

19.2.1 A conservative vibratory system has the following equation of conservation of energy.

$$m(\ell \dot{\theta})^2 - mg\ell(1 - \cos \theta) + K(a\theta)^2 = E_0$$

where E_0 is a constant.

- a) Obtain the differential equation of motion of this system by differen-

tiating this energy equation with respect to t .

- b) Determine the circular frequency of small oscillations of the system in part (a). HINT: (Let $\sin \theta \cong \theta$ and $\cos \theta \cong 1 - \theta^2/2$).

18.2.1

Given, the equation of conservation of energy:

$$m(\ell \dot{\theta})^2 - mg\ell(1 - \cos \theta) + K(a\theta)^2 = E_0$$

Differentiating w.r.t. time

$$2m\ell \dot{\theta} \ddot{\theta} - mg\ell(+\sin \theta) \dot{\theta} + 2K(a\theta) a \dot{\theta} = 0$$

$$\Rightarrow \ddot{\theta} = \frac{mg\ell \sin \theta - 2Ka^2 \theta}{2m\ell}$$

Differential equation of motion is

$$\ddot{\theta} + \frac{K}{m} \left(\frac{a^2}{\ell} \right) \theta - \frac{g}{2\ell} \sin \theta$$

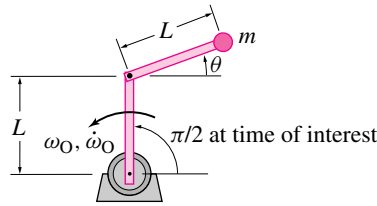
- b) As $\sin \theta \approx \theta$ for small oscillations, equation of motion can be modified as follows.

$$\ddot{\theta} + \left\{ \frac{K}{m} \left(\frac{a^2}{\ell} \right) - \frac{g}{2\ell} \right\} \theta = 0$$

Drawing analogy with $\ddot{\theta} + \tilde{\lambda} \theta = 0$, the circular frequency,

of small oscillations, $\lambda = \sqrt{\frac{K}{m} \left(\frac{a^2}{\ell} \right) - \frac{g}{2\ell}}$ Provided, $\frac{Ka^2}{m\ell} > \frac{g}{2\ell}$

19.2.2 A motor at O turns at rate ω_o , whose rate of change is $\dot{\omega}_o$. At the end of a stick connected to this motor is a frictionless hinge attached to a second massless stick. Both sticks have length L . At the end of the second stick is a mass m . For the configuration shown, what is $\ddot{\theta}$? Answer in terms of ω_o , $\dot{\omega}_o$, L , m , θ , and $\dot{\theta}$. Ignore gravity.



Problem 19.2

18.2.2

Applying LMB on 'mass, m': $\sum \underline{F} = m \underline{a}$

$$-T \hat{e}_r = m [\underline{a}_{O'} + \underline{\dot{\theta}} \times \underline{r}_{A/O'} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r}_{A/O'})]$$

$$\underline{a} = \underline{a}_{O'} + \ddot{\theta} L \hat{e}_\theta - L \dot{\theta}^2 \hat{e}_r$$

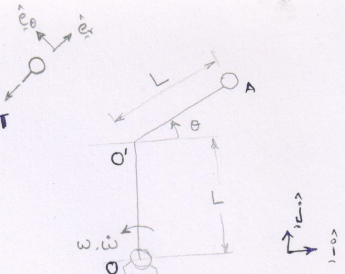
$$\underline{a}_{O'} = \underline{a}_O + \dot{\omega}_o \times \underline{r}_{O'/O} + \omega_o \times (\omega_o \times \underline{r}_{O'/O})$$

$$= -\dot{\omega}_o L \hat{i} - L \omega_o^2 \hat{j}$$

$$\therefore \{-T \hat{e}_r = m [-\dot{\omega}_o L \hat{i} - L \omega_o^2 \hat{j} + \ddot{\theta} L \hat{e}_\theta - L \dot{\theta}^2 \hat{e}_r]\}$$

$$\{ \} \cdot \hat{e}_\theta \Rightarrow 0 = m [-\dot{\omega}_o L \sin \theta - L \omega_o^2 \cos \theta + \ddot{\theta} L]$$

$$\therefore \ddot{\theta} = \omega_o^2 \cos \theta - \dot{\omega}_o \sin \theta //$$



$$\hat{e}_r = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{e}_\theta = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

