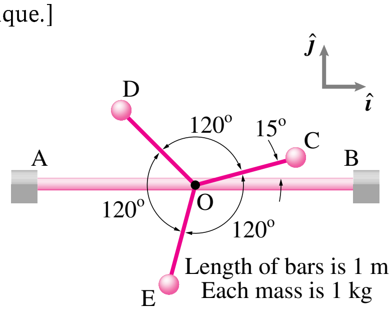
20.5 Dynamic balance

20.5.1 Is $\vec{\omega} = (2\hat{\imath} + 5\hat{k})$ rad/s an eigenvector of $[I^{cm}] = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 16 & -4 \\ 0 & -4 & 8 \end{bmatrix}$ kg m^2?

20.5.2
Find the eigenvalues and eigenvectors of
$[I^{cm}] = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & -3 \\ 0 & -3 & 10 \end{bmatrix}$ kg m^2.

20.5.3 Three equal length (1 m) massless bars are welded to the rigid shaft AB which spins at constant rate $\vec{\omega} = \omega\hat{\imath}$. Three equal masses (1 kg) are attached to the ends of the bars. At the instant of interest the masses are all in the xy plane as shown. Where should what masses be added to make the system shown dynamically balanced? [The solution is not unique.]



Problem 20.5.3

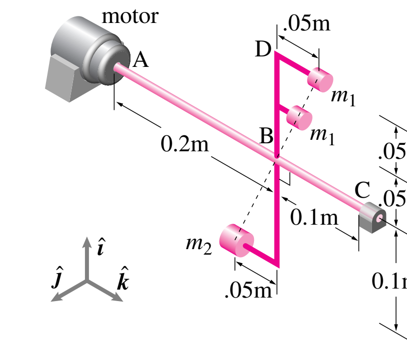
20.5.4 Dynamic balance of a system of particles. Three masses are mounted on a massless shaft AC with the help of massless rigid rods. Mass $m_1 = 1$ kg and mass $m_2 = 1.5$ kg. The shaft rotates with a constant angular velocity $\vec{\omega} = 5\,\text{rad/s}\,\hat{k}$. At the instant shown, the three masses are in the xz-plane. Assume frictionless bearings and ignore gravity.

a) Find the net reaction force (the total of all the forces) on the shaft. Is the system statically balanced?

b) Find the rate of change of angular momentum $\dot{\vec{H}}_{/A}$, using $\dot{\vec{H}}_{/A} = \sum_i \vec{r}_{i/A} \times m_i \vec{a}_i$.

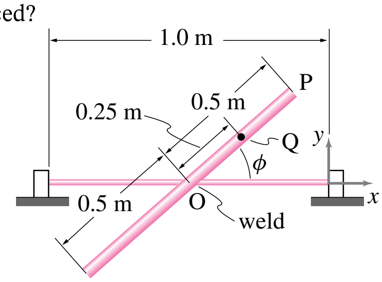
c) Find the angular momentum about point A, $\vec{H}_{/A}$. Use this intermediate result to find the rate of change of angular momentum $\dot{\vec{H}}_{/A}$, using $\dot{\vec{H}}_{/A} = \vec{\omega} \times \vec{H}_{/A}$ and verify the result obtained in (b).

d) What is the value of the resultant unbalanced torque on the shaft? Propose a design to balance the system by adding an appropriate mass in an appropriate location.



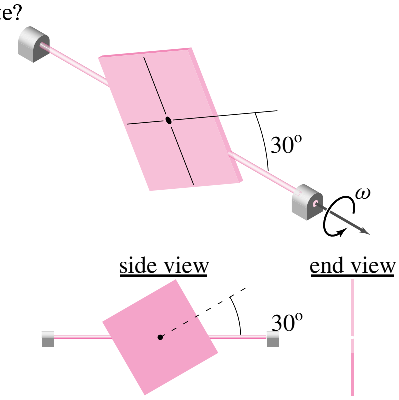
Problem 20.5.4

20.5.5 For what orientation of the rod ϕ is the system shown dynamically balanced?



Problem 20.5.5

20.5.6 A uniform square plate with 1 m sides and 1 kg mass is mounted (as shown) with a shaft that is in the plane of the square but not parallel to the sides. What is the net force and moment required to maintain rotation at a constant rate?



Problem 20.5.6

20.5.7 An equilateral uniform triangular plate with sides of length ℓ plate spins at constant rate ω about an axis through its center of mass. The axis is coplanar with the triangle. The angle ϕ is arbitrary. When $\phi = 0°$, the base of the triangle is parallel to the axis of rotation. Find the net force and moment required to maintain rotation of the plate at a constant rate for any ϕ. For what angle ϕ is the plate dynamically balanced?

