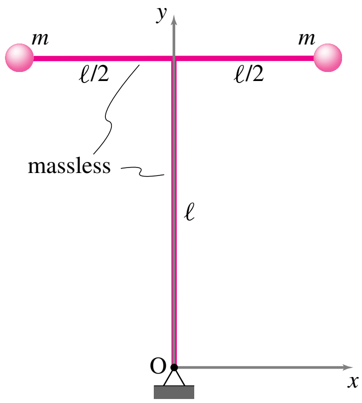
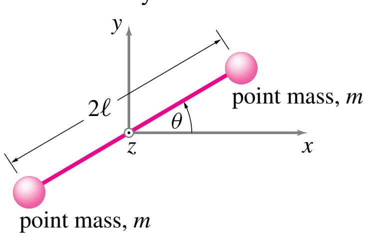
20.3 Moment of inertia matrices: $[I^{\text{cm}}]$ and $[I^{\text{O}}]$

20.3.1 Find the moment of inertia matrix $[\boldsymbol{I}^{\text{O}}]$ of the system shown in the figure, given that $m = 0.25\,\text{kg}$ and $\ell = 0.5\,\text{m}$. Which components in the matrix $[\boldsymbol{I}^{\text{O}}]$ will change if one mass is removed and the other one is doubled?



Problem 20.3.1

20.3.2 A dumbbell (two masses connected by a massless rod of length ℓ) lies in the xy plane. Compute the moments and products of inertia of this body about the x, y, and z axes. What are the principle axes of this body?



Problem 20.3.2

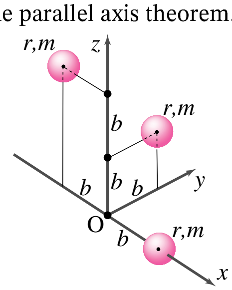
20.3.3 Equivalent dumbbell. It is a great fact that, for the purposes of dynamics, every rigid body can be replaced by a massless rigid structure holding six masses. The structure looks like a child's jack. It has three orthogonal bars with three pairs of point masses along the bars. The center of the jack is at the center of mass. The bars are in the directions of the eigenvectors of the moment of inertia matrix of the body (the principle directions). For flat objects one only needs four point masses. Where should one put four masses on a massless disk to make it equivalent to a uniform and 'massful' disk in problem 20.3.6(a). That is, what set of four masses has the same total mass, the same center of mass location, and the same moment of inertia matrix as a uniform disk?

20.3.4 Three identical solid spheres of mass m and radius r are situated at $(b, 0, 0)$, $(0, b, b)$, and $(-b, 0, 2b)$ relative to the origin O. Find all nine components of $[\boldsymbol{I}^{\text{O}}]$ of the system of three spheres.

Hint: $[\boldsymbol{I}^{\text{cm}}]$ for each sphere is

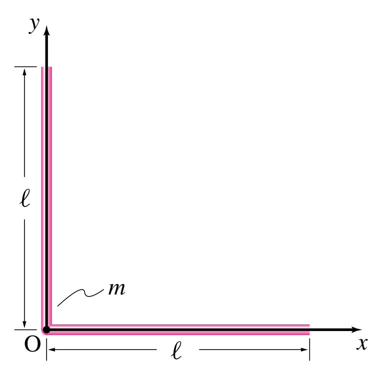
$$\begin{bmatrix} \frac{2}{5}mr^2 & 0 & 0 \\ 0 & \frac{2}{5}mr^2 & 0 \\ 0 & 0 & \frac{2}{5}mr^2 \end{bmatrix}.$$

Use the parallel axis theorem.



Problem 20.3.4

20.3.5 Find the moment of inertia matrix $[\boldsymbol{I}^{\text{O}}]$ for the 'L'-shaped rod shown in the figure. The mass of the rod is $m = 1.5\,\text{kg}$ and each leg is $\ell = 0.4\,\text{m}$ long. You may ignore the width of the rod.