18.2.4

Let $\underline{T}$ be the net force acting on the mass at 'P'.Using L.M.B. $\sum \underline{F} = m \underline{a}$

$$\underline{T} = \underline{F}(-\hat{i}) + mg(-\hat{j}) = m \underline{a}_P$$

$$\underline{a}_P = \underline{a}_B + \underline{\dot{\omega}}_{BC} \times \underline{r}_2 + \underline{\omega}_{BC} \times (\underline{\omega}_{BC} \times \underline{r}_2)$$

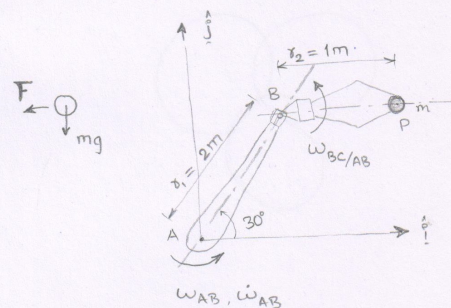
$$= \underline{\dot{\omega}}_{AB} \times \underline{r}_1 + \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_1) + \underline{\dot{\omega}}_{AB} \hat{j} - \omega_{BC}^2 \hat{i}$$

$$\underline{a}_P = 2\dot{\omega}_{AB} r_1 \hat{e}_\theta - r_1 \omega_{AB}^2 \hat{e}_r - \omega_{BC}^2 \hat{i} + \dot{\omega}_{AB} \hat{j}$$

$$\therefore \underline{T} = m(-r_1 \dot{\omega}_{AB} \sin 30^\circ - \omega_{BC}^2 - r_1 \omega_{AB}^2 \cos 30^\circ) \hat{i} + m(r_1 \dot{\omega}_{AB} \cos 30^\circ - r_1 \omega_{AB}^2 \sin 30^\circ + \dot{\omega}_{AB}) \hat{j}$$

$$\text{Given, } r_1 = 2\text{ m, } \omega_{AB} = 5 \text{ rad/s, } \dot{\omega}_{AB} = 2 \text{ rad/s}^2; m = 1\text{ kg, } \omega_{BC} = \omega_{BC/AB} + \omega_{AB} = 8 \text{ rad/s}$$

$$\Rightarrow \underline{T} = -109.3 \hat{i} - 19.5 \hat{j} //$$



$$\hat{e}_r = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}$$

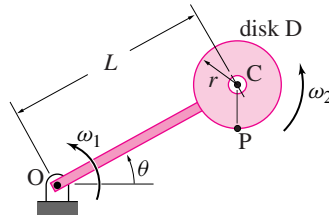
$$\hat{e}_\theta = -\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j}$$

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19.2.3 In problem ??, find

- the angular momentum about point O ,
- the rate of change of angular momentum of the disk about point O ,
- the angular momentum about point C ,
- the rate of change of angular momentum of the disk about point C .

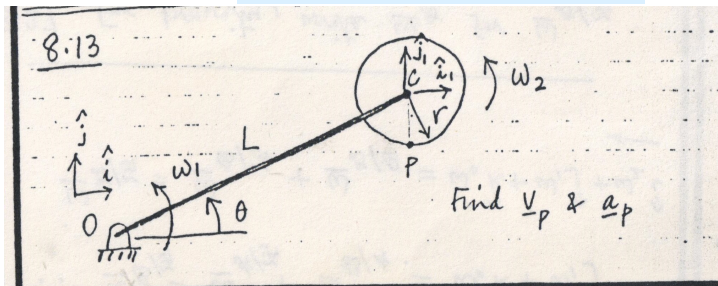
Assume the rod is massless and the disk has mass m .