Problem 20.5.7

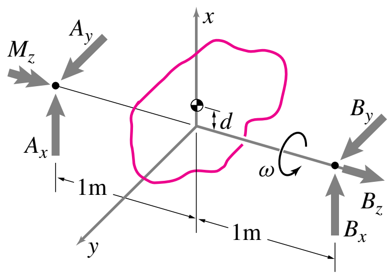
20.5.8 An abstract problem. This problem concerns a system rigidly attached to a massless rod spinning about a fixed axis at a fixed rate. That is, whether or not the system is a rigid body, it is moving as if it were a rigid body. Every point in the system is moving in circles about the same axis at the same number of radians per second. The position of a particle of the system relative to the axis is $\vec{R}$. The position relative to some point C is $\vec{r}_C$. In all cases, the body rotates about the axis shown. The total mass of all the objects is M_{tot}. Point O is at the center of mass of the system and is at a distance d from the rotation axis and is on the x-axis at the moment of interest. Assume there is no gravity. At the point A, the rod is free to slide in the z-direction. But, at point B, the rod is constrained from moving in the z-direction. The FBD of the system is shown in the figure. Referring to the free-body diagram, there are six scalar unknown reactions in this 3 dimensional problem.

a) **The general case:** Imagine that you know $\dot{\vec{H}}_{/O} = \dot{H}_{0x}\hat{\imath} + \dot{H}_{0y}\hat{\jmath} + \dot{H}_{0z}\hat{k}$ and also $\dot{\vec{L}} = \dot{L}_x\hat{\imath} + \dot{L}_y\hat{\jmath} + \dot{L}_z\hat{k}$.

How would you find the six unknown reactions A_x, A_y, M_{Az}, B_x, B_y, B_z? [Reduce the problem to one of solving six equations in six unknowns.]

b) **The General Result:** Is M_{Az} necessarily equal to zero? Prove your result or find a counter-example. *

c) **Static Balance:**. If $d = 0$ is it true that all reaction forces are equal to zero? Prove your result or find a counter example. *



Problem 20.5.8

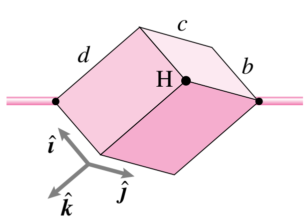
20.5.9 A uniform block spins at constant angular velocity. The magnitude of the angular velocity is ω. The center of mass G is on the axis of rotation. The $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ directions are parallel to the sides with lengths b, c and d, respectively. Answer in terms of some or all of ω, m, b, c, d, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

a) Find $\vec{v}_H$.

b) Find $\vec{a}_H$.

c) What is the *sum* of all the moments about point G of all the reaction forces at A and B?

d) Are there any values of b, c, and d for which this block is dynamically balanced for rotation about an axis through a pair of corners? Why?



Problem 20.5.9

20.5.10 Dynamic Balance. Consider each of the objects shown in turn (so to speak), each spinning about the z axis which is horizontal. In each case the center of mass is half way between the bearings (not shown). Gravity is in the $-\hat{\jmath}$ direction. State whether it is possible to spin the object at constant rate with constant bearing reactions of $mg\hat{\jmath}/2$ *and* no other applied torques ('yes') or might not be ('no'). Give a brief explanation of your answer (enough to distinguish an informed answer from a guess).

a) Axis along centerline of a uniform rectangular plate.

b) Axis orthogonal to uniform rectangular plate and through its centroid.

c) Axis orthogonal to uniform rectangular plate and *not* through its centroid.

d) Axis orthogonal to uniform but totally irregular shaped plate and through its centroid.

e) Axis along the diagonal of a uniform rhombus.

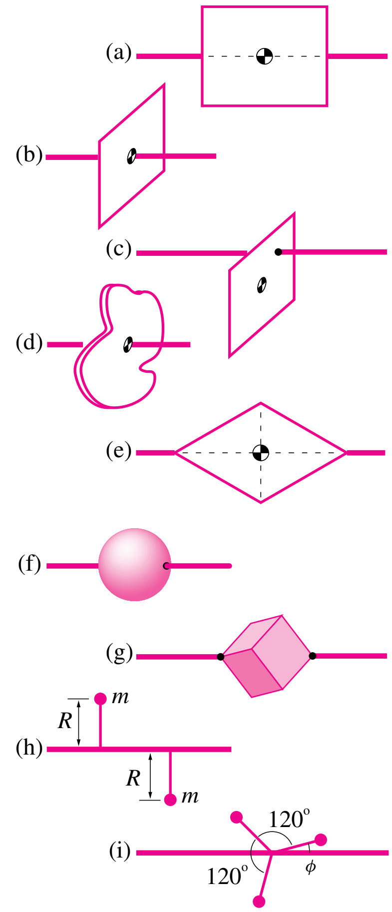
f) Axis through the center of a uniform sphere.

