and $\hat{\boldsymbol{\lambda}}_{AC}$), we can carry out the dot product and get the values of T_1 and T_2.

Matrix equation The two scalar equations obtained from eqn. (5.6) can be written in the matrix form as

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ mg \end{pmatrix}.$$

Using $mg = (10\,\text{kg})\cdot(9.81\,\text{m/s}^2) = 98.1\,\text{N}$, and solving the above matrix equation (see Sample 1.28 on page 123 and Sample 1.30 on page 125), we get

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 46.24 \\ 73.12 \end{pmatrix}\text{N}$$

which is, of course, the same result as we got above.

SAMPLE 5.3 **Tensions in a paraglider's ropes.** A paraglider is held by two ropes that in turn connect to the parachute with many ropes. The angles that the two ropes make with the horizontal (or vertical) are not necessarily the same for each rope. Assume that the right rope makes an angle θ_1 with the horizontal and the left one makes an angle θ_2 (in flight, these angles will vary with time). Also assume that the paraglider is descending at some uniform velocity. Find the tensions in the two ropes in terms of the weight of the person and the harness (say, mg), and the angles θ_1 and θ_2.

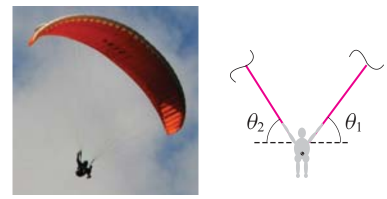


Figure 5.13

Solution The free-body diagram of the glider is shown in fig. 5.14. We assume that the tensions T_1, T_2, and the weight mg are all in one vertical plane. Since the paraglider has a uniform velocity (no acceleration), we can use the force-balance equation for static equilibrium, $\sum \vec{F} = \vec{0}$:

$$T_1\hat{\boldsymbol{\lambda}}_1 + T_2\hat{\boldsymbol{\lambda}}_2 - mg\,\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}} \qquad (5.7)$$

where $\hat{\boldsymbol{\lambda}}_1 = \cos\theta_1\hat{\boldsymbol{\imath}} + \sin\theta_1\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{\lambda}}_2 = -\cos\theta_2\hat{\boldsymbol{\imath}} + \sin\theta_2\hat{\boldsymbol{\jmath}}$ are the unit vectors along the two ropes, respectively. Thus the vector equation of equilibrium is:

$$T_1(\cos\theta_1\hat{\boldsymbol{\imath}} + \sin\theta_1\hat{\boldsymbol{\jmath}}) + T_2(-\cos\theta_2\hat{\boldsymbol{\imath}} + \sin\theta_2\hat{\boldsymbol{\jmath}}) - mg\,\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}. \qquad (5.8)$$

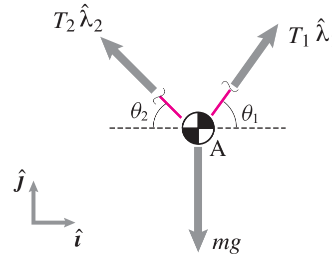


Figure 5.14: The free-body diagram of the paraglider considering the ropes to be at different angles with the horizontal.

From this equation, we get two independent scalar equations by separating the x and y components:

$$\sum F_x = 0 \quad \Rightarrow \quad T_1\cos\theta_1 - T_2\cos\theta_2 = 0$$
$$\sum F_y = 0 \quad \Rightarrow \quad T_1\sin\theta_1 + T_2\sin\theta_2 - mg = 0.$$

Solving these two equations simultaneously we get,

$$T_1 = \frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}mg \quad \text{and} \quad T_2 = \frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}mg.$$

$$\boxed{T_1 = \frac{mg\cos\theta_2}{\sin(\theta_1+\theta_2)} \text{ and } T_2 = \frac{mg\cos\theta_1}{\sin(\theta_1+\theta_2)}}$$

Another vector method: We can find T_1 and T_2 directly from eqn. (5.7) by taking cross products with $\hat{\boldsymbol{\lambda}}_2$ and $\hat{\boldsymbol{\lambda}}_1$, respectively:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_2 \times (T_1\hat{\boldsymbol{\lambda}}_1 + T_2\hat{\boldsymbol{\lambda}}_2 - mg\,\hat{\boldsymbol{\jmath}}) &= \vec{\mathbf{0}} \\
\Rightarrow \quad T_1\underbrace{(\hat{\boldsymbol{\lambda}}_2 \times \hat{\boldsymbol{\lambda}}_1)}_{-\sin(\theta_1+\theta_2)\hat{\boldsymbol{k}}} - mg\underbrace{(\hat{\boldsymbol{\lambda}}_2 \times \hat{\boldsymbol{\jmath}})}_{-\cos\theta_2\hat{\boldsymbol{k}}} &= \vec{\mathbf{0}} \\
\Rightarrow \quad T_1 &= \frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}mg
\end{aligned}$$

Similarly, $\lambda_2\times$[eqn. (5.7)] gives us

$$T_2 = \frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}mg.$$

Thus we get the same results as we got above by solving the two scalar equations simultaneously.

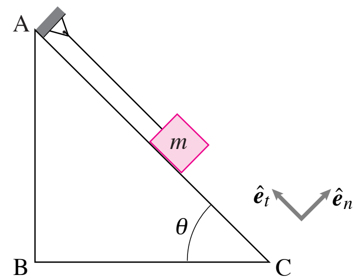


Figure 5.15: A mass-particle on an inclined plane.

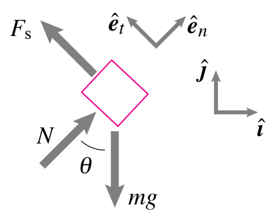


Figure 5.16: Free-body diagram of the mass and the geometry of force vectors.

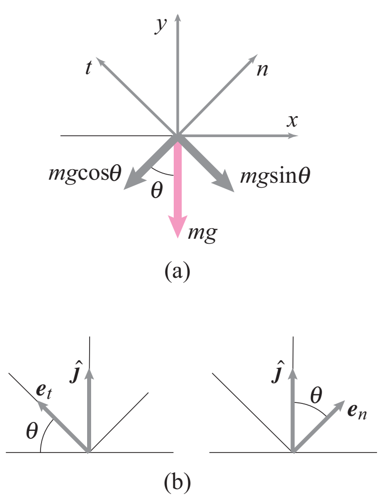


Figure 5.17: (a) Components of mg along t and n directions. (b) The mixed basis dot products: $\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{e}}_t = \sin\theta$ and $\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{e}}_n = \cos\theta$

SAMPLE 5.4 A single string holding a mass on a frictionless incline. A block of mass m rests on a frictionless inclined plane with the help of a string that connects the mass to a fixed support at A. Find the force in the string.

Solution The free-body diagram of the mass is shown in Fig. 5.16. The string force F_s and the normal reaction of the plane N are unknown forces. The force-balance equation, $\sum \vec{F} = \vec{\mathbf{0}}$, is

$$\vec{F}_s + \vec{N} + m\vec{g} = \vec{\mathbf{0}}.$$

We can express the forces in terms of their components in various ways and then dot the vector equation with appropriate unit vectors to get two independent scalar equations. For example, we write the force-balance equation using mixed basis vectors $\hat{\boldsymbol{e}}_t$ and $\hat{\boldsymbol{e}}_n$, and $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$:

$$F_s\hat{\boldsymbol{e}}_t + N\hat{\boldsymbol{e}}_n - mg\hat{\boldsymbol{\jmath}} = \vec{\mathbf{0}}. \tag{5.9}$$

We can now find F_s directly by taking the dot product of the above equation with $\hat{\boldsymbol{e}}_t$ since the other unknown N is in the $\hat{\boldsymbol{e}}_n$ direction and $\hat{\boldsymbol{e}}_n \cdot \hat{\boldsymbol{e}}_t = 0$:

$$\begin{aligned} \{\text{eqn. } (5.9)\} \cdot \hat{\boldsymbol{e}}_t \quad &\Rightarrow \quad F_s - mg\overbrace{(\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{e}}_t)}^{\sin\theta} = 0 \\ &\Rightarrow \quad F_s = mg\sin\theta. \end{aligned}$$

$$\boxed{F_s = mg\sin\theta}$$

. . .