Problem 20.3.5

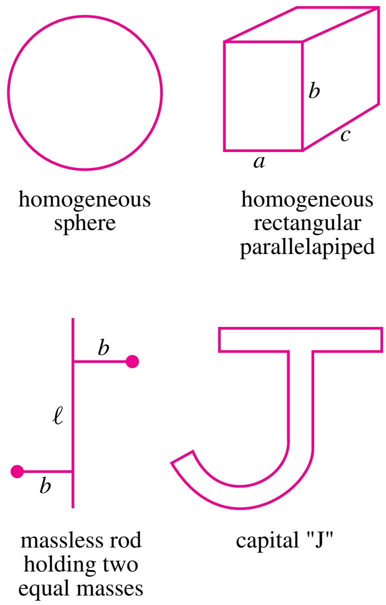
20.3.6 Moment of Inertia Matrix. This problem concerns a uniform flat disk of radius R_0 with mass $M_d = M_{disc}$. In the two parts of this problem, you are to calculate the moment of inertia matrix for the disk. In the first part, do it by direct integration and, in the second part, by using a change of basis.

a) Using a coordinate system that is lined up with the disk so that the z axis is normal to the disk and the origin of the coordinate system is at the center of mass (the center) of the disk, calculate the moment of inertia matrix for the disk. [Hint: use polar coordinates, and use $dm = (M/\pi R_0^2)\,dA$, and $dA = R\,dR\,d\theta$.]

b) Use a coordinate system $x'y'z'$ aligned as follows: the origin of the coordinate system is at the center of the disk, the z' axis makes an angle ϕ with the z axis which is normal to the plane of the disk (measured from the z axis towards the z' axis), the disk intersects the $x'y'$ plane along the y' axis. The y axis and the y' axis are coincident. Calculate the moment of inertia matrix for the disk. Alternate method: use the solution from (a) above with a change of basis.

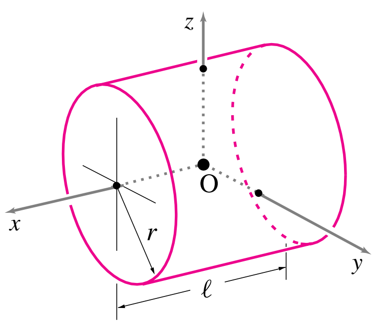
Problem 20.3.6

20.3.7 For each of the shapes below, mark the approximate location of the center of mass and the principle axes about the center of mass



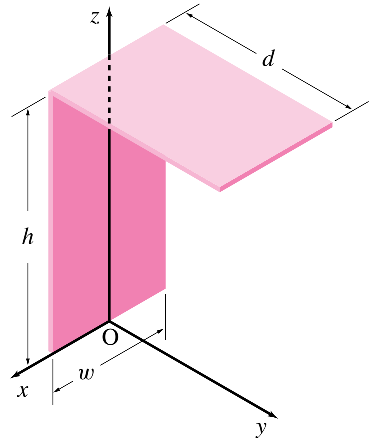
Problem 20.3.7

20.3.8 Find the moment of inertia matrix $[\boldsymbol{I}^{\text{cm}}]$ for the uniform solid cylinder of length ℓ and radius r shown in the figure.



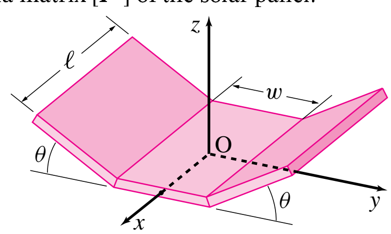
Problem 20.3.8

20.3.9 An 'L'-shaped structure consists of two thin uniform rectangular plates each of width $w = 200\,\text{mm}$. The vertical plate has height $h = 400\,\text{mm}$ and the horizontal plate has depth $d = 300\,\text{mm}$. The plates are welded at right angles along the side measuring 200 mm. The total mass of the structure is $m = 0.7\,\text{kg}$. Find the moment of inertia matrix $[\boldsymbol{I}^{\text{O}}]$.



Problem 20.3.9

20.3.10 A solar panel consists of three rectangular plates each of mass $m = 1.2\,\text{kg}$, length $\ell = 1$ and width $w = 0.5\,\text{m}$ The two side panels are inclined at 30° with the plane of the middle panel. Evaluate all components of the moment of inertia matrix $[\boldsymbol{I}^{\text{O}}]$ of the solar panel.



Problem 20.3.10