Problem 19.3

Answer:

- $\dot{\mathbf{H}}_O = [mL^2\omega_1 + \frac{1}{2}mr^2(\omega_1 + \omega_2)]\hat{\mathbf{k}}$.
- $\dot{\mathbf{H}}_O = \mathbf{0}$.
- $\dot{\mathbf{H}}_C = \frac{1}{2}mr^2(\omega_1 + \omega_2)\hat{\mathbf{k}}$.
- $\dot{\mathbf{H}}_C = \mathbf{0}$.



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Attach a moving frame at point C , with unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$, called $\mathcal{B}$.

a) First, assume that the moving frame does not rotate (it just translates; unit vectors don't change direction).

$$\omega_{\text{disk}/\mathcal{B}} = (\omega_1 + \omega_2)\hat{\mathbf{k}}$$

$$\therefore \mathbf{v}_{P/\mathcal{B}} = (\omega_1 + \omega_2)\hat{\mathbf{k}} \times \mathbf{r}_{CP}$$

$$= (\omega_1 + \omega_2)r\hat{\mathbf{k}} \times (-\hat{\mathbf{j}}_1)$$

$$= (\omega_1 + \omega_2)r\hat{\mathbf{i}} \quad \leftarrow$$

$$\mathbf{v}_C = \omega_1\hat{\mathbf{k}} \times L(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$$

$$= \omega_1 L(\cos\theta\hat{\mathbf{j}} - \sin\theta\hat{\mathbf{i}})$$

$$\mathbf{v}_P = \mathbf{v}_C + \mathbf{v}_{P/\mathcal{B}} + \omega_{\mathcal{B}}^0 \times \mathbf{r}_{CP}$$

$$\mathbf{v}_P = \omega_1 L(\cos\theta\hat{\mathbf{j}} - \sin\theta\hat{\mathbf{i}}) + (\omega_1 + \omega_2)r\hat{\mathbf{i}}$$

$$\quad \quad \quad \omega_1 L \cos\theta \hat{\mathbf{j}} - \omega_1 L \sin\theta \hat{\mathbf{i}} + (\omega_1 + \omega_2)r\hat{\mathbf{i}}$$

$$\mathbf{a}_{P/\mathcal{B}} = (\omega_1 + \omega_2)\hat{\mathbf{k}} \times [(\omega_1 + \omega_2)\hat{\mathbf{k}} \times \mathbf{r}_{CP}]$$

$$= (\omega_1 + \omega_2)^2 r\hat{\mathbf{j}}$$

$$\mathbf{a}_C = \omega_1\hat{\mathbf{k}} \times [\omega_1\hat{\mathbf{k}} \times L(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})]$$

$$\mathbf{a}_P = \mathbf{a}_C + \mathbf{a}_{P/\mathcal{B}} + \text{bunch of zeros}$$

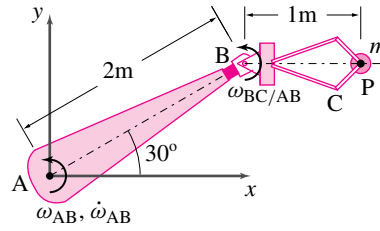
$$\mathbf{a}_P = -\omega_1^2 L(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}) + (\omega_1 + \omega_2)^2 r\hat{\mathbf{j}}$$

b) Next, consider the situation where moving frame rotates with the rod, i.e., $\omega_{\mathcal{B}} = \omega_1\hat{\mathbf{k}}$ and $\omega_{\text{disk}/\mathcal{B}} = \omega_2\hat{\mathbf{k}}$

$$\therefore \mathbf{v}_{P/\mathcal{B}} = \omega_2\hat{\mathbf{k}} \times \mathbf{r}_{CP} = \omega_2 r\hat{\mathbf{i}}$$

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19.2.4 Robot arm, 2-D. The robot arm AB is rotating about point A with $\omega_{AB} = 5 \text{ rad/s}$ and $\dot{\omega}_{AB} = 2 \text{ rad/s}^2$. Meanwhile the forearm BC is rotating at a constant angular speed with respect to AB of $\omega_{BC/AB} = 3 \text{ rad/s}$. Gravity cannot be neglected. At the instant shown, find the net force acting on the object P which has mass $m = 1 \text{ kg}$.