Note: We can also find N from a single equation by taking the dot product of eqn. (5.9) with $\hat{\boldsymbol{n}}$:

$$\begin{aligned} \{\text{eqn. } (5.9)\} \cdot \hat{\boldsymbol{e}}_n \quad &\Rightarrow \quad N - mg\overbrace{(\hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{e}}_n)}^{\cos\theta} = 0 \\ &\Rightarrow \quad N = mg\cos\theta. \end{aligned}$$

Scalar approach: We resolve all forces into their $\hat{\boldsymbol{e}}_t$ and $\hat{\boldsymbol{e}}_n$ components and then sum the forces. Here, F_s is along the plane and therefore, has no component perpendicular to the plane. Force N is perpendicular to the plane and therefore, has no component along the plane. We resolve the weight mg into two components: (1) $mg\cos\theta$ perpendicular to the plane (along $\hat{\boldsymbol{e}}_n$) and (2) $mg\sin\theta$ along the plane (along $\hat{\boldsymbol{e}}_t$). Now we can sum the forces:

$$\begin{aligned} &\sum F_t = 0 \quad &\Rightarrow \quad &F_s - mg\sin\theta = 0; \\ \text{and} \quad &\sum F_n = 0 \quad &\Rightarrow \quad &N - mg\cos\theta = 0 \end{aligned}$$

which, of course, is essentially the same as the equations obtained above.

SAMPLE 5.5 **A particle in 3D.** A particle of mass 1 kg is attached to two strings tied at points C and D shown in the figure. Another string, AB, attached to the particle, passes over a pulley and is used to hold the particle in equilibrium under gravity such that it loses contact with the ground at point A. Find the tension in string AB.

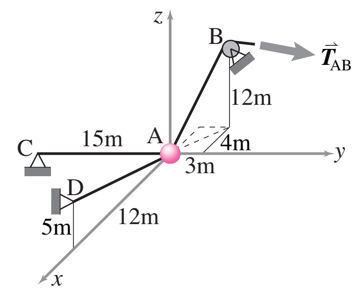


Figure 5.18

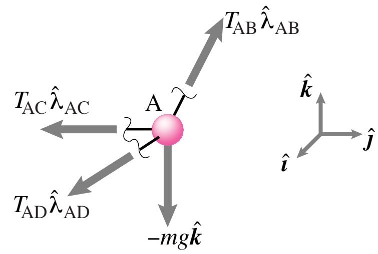


Figure 5.19

Solution The free-body diagram of the particle is shown in fig. 5.19. Assuming the tensions in strings AB, AC, and AD to be T_{AB}, T_{AC}, and T_{AD} respectively, we can represent the string forces acting on the particle as $T_{AB}\hat{\boldsymbol{\lambda}}_{AB}$, $T_{AC}\hat{\boldsymbol{\lambda}}_{AC}$, and $T_{AD}\hat{\boldsymbol{\lambda}}_{AD}$, where the $\hat{\boldsymbol{\lambda}}$'s are the unit vectors along the strings.

The force balance on the particle gives us

$$T_{AB}\hat{\boldsymbol{\lambda}}_{AB} + T_{AC}\hat{\boldsymbol{\lambda}}_{AC} + T_{AD}\hat{\boldsymbol{\lambda}}_{AD} - mg\hat{\boldsymbol{k}} = 0. \tag{5.10}$$

This is the equation we need to solve to find T_{AB}. We show various methods below that you can use to get T_{AB}.

1. **Brute force (by hand).**
   From the given figure, the unit vectors are:

$$\begin{aligned}
\hat{\boldsymbol{\lambda}}_{AB} &= \frac{-4\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} + 12\hat{\boldsymbol{k}}}{\sqrt{4^2+3^2+12^2}} = -\frac{4}{13}\hat{\boldsymbol{\imath}} + \frac{3}{13}\hat{\boldsymbol{\jmath}} + \frac{12}{13}\hat{\boldsymbol{k}} \\
\hat{\boldsymbol{\lambda}}_{AC} &= -\hat{\boldsymbol{\jmath}} \\
\hat{\boldsymbol{\lambda}}_{AD} &= \frac{12\hat{\boldsymbol{\imath}} + 5\hat{\boldsymbol{k}}}{\sqrt{12^2+5^2}} = \frac{12}{13}\hat{\boldsymbol{\imath}} + \frac{5}{13}\hat{\boldsymbol{k}}.
\end{aligned}$$

   Substituting these vectors in eqn. (5.10) and equating the x, y and z components of the equation to zero separately, we get

$$\begin{aligned}
-\frac{4}{13}T_{AB} + \frac{12}{13}T_{AD} &= 0 \\
\frac{3}{13}T_{AB} - T_{AC} &= 0 \\
\frac{12}{13}T_{AB} + \frac{5}{13}T_{AD} &= mg.
\end{aligned} \tag{5.11}$$

   We can solve the three equations simultaneously to get

$$T_{AB} = \frac{39}{41}mg, \quad T_{AC} = \frac{9}{41}mg, \quad \text{and} \quad T_{AD} = \frac{13}{41}mg.$$

   Substituting $m = 1\,\text{kg}$ and $g = 9.81\,\text{m/s}^2$, we get the required values.

$$T_{AB} = 9.33\,\text{N}, \quad T_{AC} = 2.15\,\text{N}, \quad T_{AD} = 3.11\,\text{N}$$

2. **Systematically set up matrix equations.** Eqn. 5.10 can be written in matrix form as

$$\underbrace{\begin{bmatrix} [\hat{\boldsymbol{\lambda}}_{AB}]'_{xyz} & [\hat{\boldsymbol{\lambda}}_{AC}]'_{xyz} & [\hat{\boldsymbol{\lambda}}_{AD}]'_{xyz} \end{bmatrix}}_{3\times 3 \text{ matrix}} \begin{bmatrix} T_{AB} \\ T_{AB} \\ T_{AB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

   where $[\hat{\boldsymbol{\lambda}}_{AB}]'_{xyz}$ is a column of 3 numbers, namely the x, y, and z components of $\hat{\boldsymbol{\lambda}}_{AB}$; similarly for the other two columns of the 3×3 matrix. This matrix equation is then ready to hand to a calculator or computer for a matrix solution. Thus, eqn. (5.11) can be written as,

$$\begin{bmatrix} -4/13 & 0 & 12/13 \\ 3/13 & -1 & 0 \\ 12/13 & 0 & 5/13 \end{bmatrix} \begin{pmatrix} T_{AB} \\ T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix}.$$

Using the pseudo code shown on the side ③ we solve the equations on a computer and get,

```
T = [9.33   2.15   3.11]
```

which is the solution that we obtained above by hand calculation.

3. **Computer solution.** All the math can be handed to a computer by a sequence of commands like this, working from the knowns to the unknowns (see page 22), all in consistent units:

```
% Get all the knowns into the computer
  rA = [0 0 0]' ;   rB = [-4 3 12]'
  rC = [0 -15 0]' ;   rD = [12 0 5]'
  m = 1 ;  g = 9.81 ;
% Make relative position vectors
  rAB = rB- rA;   rAC = rC - rA;  rAD = rD - rA
% Make unit vectors
  lamdaAB = rAB/magnitude(rAB);
  lamdaAC = rAC/magnitude(rAC);
  lamdaAD = rAD/magnitude(rAD);
% Set up and solve the matrix equation
  M = [lamdaAB  lamdaAC  lamdaAD]  %3x3 matrix
  F = [0 0 mg]'   %column vector of known force
  solve {M T = F} for T
```

The column of numbers `T` will be the tensions in the 3 cables. Using this pseudo-code on a computer, we get `T = [9.33   2.15   3.11]` again.

