CHAPTER 20

Fixed-axis rotation in 3D

The ideas of chapter 10 on 2D circular motion are extended to fixed axis rotation in 3D. The key difference here is the non-trivial use of the cross product for calculating velocities and accelerations. Fixed axis rotation is the simplest motion with which one can introduce the full moment of inertia matrix, where the diagonal terms are analogous to the scalar 2D moment of inertia and the off-diagonal terms have a "centripetal" interpretation. The main new application is dynamic balance.

In chapter 10 we discussed the motions of particles and rigid bodies that lie in a plane and rotate about a fixed axis perpendicular to that plane. In this chapter we again are going to think about fixed axis rotation, but now in three dimensions. The axis of rotation might be in any skew direction and the rotating bodies might be arbitrarily complicated three-dimensional shapes. As a cartoon, imagine a rigid body skewered with a rigid rod and then turned by a motor that speeds up and slows down. More practically think of the crankshaft in a car engine (fig. 20.1). Other

Figure 20.1: A car crankshaft is a complex three-dimensional object which is well approximated for many purposes as rotating about a fixed axis. The relative timing of the reciprocating pistons is controlled by this complex shape. "Connecting rods" are pinned to the cylinders at one end and to the short offset cylinders on the crankshaft at the other.

applications include accelerating or decelerating shafts of all kinds, gears, turbines, flywheels, pendula, and swinging doors.

To understand this motion we need to take a little more care with the kinematics because it now involves three dimensions, although in some sense the basic ideas are unchanged from the previous two-dimensional chapter. The three-dimensional mechanics naturally gets more involved.

This one special motion, rotation about a fixed axis, serves as our introduction to three-dimensional rigid body mechanics.

As for all motions of all systems, the momentum balance equations apply to any system or any part of a system that has fixed axis rotation. So our mechanics results will be based on these familiar equations:

$$\text{Linear momentum balance:} \qquad \sum \vec{\boldsymbol{F}}_i = \dot{\vec{\boldsymbol{L}}},$$

$$\text{Angular momentum balance:} \qquad \sum \vec{\boldsymbol{M}}_{i/O} = \dot{\vec{\boldsymbol{H}}}_{/\mathrm{O}}.$$

and

$$\text{Power balance:} \qquad P = \dot{E}_{\mathrm{K}}.$$

As always, we will evaluate the left hand sides of the momentum equations using the forces and moments in the free-body diagram. We evaluate the right hand sides of these equations using our knowledge of the velocities and accelerations of the various mass points.

The chapter starts with a discussion of kinematics. Then we consider the mechanics of systems with fixed axis rotation. The moment of inertia matrix is then introduced followed by a section where the moment of inertia is used as a shortcut in the evaluation of $\dot{\vec{\boldsymbol{H}}}_{/\mathrm{O}}$. Finally we discuss dynamic balance, an important genuinely three-dimensional topic in machine design.