Problem 19.4

Answer:

$$\vec{F} = m\vec{a} = -109.3 \text{ N}\hat{i} - 19.54 \text{ N}\hat{j}.$$

18.2.2

Applying LMB on 'mass, m': $\sum \vec{F} = m\vec{a}$

$$-T\hat{e}_r = m[\ddot{r}_{A/O'} + \ddot{\theta} \times r_{A/O'} + \dot{\theta} \times (\dot{\theta} \times r_{A/O'})]$$

$$\ddot{r} = \ddot{r}_{O'} + \ddot{\theta} L \hat{e}_\theta - L \ddot{\theta} \hat{e}_r$$

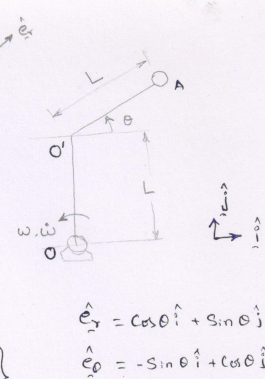
$$\ddot{r}_{O'} = \ddot{r}_{O'} + \dot{\omega}_0 \times r_{O'/O} + \omega_0 \times (\omega_0 \times r_{O'/O})$$

$$= -\dot{\omega}_0 L \hat{i} - L \dot{\omega}_0 \hat{j}$$

$$\therefore \{-T\hat{e}_r = m[-\dot{\omega}_0 L \hat{i} - L \dot{\omega}_0 \hat{j} + \ddot{\theta} L \hat{e}_\theta - L \ddot{\theta} \hat{e}_r]\}$$

$$\{ \} \cdot \hat{e}_\theta \Rightarrow 0 = m[-\dot{\omega}_0 L (\sin \theta) - L \dot{\omega}_0 \cos \theta + \ddot{\theta} L]$$

$$\therefore \ddot{\theta} = \dot{\omega}_0 \cos \theta - \dot{\omega}_0 \sin \theta //$$



$$\hat{e}_r = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{e}_\theta = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

18.2.4

Let $\vec{T}$ be the net force acting on the mass at 'P'.Using L.M.B. $\sum \vec{F} = m\vec{a}$

$$\vec{T} = \vec{F}(-\hat{i}) + mg(-\hat{j}) = m\vec{a}_P$$

$$\vec{a}_P = \vec{a}_B + \dot{\omega}_{BC} \times \vec{r}_2 + \dot{\omega}_{BC} \times (\dot{\omega}_{BC} \times \vec{r}_2)$$

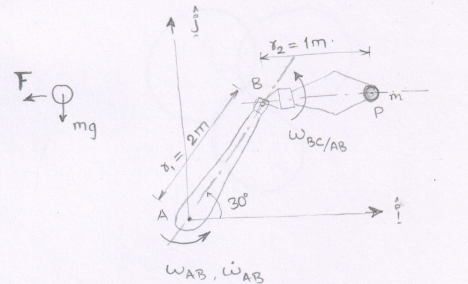
$$= \dot{\omega}_{AB} \times \vec{r}_1 + \dot{\omega}_{AB} \times (\dot{\omega}_{AB} \times \vec{r}_1) + \dot{\omega}_{AB} \hat{j} - \dot{\omega}_{BC} \hat{i}$$

$$\vec{a}_P = 2\dot{\omega}_{AB} \vec{r}_1 \hat{e}_\theta - \vec{r}_1 \dot{\omega}_{AB} \hat{e}_r - \dot{\omega}_{BC} \hat{i} + \dot{\omega}_{AB} \hat{j}$$

$$\therefore \vec{T} = m(-\vec{r}_1 \dot{\omega}_{AB} \sin 30^\circ - \dot{\omega}_{BC} - \vec{r}_1 \dot{\omega}_{AB} \cos 30^\circ) \hat{i} + m(\vec{r}_1 \dot{\omega}_{AB} \cos 30^\circ - \vec{r}_1 \dot{\omega}_{AB} \sin 30^\circ + \dot{\omega}_{AB}) \hat{j}$$