4. **Be tricky to get one equation in one unknown.** Since we are interested only in T_{AB}, we can get rid of the terms we don't know or care about. ④ The vector $\vec{r}_{AC} \times \vec{r}_{AD}$ is orthogonal to both $\vec{r}_{AC}$ and $\vec{r}_{AD}$, so it is orthogonal to $\hat{\lambda}_{AC}$ and $\hat{\lambda}_{AD}$. So taking the dot product of both sides of eqn. (5.10) with $\vec{r}_{AC} \times \vec{r}_{AD}$, we get

$$(\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \{T_{AB}\hat{\lambda}_{AB} + T_{AC}\hat{\lambda}_{AC} + T_{AD}\hat{\lambda}_{AD} - mg\hat{k}\} = (\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \vec{0}$$

$$((\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \hat{\lambda}_{AB})\, T_{AB} = mg\,((\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \hat{k})$$

$$T_{AB} = \frac{mg(\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \hat{k}}{(\vec{r}_{AC} \times \vec{r}_{AD}) \cdot \hat{\lambda}_{AB}}.$$

Since, $\vec{r}_{AC} \times \vec{r}_{AD} = (-15\hat{j})\,\mathrm{m} \times (12\hat{\imath} + 5\hat{k})\,\mathrm{m} = (180\hat{k} - 75\hat{\imath})\,\mathrm{m}^2$, substituting this cross product and other known quantities, we get

$$\begin{aligned} T_{AB} &= \frac{mg \cdot 180}{180 \cdot 12/13 + 75 \cdot 4/13} \\ &= 0.95\, mg \\ &= 9.33\,\mathrm{N}. \end{aligned}$$

$$\boxed{T_{AB} = 9.33\,\mathrm{N}}$$

③ Pseudo-code:

```
Let m=1, 9=9.81
A = [ -4/13 0 12/13
       3/13 -1 0
       12/13 0 5/13 ]
b = [ 0 0 m*g]'
solve A*T = b for T
```

④ Another way of doing this is by taking **Moment about an axis.** This approach is similar in spirit to the previous approach. Instead of the equilibrium eqn. (5.10) we could have used moments about axis CD to 'kill off' the tensions in ropes AC and AD (they have no moment about that axis), like this,

$$\sum M_{\text{axis CD}} = 0$$

$$\vec{r}_{CD} \cdot \{\vec{r}_{DA} \times \{T_{AB}\hat{\lambda}_{AB} - mg\hat{k}\}\} = 0$$

$$T_{AB} = \frac{mg\vec{r}_{CD} \cdot (\vec{r}_{DA} \times \hat{k})}{\vec{r}_{CD} \cdot (\vec{r}_{DA} \times \hat{\lambda}_{AB})}.$$

Again we have found one equation for one unknown, T_{AB}. All the quantities on the right can be evaluated to give T_{AB}.

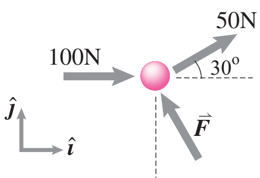
5.1 Static equilibrium of a particle

Preparatory Problems

5.1.1 What is a particle?

5.1.2 What are the equations of equilibrium for a particle (also called "*equilibrium conditions*", "*force balance*", or "*linear momentum balance for statics*"?

5.1.3 The particle shown in the figure is in static equilibrium. Find the unknown force $\vec{F}$.



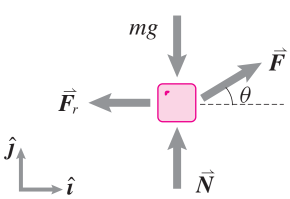
Problem 5.1.3

5.1.4 Four forces act on a block as shown in the figure and hold it in static equilibrium. Assume that the magnitude of force $\vec{F}$ is known and it is F.

a) Write the scalar equations of equilibrium in the x and y directions.

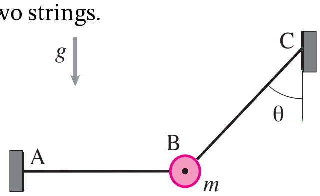
b) Solve for $\vec{N}$ and $\vec{F}_r$ in terms of F, mg, and θ.

c) What is $\vec{F}_r$ when $\theta = 90°$?



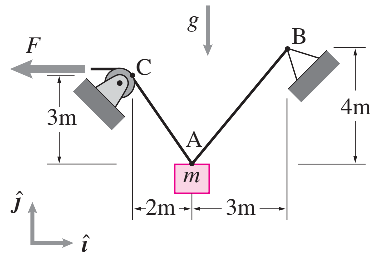
Problem 5.1.4

5.1.5 A particle of mass $m = 2\,\text{kg}$ hangs from strings AB and AC as shown. AB is horizontal and $\theta = 45^{\circ}$. Find the tension in the two strings.



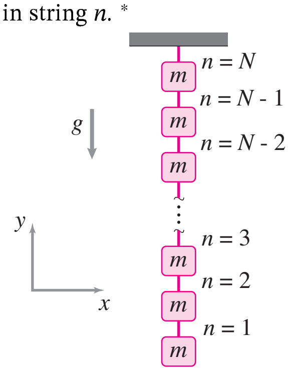
Problem 5.1.5

5.1.6 What force should be applied to the end of the string over the pulley at C so that the mass at A is at rest in the configuration shown?



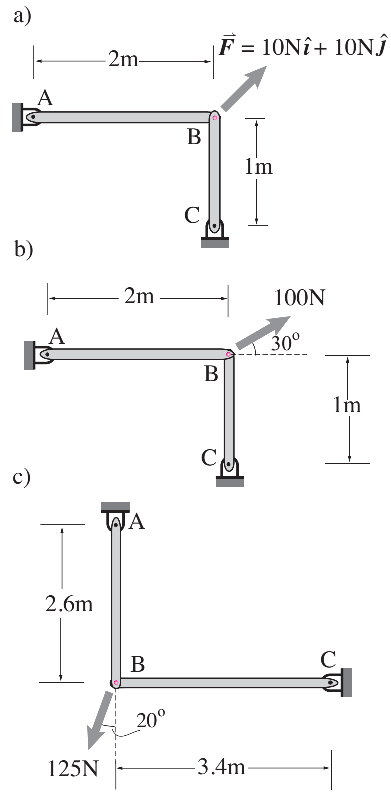
Problem 5.1.6

5.1.7 N small blocks each of mass m hang vertically as shown, connected by N inextensible strings. Find the tension T_n in string n. *



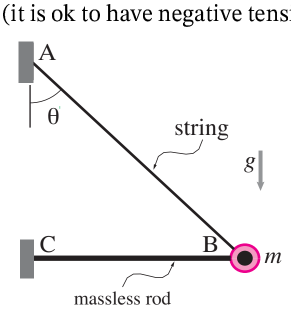
Problem 5.1.7

5.1.8 For each situation below, assume static equilibrium under the applied force and find the tensions in the two rods.



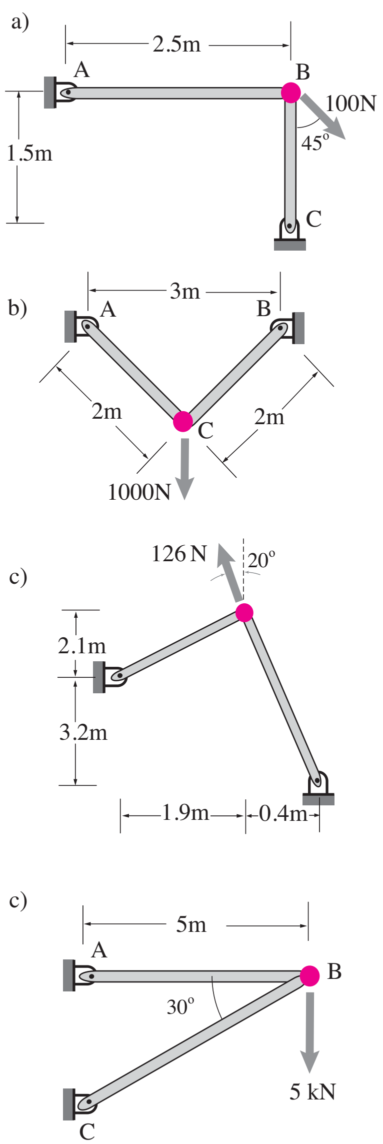
Problem 5.1.8

5.1.9 A particle of mass m = 5 kg at the end of a horizontal massless rod CB of length 1.2 m is held in place with the help of a string AB that makes an angle $\theta = 45^{\circ}$ with the vertical in the equilibrium position. Find the tension in the bar CB (it is ok to have negative tension).



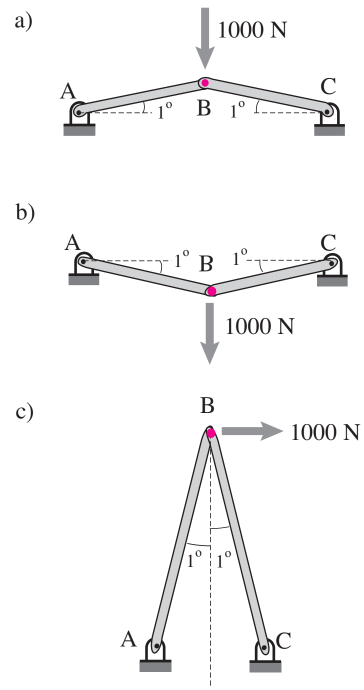
Problem 5.1.9

5.1.10 For each structure shown below, find the tension in each rod. (Note the tension can be less than zero.)



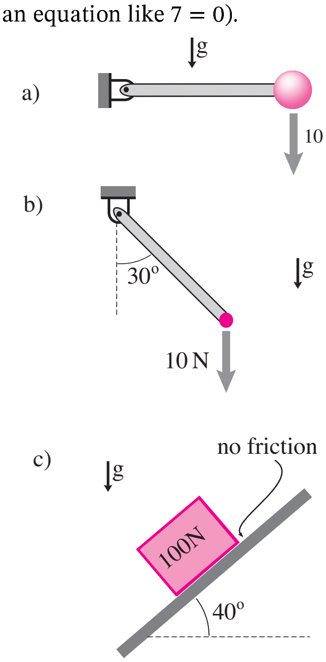
Problem 5.1.10

5.1.11 In the following structures, a pin connects two thin bars that are very nearly either horizontal or vertical. Find the tensions in each rod under the applied loads. (Note the tension is less than zero for some of the rods.)