g) Axis through the diagonal of a uniform cube.

h) Axis with two equal masses attached as shown.

i) Axis with three equal masses attached as shown, all in a common plane with the axis, with ϕ not equal to 30°.

j) An arbitrary rigid body which has center of mass on the axis and two of its moment-of-inertia eigenvectors perpendicular to the axis. (no picture)



Problem 20.5.10

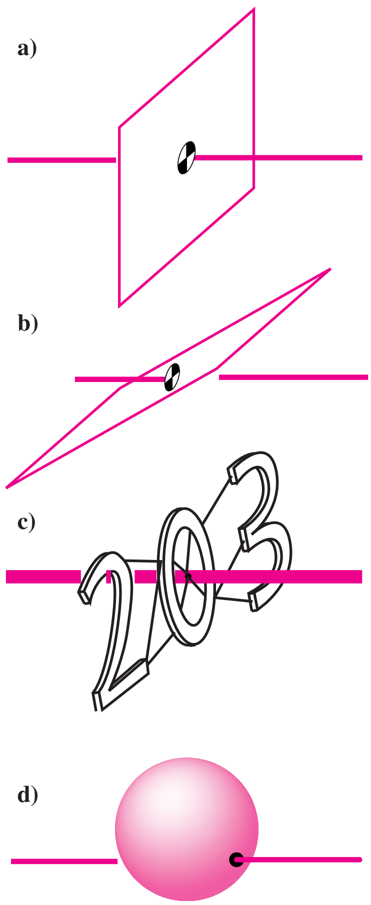
20.5.11 Static and Dynamic Balance A series of bodies, each of uniform density and each with total mass m, rotate at a constant angular speed ω about a fixed horizontal axis. Ignore gravity. For each body state whether the body is (i) *statically* balanced and whether it is (ii) *dynamically* balanced. Give clear arguments using words or equations to support your claims. (iii, iv) For each body you must *either* (a) add one point mass m or (b) add two point masses each of mass $m/2$ (your choice) that *maintain* static and dynamic balance if they are balanced, or that *make* the bodies statically and dynamically balanced. Justify your placement with words and/or equations. The masses need not be added to the bodies, but could be attached off the bodies by structures with negligible mass. [Hint: none of the placements are unique. You may draw a side view if that helps clarify your placement.]

a) A rectangular plate (height h, length ℓ) mounted with the axle perpendicular to the plate and through its center.

b) The same plate as in (a) above but mounted at an angle $\phi \neq \pi/2$ from the shaft.

c) The numerals '203' cut out of a plate and connected by massless rods. Each letter has mass $m/3$ and the three center-of-mass points of the individual letters are colinear and equally spaced. The shaft goes through the center of the '0' and is perpendicular to the plane of the letters.

d) A sphere with radius R where the shaft passes a distance $d < R$ from the center.

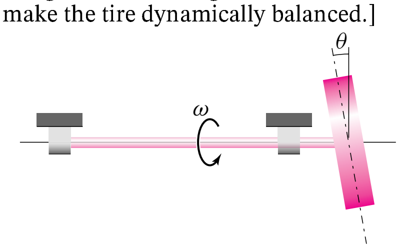


Problem 20.5.11

20.5.12 Dynamic tire balancing. Where should you place what weights in order to dynamically balance the tire?

Assume that a car tire can be modeled as a uniform disk of radius one foot and mass 20 lbm. Assume the tire has been mounted crooked by one degree (the plane of the disk makes an 89° angle with the car axle). This crooked wheel is *statically* balanced since its center of mass is on the axis of rotation. But it is dynamically unbalanced since it wobbles and will wobble the car. Car shops put little weights on unbalanced wheels to balance them. They figure out where to put the weights by using a little machine that measures the wheel wobble. But you don't need such a machine for this problem because you have been told where all the mass is located.

There are many approaches to solving this problem and there are also many correct solutions. You should find any correct solution. [one approach: guess a good location to put two masses and then figure out how big they have to be to make the tire dynamically balanced.]



Problem 20.5.12