$$\text{Given: } \vec{r}_1 = 2\text{m}, \omega_{AB} = 5 \text{ rad/s}, \dot{\omega}_{AB} = 2 \text{ rad/s}^2; m = 1\text{kg}, \omega_{BC} = \omega_{BC/AB} + \omega_{AB} = 8 \text{ rad/s}$$

$$\Rightarrow \vec{T} = -109.3 \hat{i} - 19.5 \hat{j}$$



$$\hat{e}_r = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}$$

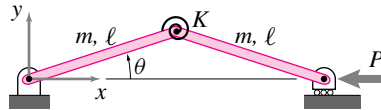
$$\hat{e}_\theta = -\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j}$$

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19.2.5 A crude model for a column, shown in the figure, consists of two identical rods of mass m and length ℓ with hinge connections, a linear torsional spring of stiffness K attached to the center hinge (the spring is relaxed when $\theta = 0$), and a load P applied at the top end.

quency for small motions θ .

- c) Check the stability of the straight equilibrium state $\theta = 0$ for all $P \geq 0$ via minimum potential energy. How do these results compare with those from part(b)?



Problem 19.5

- a) Obtain the exact nonlinear equation of motion.
b) Obtain the squared natural fre-

18.2.5

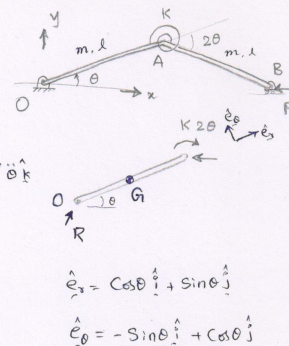
Applying A.M.B about 'O': $\sum \vec{M}_O = \vec{H}_O$

$$\vec{r}_A \times \vec{P} - K(2\theta)\hat{k} = \vec{r}_G \times m\vec{a}_G + \frac{1}{12}m\ell^2\ddot{\theta}\hat{k}$$

$$1\dot{\vec{e}}_2 \times P(-\hat{i}) - 2K\theta\hat{k} = \frac{\ell}{2}\vec{e}_2 \times m\left[-\omega^2\frac{\ell}{2}\vec{e}_2 + \ddot{\theta}\frac{\ell}{2}\vec{e}_2\right] + \frac{1}{12}m\ell^2\ddot{\theta}\hat{k}$$

$$\Rightarrow \left\{ (Pl \sin\theta - 2K\theta)\hat{k} = \left(\frac{m\ell^2}{4}\hat{k} + \frac{m\ell^2}{12}\hat{k} \right) \ddot{\theta} \right\}$$

$$\left\{ \right\} \cdot \hat{k} \Rightarrow \frac{1}{3}m\ell^2\ddot{\theta} + 2K\theta - Pl \sin\theta = 0$$



$$\begin{aligned}\hat{e}_2 &= \cos\theta\hat{i} + \sin\theta\hat{j} \\ \dot{\hat{e}}_2 &= -\sin\theta\dot{\theta}\hat{i} + \cos\theta\dot{\theta}\hat{j}\end{aligned}$$

b) For small oscillations, $\sin\theta \approx \theta$

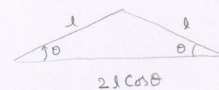
$$\Rightarrow \frac{1}{3}m\ell^2\ddot{\theta} + (2K - Pl)\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{3(2K - Pl)}{m\ell^2}\theta = 0$$

$$\therefore \text{Squared natural frequency, } \lambda = \frac{3(2K - Pl)}{m\ell^2}$$

c) Energy stored in the spring, E_{spring}

$$E_{\text{spring}} = \frac{1}{2}K(2\theta)^2 = 2K\theta^2$$



Work done by the load 'P' in moving a distance, d , is

$$W = P \cdot d = P(2\ell - 2\ell \cos\theta)$$

$$\Rightarrow W = 2Pl(1 - \cos\theta) = 2Pl\left(1 - \left(1 - \frac{\theta^2}{2} + \dots\right)\right) \approx Pl\theta^2$$

$$\therefore \text{Potential Energy} = E_{\text{spring}} - W$$

System becomes unstable if $W - E_{\text{spring}} > 0$.