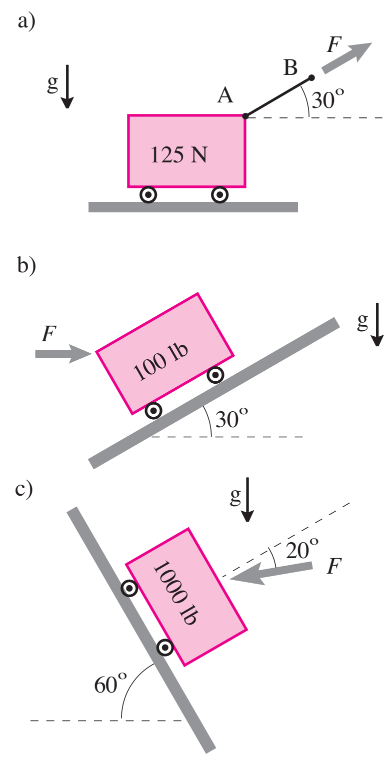
Problem 5.1.11

5.1.12 For each situation shown below, equilibrium is not possible. Write the vector equation for force balance and show that it has no solutions (*i.e.,* leads to an equation like 7 = 0).



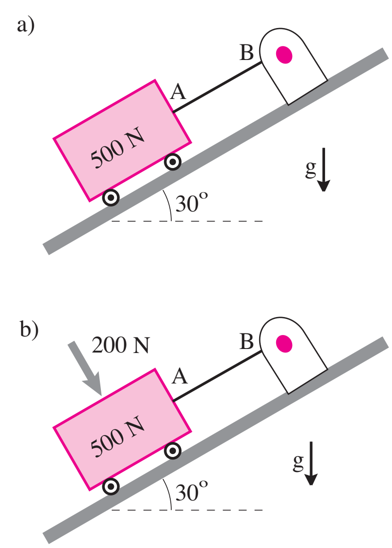
Problem 5.1.12

5.1.13 Assume no sliding friction (μ = 0). Assume equilibrium. Find all reactions, tensions, and forces. *



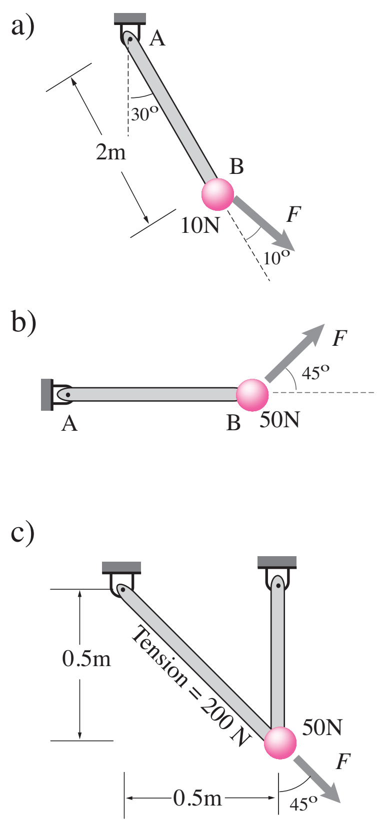
Problem 5.1.13

5.1.14 In each of the two cases given below, find the tension in the string AB assuming the block to be at rest and the ramp to be frictionless.



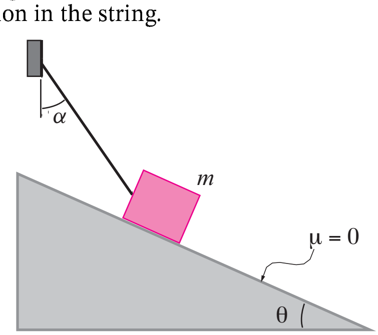
Problem 5.1.14

5.1.15 Find the unknown forces and tensions in each structure shown below.



Problem 5.1.15

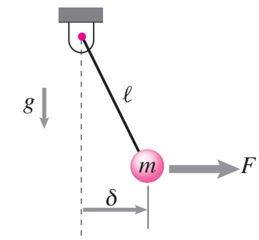
5.1.16 A block of mass $m = 5\,\text{kg}$ rests on a frictionless inclined plane as shown in the figure. Let $\theta = \alpha = 30^\circ$. Find the tension in the string.



Problem 5.1.16

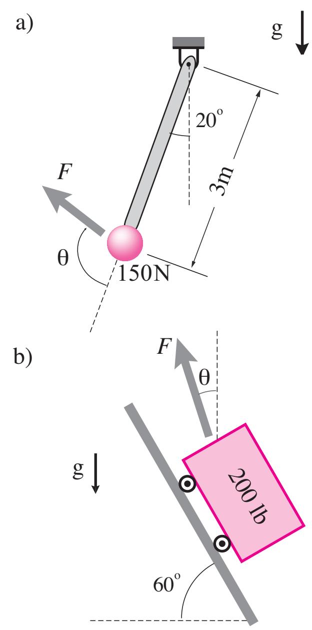
More-Involved Problems

5.1.17 For small δ what is the relation between F and δ (and g and ℓ) for a static pendulum?



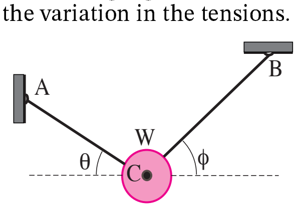
Problem 5.1.17

5.1.18 In the situations shown in the figures, find the value of θ that minimizes F. What is the corresponding value of F in each case?



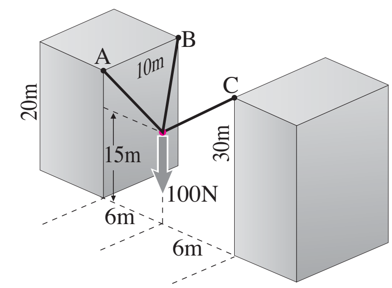
Problem 5.1.18

5.1.19 An object of weight $W = 10\,\text{N}$ is held in equilibrium in the vertical plane by two strings AC and BC. Let $\theta = 30^\circ$ and $0 \le \phi \le 90^\circ$. Find and plot the tension in the two strings against ϕ and comment on the variation in the tensions.



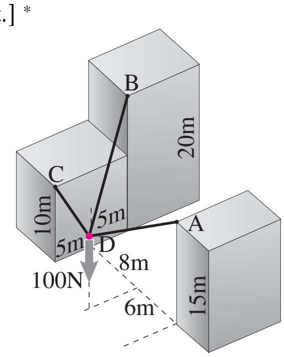
Problem 5.1.19

5.1.20 Find the tensions in the three strings shown in the figure.



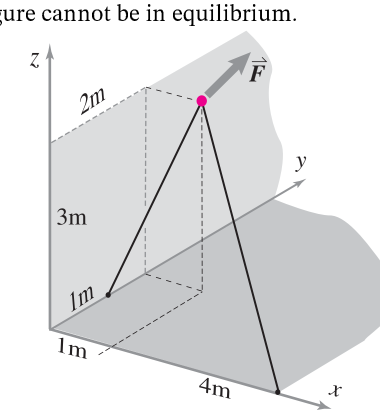
Problem 5.1.20

5.1.21 Find the tensions in the three strings shown in the figure. String CD is horizontal and the force at D is 100 N straight down. [Hint: this problem has a trick to it.] *



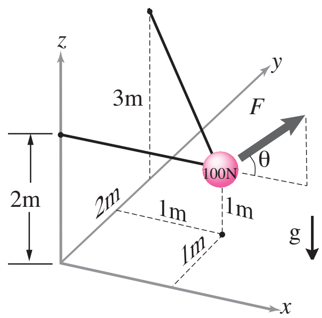
Problem 5.1.21

5.1.22 Show that the particle acted upon by the given force $\vec{F} = (3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})\,\text{N}$, and held by the two bars as shown in the figure cannot be in equilibrium.

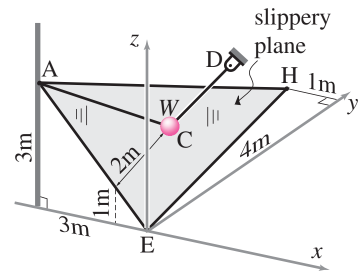


Problem 5.1.22

5.1.23 In the figure shown, the force $\vec{F}$, in the x-z plane, acts on the particle (weighing 100 N). Find F as a function of θ for equilibrium of the particle. Find the value of θ for which the required force is smallest.



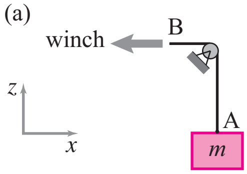
Problem 5.1.23



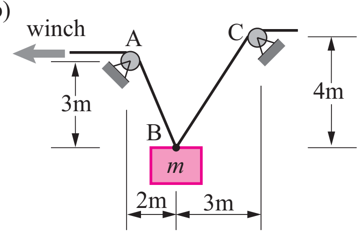
Problem 5.1.25

5.1.24 For the three cases (a), (b), and (c), below, find the tension in the string AB. In all cases the strings hold up the mass $m = 3\,\mathrm{kg}$. You may assume the local gravitational constant is $g = 10\,\mathrm{m/s^2}$. In all cases the winches are pulling in the string so that the velocity of the mass is a constant $4\,\mathrm{m/s}$ upwards (in the $\hat{\boldsymbol{k}}$ direction). [Note that in problems (b) and (c), in order to pull the mass up at constant rate the winches must pull in the strings at an unsteady speed.] *

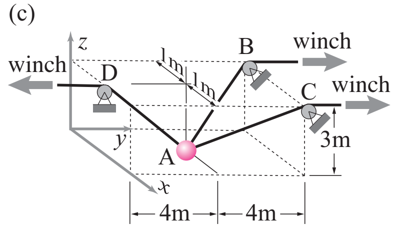
(a)



(b)



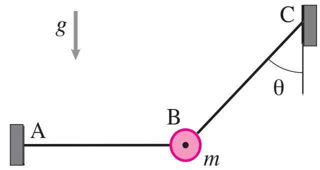
(c)



Problem 5.1.24

5.1.25 A block of weight W , held by two strings AC and DC, rests on a slippery plane AEH. String CD is parallel to EH. Find the tensions in the two strings and the reaction of the plane. You may approximate AC to lie in the plane AEH.

5.1.5 A particle of mass $m = 2\,\text{kg}$ hangs from strings AB and AC as shown. AB is horizontal and $\theta = 45^{\circ}$. Find the tension in the two strings.



Problem 5.5

Ans: 4.1.5

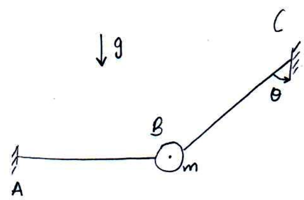


Fig (a)

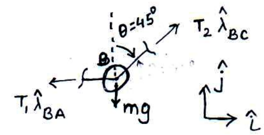


Fig (b): FBD of the mass m.

The free body diagram of the mass 'm' is given in Fig (b).

Force balance, $\Sigma \vec{F} = 0$, gives

$$T_1 \hat{\lambda}_{BA} + T_2 \hat{\lambda}_{BC} + mg(-\hat{j}) = \vec{0} \quad \text{——— (i)}$$

$\hat{\lambda}_{BA}$ and $\hat{\lambda}_{BC}$ are unit vectors in BA and BC directions, and are given by

$$\hat{\lambda}_{BA} = -\hat{i}$$

$$\hat{\lambda}_{BC} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j} = \frac{1}{\sqrt{2}}\left[\hat{i} + \hat{j}\right]$$

Substituting in equation (i) we get,

$$-T_1\,\hat{i} + \frac{T_2}{\sqrt{2}}(\hat{i} + \hat{j}) - mg\,\hat{j} = \vec{0} \quad \text{——— (ii)}$$

Seperating, x and y components of equation (ii). we get,

$$T_2 = \sqrt{2}\,T_1$$

and $\quad T_2 = \sqrt{2}\,mg$

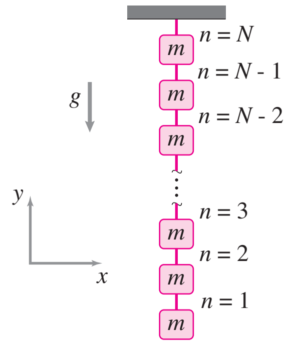
Substituting, $m = 2\,\text{kg}$ and $g = 9.8\,\text{m/sec}^2$ we get

$$\boxed{\begin{aligned} T_2 &= 27.7\,\text{N} \\ T_1 &= 19.6\,\text{N} \end{aligned}}$$

T_2 — Tension in string 'AB'

T_1 — Tension in string 'BC'

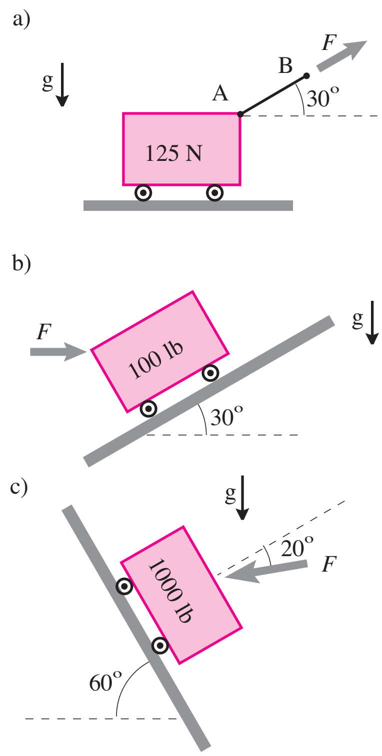
5.1.7 N small blocks each of mass m hang vertically as shown, connected by N inextensible strings. Find the tension T_n in string n.



Problem 5.7

Answer:
$T_{N-1} = (N-1)Nmg$, $T_2 = 2mg$, $T_1 = mg$, and in general $T_n = nmg$

5.1.13 Assume no sliding friction ($\mu = 0$). Assume equilibrium. Find all reactions, tensions, and forces.

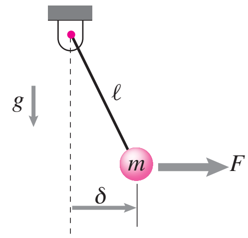


Problem 5.13

Answer:
(a) is nonsense; others are fine

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

5.1.17 For small δ what is the relation between F and δ (and g and ℓ) for a static pendulum?



Problem 5.17

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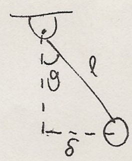
pfigure - spring 3

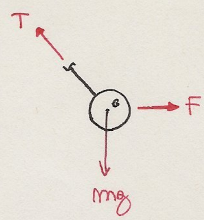
$\vec{T} = T\,\hat{\lambda}$

$\vec{F} = F\,\hat{\imath}$

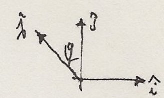
$\vec{W} = -mg\,\hat{\jmath}$



F.B.D.



$\sum \vec{F}_i = 0 \quad \Rightarrow T\hat{\lambda} + F\hat{\imath} - mg\hat{\jmath} = 0 \quad (1)$



$\{(1)\}\cdot\hat{\jmath} \quad \Rightarrow T\,\hat{\lambda}\cdot\hat{\jmath} = mg \quad \Rightarrow T\cos\theta = mg$

$\{(1)\}\cdot\hat{\imath} \quad \Rightarrow T\,\hat{\lambda}\cdot\hat{\imath} + F = 0 \Rightarrow T\cos(90^\circ+\theta) + F = 0 \quad \Rightarrow$

$\Rightarrow -T\sin(\theta) + F = 0 \quad \Rightarrow F = T\sin(\theta) \quad \Rightarrow F = mg\tan(\theta)$

$F = mg\tan(\theta) \simeq \boxed{mg\cdot\frac{\delta}{\ell}}$ for small δ

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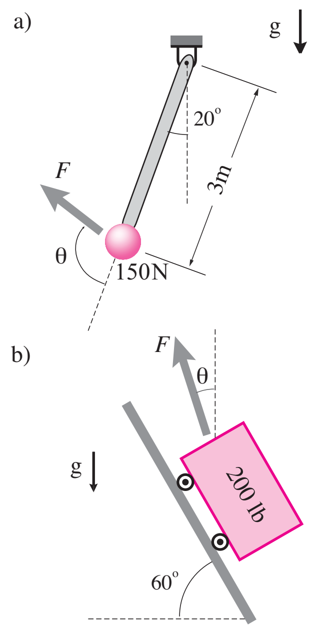
5.1.18 In the situations shown in the figures, find the value of θ that minimizes F. What is the corresponding value of F in each case?



Problem 5.18

Problem $\boxed{4.1.17}$

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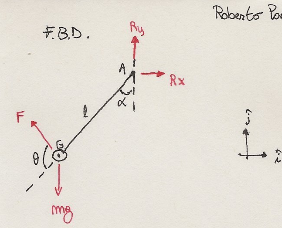
a) $\alpha = 20°$; $l = 3m$



$\sum \vec{M}_{/A} = \vec{0} \Rightarrow$

$\Rightarrow \vec{r}_{G/A} \times \vec{F} + \vec{r}_{G/A} \times (-mg \cdot \hat{j}) = 0 \Rightarrow$

$\Rightarrow l \cdot \sin\theta \cdot F - l \cdot \sin\alpha \cdot mg = 0 \Rightarrow$

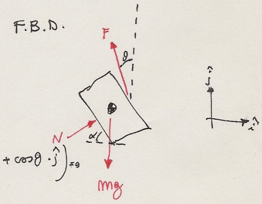
$\Rightarrow F \cdot \sin\theta = mg \cdot \sin\alpha \Rightarrow F = mg \cdot \frac{\sin\alpha}{\sin\theta}$

$\frac{dF}{d\theta} = 0 \Rightarrow mg \cdot \sin\alpha \cdot \frac{d}{d\theta}\left[\sin\theta^{-1}\right] = 0 \Rightarrow (-1) \cdot \frac{1}{\sin^2\theta} \cdot \cos\theta = 0 \Rightarrow$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \begin{cases} \boxed{\pi/2} \\ \cancel{\frac{3}{2}\pi} \to \text{no equilibrium} \end{cases}$

minimum force : $\boxed{F = mg \cdot \sin\alpha}$

b) no friction; N is the result of the two support wheels



$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{N} + \vec{W} + \vec{F} = 0 \Rightarrow$

$\Rightarrow N(\sin\alpha \cdot \hat{i} + \cos\alpha \cdot \hat{j}) - mg \cdot \hat{j} + F(-\sin\theta \cdot \hat{i} + \cos\theta \cdot \hat{j}) = 0$

$\Rightarrow \{\ldots\} \cdot \hat{i} \Rightarrow N \cdot \sin\alpha = F \cdot \sin\theta \Rightarrow N = F \cdot \frac{\sin\theta}{\sin\alpha}$

$\{\ldots\} \cdot \hat{j} \Rightarrow F \cdot \sin\theta \cdot \frac{\cos\alpha}{\sin\alpha} - mg + F \cdot \cos\theta = 0 \Rightarrow$

$\Rightarrow F\left(\frac{\sin\theta}{tg\alpha} + \cos\theta\right) = mg \Rightarrow F = \frac{mg}{\frac{\sin\theta}{tg\alpha} + \cos\theta}$

$\frac{dF}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[(\sin\theta \cdot cotg\alpha + \cos\theta)^{-1}\right] = 0 \Rightarrow \cancel{(-1)}\cancel{(\sin\theta \cdot cotg\alpha + \cos\theta)^{-2}}(\cos\theta \cdot cotg\alpha - \sin\theta) = 0$

$\Rightarrow \cos\theta \cdot \frac{1}{tg\alpha} = \sin\theta \Rightarrow tg\theta = \frac{1}{tg\alpha} \Rightarrow \boxed{\begin{cases} \theta_{min} = 30° \\ F_{min} = mg \cdot \frac{\sqrt{3}}{2} \end{cases}}$

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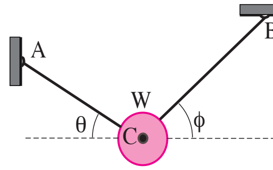
5.1.19 An object of weight $W = 10\,\text{N}$ is held in equilibrium in the vertical plane by two strings AC and BC. Let $\theta = 30^\circ$ and $0 \le \phi \le 90^\circ$. Find and plot the tension in the two strings against ϕ and comment on the variation in the tensions.

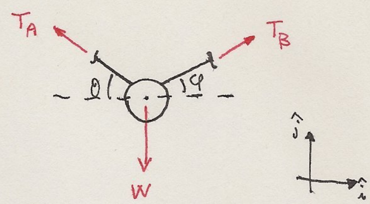


Problem 5.19

Problem $\boxed{4.1.18}$

Roberto Poobrani

F.B.D.



$\vec{T}_A = T_A \cdot \hat{\lambda}_A \quad ; \quad \hat{\lambda}_A = (-\cos\theta \cdot \hat{i} + \sin\theta \hat{j})$

$\vec{T}_B = T_B \cdot \hat{\lambda}_B \quad ; \quad \hat{\lambda}_B = (\cos\varphi \cdot \hat{i} + \sin\varphi \cdot \hat{j})$

$\vec{W} = -mg \cdot \hat{j}$

$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{T}_A + \vec{T}_B + \vec{W} = \vec{0} \quad (1)$

$\{1\} \cdot \hat{i} \Rightarrow -T_A \cdot \cos\theta + T_B \cdot \cos\varphi = 0 \quad (2)$

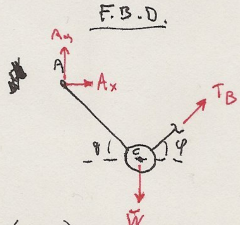
$\{1\} \cdot \hat{j} \Rightarrow T_A \cdot \sin\theta + T_B \cdot \sin\varphi - mg = 0 \quad (3)$

$(2) \Rightarrow T_B = T_A \cdot \dfrac{\cos\theta}{\cos\varphi}$, (3) $\Rightarrow T_A \cdot \sin\theta + T_A \cdot \cos\theta \cdot \operatorname{tg}\varphi = mg \Rightarrow$

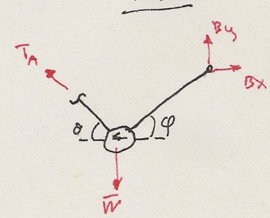
$\Rightarrow T_A = mg \cdot \dfrac{1}{\sin\theta + \cos\theta \cdot \operatorname{tg}\varphi} \quad ; \quad T_B = mg \cdot \dfrac{1}{\cos\varphi \cdot \operatorname{tg}\theta + \sin\varphi}$

alternative solution:

F.B.D.



F.B.D.



$\sum \vec{M}_{/A} = \vec{0} \Rightarrow$

$\Rightarrow \vec{r}_{/A} \times \vec{W} + \vec{r}_{C/A} \times \vec{T}_B = \vec{0}$

$\Rightarrow -mg \cdot \cancel{\ell} \cdot \cos\theta + T_B \cdot \cancel{\ell} \cdot \sin(\theta + \varphi) = 0$

$\Rightarrow T_B = mg \cdot \dfrac{\cos\theta}{\sin(\theta + \varphi)}$

$\sum \vec{M}_{/B} = \vec{0} \Rightarrow \vec{r}_{C/B} \times \vec{W} + \vec{r}_{C/B} \times \vec{T}_A = 0 \Rightarrow -mg\cancel{\ell}\cos\varphi + T_A\cancel{\ell}\sin(\theta+\varphi) = 0$

$\Rightarrow T_A = mg \cdot \dfrac{\cos\varphi}{\sin(\theta + \varphi)}$

they are equivalent to those above, but easier to understand

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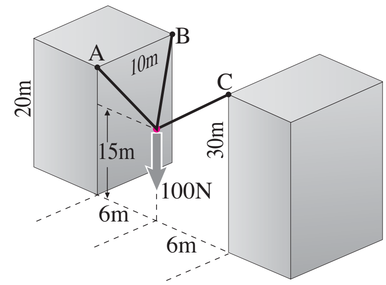
5.1.20 Find the tensions in the three strings shown in the figure.



Problem 5.20

4.1.20 :

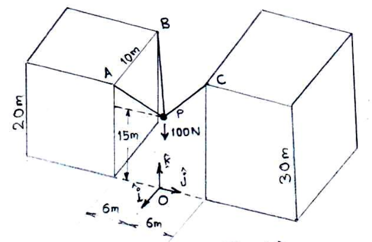


Fig (i)

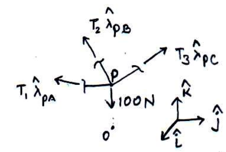


Fig (ii) FBD of point 'P'.

Let us call the point of application of the 100 N external force as 'P'.

If we assume origin of the cooridinate system as shown in Fig (i) and (ii) by the symbol 'O', the cooridinates of points P, A, B, and C can be given as,

P: (0, 0, 15) m

A: (0, -6, 20) m

B: (-10, -6, 20) m

C: (0, 6, 30) m

The unit vectors of direction PA, PB and PC can be given as

$$\hat{\lambda}_{PA} = \frac{\vec{r}_{A/O} - \vec{r}_{P/O}}{|\vec{r}_{A/O} - \vec{r}_{P/O}|} = \frac{0\hat{\imath} - 6\hat{\jmath} + 5\hat{k}}{\sqrt{6^2+5^2}} = \frac{1}{\sqrt{61}}(-6\hat{\jmath} + 5\hat{k})$$

$$\hat{\lambda}_{PB} = \frac{\vec{r}_{B/O} - \vec{r}_{P/O}}{|\vec{r}_{B/O} - \vec{r}_{P/O}|} = \frac{-10\hat{\imath} - 6\hat{\jmath} + 5\hat{k}}{\sqrt{161}}$$

and $$\hat{\lambda}_{PC} = \frac{\vec{r}_{C/O} - \vec{r}_{P/O}}{|\vec{r}_{C/O} - \vec{r}_{P/O}|} = \frac{6\hat{\jmath} + 15\hat{k}}{\sqrt{261}}$$

By force balance, $\Sigma \vec{F} = 0$, we get

$$T_1\hat{\lambda}_{PA} + T_2\hat{\lambda}_{PB} + T_3\hat{\lambda}_{PC} - 100\,\hat{k}\,\text{N} = \vec{0} \quad \text{——— (i)}$$

By doing dot product of equ (i) with $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ we get,

$$\text{(i)}\cdot\hat{\imath} \Rightarrow T_2\left(\frac{-10}{\sqrt{161}}\right) = 0\text{ N} \Rightarrow \boxed{T_2 = 0\text{ N}} \quad \text{——— (ii.a)}$$

$$\text{(ii)}\cdot\hat{\jmath} \Rightarrow T_1\left(\frac{-6}{\sqrt{61}}\right) + T_2\left(-\frac{6}{\sqrt{161}}\right) + T_3\left(\frac{6}{\sqrt{261}}\right) = 0\text{ N} \Rightarrow T_1 = T_3\sqrt{\frac{61}{261}} \quad \text{——— (ii.b)}$$

$$\text{(iii)}\cdot\hat{k} \Rightarrow T_1\left(\frac{5}{\sqrt{61}}\right) + T_2\left(\frac{5}{\sqrt{161}}\right) + T_3\left(\frac{15}{\sqrt{261}}\right) - 100\text{ N} = 0$$

$$\Rightarrow T_3\left(\frac{5}{\sqrt{261}}\right) + T_3\left(\frac{15}{\sqrt{261}}\right) = 100\text{ N} \Rightarrow \boxed{T_3 = 5\sqrt{261}\text{ N}} \quad \text{——— (ii.c)}$$

From (ii.b) $\boxed{T_1 = 5\sqrt{61}\text{ N}}$

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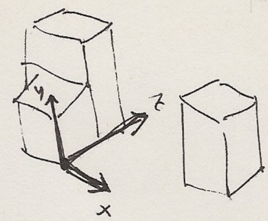
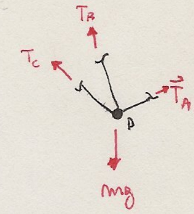




positions: $\vec{r}_A = (14; 15; 5)$ (x; y; z)

$\vec{r}_B = (0; 20; 10)$

$\vec{r}_C = (0; 10; 0)$

$\vec{r}_D = (8; 10; 0)$

$$\vec{W} = -mg\,\hat{j} = 100N\,(0; -1; 0)$$

$$\vec{T}_A = T_A \hat{\lambda}_A \;;\quad \hat{\lambda}_A = \frac{\bar{r}_A - \bar{r}_D}{|\bar{r}_A - \bar{r}_D|} = \frac{1}{\sqrt{86}}(6; 5; 5)$$

$$\vec{T}_B = T_B \hat{\lambda}_B \;;\quad \hat{\lambda}_B = \frac{\bar{r}_B - \bar{r}_D}{|\bar{r}_B - \bar{r}_D|} = \frac{1}{\sqrt{264}}(-8; 10; 10)$$

$$\vec{T}_C = T_C \hat{\lambda}_C \;;\quad \hat{\lambda}_C = \frac{\bar{r}_C - \bar{r}_D}{|\bar{r}_C - \bar{r}_D|} = \frac{1}{8}(-8; 0; 0)$$

$$\sum \vec{F} = \vec{0} \Rightarrow \bar{W} + \bar{T}_A + \bar{T}_B + \bar{T}_C = 0 \Rightarrow$$

$$\Rightarrow \hat{\lambda}_A T_A + \hat{\lambda}_B T_B + \hat{\lambda}_C T_C = mg\,\hat{j}$$

$$\Rightarrow \underbrace{\begin{bmatrix} \frac{6}{\sqrt{86}} & -\frac{8}{\sqrt{264}} & -1 \\ \frac{5}{\sqrt{86}} & \frac{10}{\sqrt{264}} & 0 \\ \frac{5}{\sqrt{86}} & \frac{10}{\sqrt{264}} & 0 \end{bmatrix}}_{\bar{\bar{A}}} \cdot \underbrace{\begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix}}_{\bar{T}} = \underbrace{\begin{pmatrix} 0 \\ 100N \\ 0 \end{pmatrix}}_{\bar{k}} \Rightarrow$$

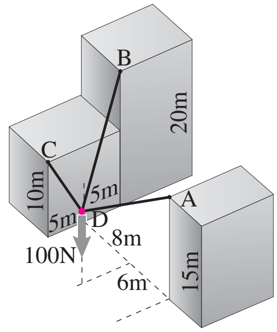
$$\Rightarrow \bar{\bar{A}}^{-1}\bar{\bar{A}}\,\bar{T} = \bar{\bar{A}}^{-1}\bar{k} \Rightarrow \bar{T} = \bar{\bar{A}}^{-1}\bar{k}$$

$$\det(\bar{\bar{A}}) = -1\,\frac{5}{\sqrt{86}}\,\frac{10}{\sqrt{264}} - (-1)\,\frac{5}{\sqrt{86}}\,\frac{10}{\sqrt{264}} = 0$$

$\bar{\bar{A}}^{-1}$ does not exist $\Rightarrow$ the system has no solution $\Rightarrow \sum \bar{F} \neq \vec{0}$

$\Rightarrow$ there is no equilibrium!

5.1.21 Find the tensions in the three strings shown in the figure. String CD is horizontal and the force at D is 100 N straight down. [Hint: this problem has a trick to it.]



Problem 5.21

Answer:
The cables happen to be co-planar and the force is not in that plane. So there is no solution.

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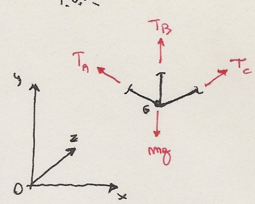
F.B.D.

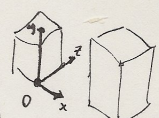


positions: $\vec{r}_G = 6\hat{i} + 15\hat{j}$ $(6;15;0)$

$\vec{r}_A = 20\hat{j}$ $(0;20;0)$

$\vec{r}_B = 20\hat{j} + 10\hat{k}$ $(0;20;10)$

$\vec{r}_C = 12\hat{i} + 30\hat{j}$ $(12;30;0)$



$\vec{T}_A = T_A \hat{\lambda}_A$; $\hat{\lambda}_A = \vec{r}_A - \vec{r}_G = \left(-\frac{6}{\sqrt{61}}; \frac{5}{\sqrt{61}}; 0\right)$

$\vec{T}_B = T_B \hat{\lambda}_B$; $\hat{\lambda}_B = \vec{r}_B - \vec{r}_G = \left(-\frac{6}{\sqrt{161}}; \frac{5}{\sqrt{161}}; \frac{10}{\sqrt{161}}\right)$

$\vec{T}_C = T_C \hat{\lambda}_C$; $\hat{\lambda}_C = \vec{r}_C - \vec{r}_G = \left(\frac{6}{\sqrt{261}}; \frac{15}{\sqrt{261}}; 0\right)$

$\vec{W} = -mg\hat{j}$

$\sum \vec{F}_i = \vec{0} \Rightarrow \vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{W} = \vec{0} \Rightarrow$

$\Rightarrow \hat{\lambda}_A T_A + \hat{\lambda}_B T_B + \hat{\lambda}_C T_C = mg\hat{j} \Rightarrow$

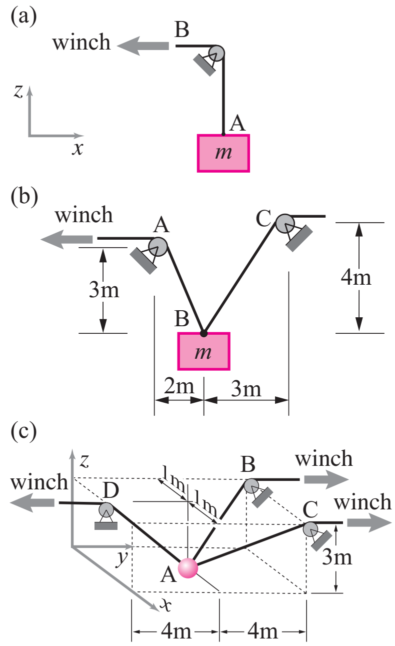
$$\Rightarrow \underbrace{\begin{bmatrix} -\frac{6}{\sqrt{61}} & -\frac{6}{\sqrt{161}} & \frac{6}{\sqrt{261}} \\ \frac{5}{\sqrt{61}} & \frac{5}{\sqrt{161}} & \frac{15}{\sqrt{261}} \\ 0 & \frac{10}{\sqrt{161}} & 0 \end{bmatrix}}_{\bar{\bar{A}}} \cdot \underbrace{\begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix}}_{T} = \underbrace{\begin{pmatrix} 0 \\ mg \\ 0 \end{pmatrix}}_{k}$$

$\Rightarrow \bar{\bar{A}}^{-1} \bar{\bar{A}}\, T = \bar{\bar{A}}^{-1} k \Rightarrow T = \bar{\bar{A}}^{-1} k$

$\det(\bar{\bar{A}}) = \frac{6}{\sqrt{261}} \cdot \frac{5}{\sqrt{61}} \cdot \frac{10}{\sqrt{161}} - \frac{-6}{\sqrt{61}} \cdot \frac{10}{\sqrt{161}} \cdot \frac{15}{\sqrt{261}} \neq 0$

A can be inverted $\Rightarrow$ ... computation ... $\Rightarrow \begin{cases} T_A = 5\sqrt{61} \\ T_B = 0 \\ T_C = 15\sqrt{29} \end{cases}$

5.1.24 For the three cases (a), (b), and (c), below, find the tension in the string AB. In all cases the strings hold up the mass $m = 3\,\text{kg}$. You may assume the local gravitational constant is $g = 10\,\text{m/s}^2$. In all cases the winches are pulling in the string so that the velocity of the mass is a constant $4\,\text{m/s}$ upwards (in the $\hat{\boldsymbol{k}}$ direction). [Note that in problems (b) and (c), in order to pull the mass up at constant rate the winches must pull in the strings at an unsteady speed.]



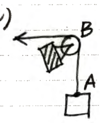
Problem 5.24

Answer:
(a) $T_{AB} = 30\,\text{N}$, (b) $T_{AB} = \frac{300}{17}\,\text{N}$,
(c) $T_{AB} = \frac{5\sqrt{26}}{2}\,\text{N}$

3.28 NOTE: In all three parts of this problem the acceleration of the mass is zero. $\therefore$ Use $(\Sigma \vec{F} = 0)$

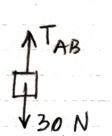
To Find: Tension in string AB for each case.

(a)



$m = 3$ kg,
$g = 10$ m/s^2

FBD



$\therefore \boxed{T_{AB} = 30\text{ N}}$

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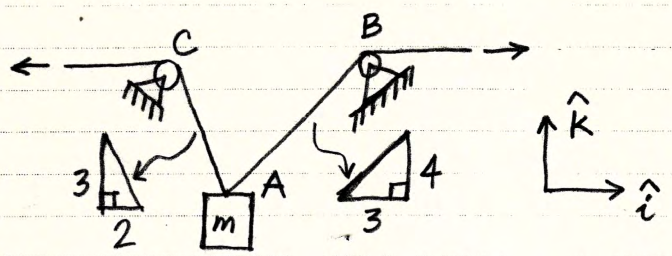
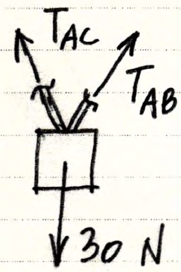
(b) Ignore dimensions of pulleys   Page 8



FBD



$$\vec{AB} = 3\hat{i} + 4\hat{k}$$

$$\hat{\lambda}_{AB} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}$$

$$\vec{AC} = -2\hat{i} + 3\hat{k}$$

$$\hat{\lambda}_{AC} = \frac{-2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{k}$$

$$\sum \vec{F} = 0$$

$$T_{AB}\hat{\lambda}_{AB} + T_{AC}\hat{\lambda}_{AC} - 30\,N\,\hat{k} = 0$$

$$\Rightarrow \sum \vec{F}\cdot\hat{i} = T_{AB}\cdot\frac{3}{5} - T_{AC}\cdot\frac{2}{\sqrt{13}} = 0 \quad -1$$

$$\sum \vec{F}\cdot\hat{k} = T_{AB}\cdot\frac{4}{5} + T_{AC}\cdot\frac{3}{\sqrt{13}} - 30\,N = 0 \quad -2$$

From 1, $T_{AC}\cdot\frac{3}{\sqrt{13}} = \frac{3}{2}\cdot T_{AB}\cdot\frac{3}{5} = \frac{9}{10}T_{AB}$

$$\therefore \left(\frac{4}{5} + \frac{9}{10}\right)T_{AB} = 30\,N$$

$$\boxed{T_{AB} = \frac{300}{17}\,N}$$

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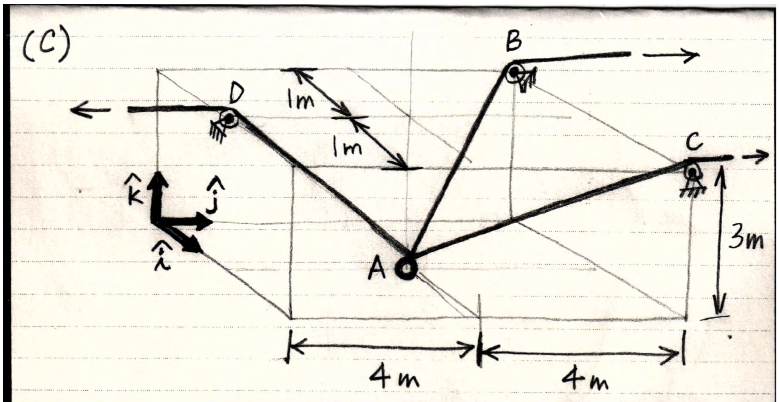

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Relevant unit vectors:

$\vec{AD} = -4\hat{j} + 3\hat{k}$ (m)

$\therefore\ \hat{\lambda}_{AD} = -\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$

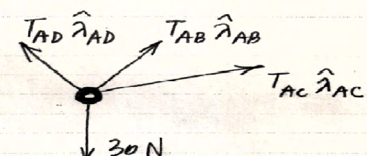
$\vec{AB} = -1.\hat{\imath} + 4\hat{j} + 3\hat{k}$ (m)

$\hat{\lambda}_{AB} = \frac{-1}{\sqrt{26}}\hat{\imath} + \frac{4}{\sqrt{26}}\hat{j} + \frac{3}{\sqrt{26}}\hat{k}$

Similarly (or by symmetry)

$\hat{\lambda}_{AC} = \frac{1}{\sqrt{26}}\hat{\imath} + \frac{4}{\sqrt{26}}\hat{j} + \frac{3}{\sqrt{26}}\hat{k}$

FBD



$\sum \vec{F} = 0$

or $T_{AD}\left(-\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}\right)$

$+ \frac{T_{AB}}{\sqrt{26}}\left(-1\hat{\imath} + 4\hat{j} + 3\hat{k}\right)$

$+ \frac{T_{AC}}{\sqrt{26}}\left(1\hat{\imath} + 4\hat{j} + 3\hat{k}\right) - 30\text{ N}\,\hat{k}$

$= 0$

Solving by equating coefficients, (omitted)

we obtain $\boxed{T_{AB} = \frac{5\sqrt{26}}{2}}$

$T_{AC} = \frac{5\sqrt{26}}{2}$

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