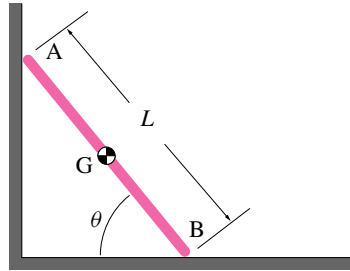
For $P \geq 0$, $E_{\text{spring}} - W > 0$ results in a stable system.

Also, $2K - Pl > 0$ satisfies the requirement in part(b)

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19.2.6 A thin uniform rod of mass m rests against a frictionless wall and on a frictionless floor. There is gravity.

- Draw a free-body diagram of the rod.
- The rod is released from rest at $\theta = \theta_0 \neq 0$. Write the equation of motion of the rod.
- Using the equation of motion, find the initial angular acceleration, $\dot{\omega}_{AB}$, and the acceleration of the center of mass, $\ddot{\mathbf{a}}_G$, of the rod.
- Find the reactions on the rod at points A and B .
- Find the acceleration of point B .
- When $\theta = \frac{\theta_0}{2}$, find $\ddot{\omega}_{AB}$ and the acceleration of point A .



Problem 19.6

Answer:

(b) $\ddot{\theta} = -\frac{3g}{2L} \cos \theta$.

(c) From the diagram, we see $\dot{\omega}_{AB} = -\ddot{\theta}(t = 0)\hat{\mathbf{k}} = \frac{3g}{2L} \cos \theta_0$, $\ddot{\mathbf{a}}_G = \frac{3}{4}g \cos \theta_0 [\sin \theta_0 \hat{\mathbf{i}} - \cos \theta_0 \hat{\mathbf{j}}]$.

(d) $\ddot{\mathbf{R}}_A = \frac{3}{4}mg \sin \theta_0 \cos \theta_0 \hat{\mathbf{i}}$, $\ddot{\mathbf{R}}_B = mg[1 - \frac{3}{4}g \cos^2 \theta_0] \hat{\mathbf{j}}$.

(e) $\ddot{\mathbf{a}}_B = \frac{3}{2}g \sin \theta_0 \cos \theta_0 \hat{\mathbf{i}}$.

(f) At $\theta = \frac{\theta_0}{2}$, $\ddot{\omega}_{AB} = \sqrt{\frac{3g}{L}(\sin \theta_0 - \sin \frac{\theta_0}{2})} \hat{\mathbf{k}}$, $\ddot{\mathbf{a}}_A = -3g \left[\frac{1}{2} \cos^2 \frac{\theta_0}{2} + \sin \frac{\theta_0}{2} (\sin \theta_0 - \sin \frac{\theta_0}{2}) \right] \hat{\mathbf{j}}$.

19.2.6

a) Free body diagram of rod:

b) Given, initial $\theta = \theta_0 \neq 0$; $\dot{\mathbf{r}}_G = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{G/B}$

$\dot{\mathbf{r}}_B = L \cos \theta \hat{\mathbf{i}}$ and $\dot{\mathbf{r}}_{G/B} = \frac{L}{2} \cos \theta (-\dot{\theta}) \hat{\mathbf{i}} + \frac{L}{2} \sin \theta \dot{\theta} \hat{\mathbf{j}}$

Apply L.M.B., $\Sigma \mathbf{F} = m \mathbf{a}_G$

$\Rightarrow \{ R_A \hat{\mathbf{i}} + R_B \hat{\mathbf{j}} - mg \hat{\mathbf{j}} = m \dot{\mathbf{r}}_G \}$

$\dot{\mathbf{r}}_G = \frac{L}{2} \cos \theta \dot{\theta} \hat{\mathbf{i}} + \frac{L}{2} \sin \theta \dot{\theta} \hat{\mathbf{j}}$

$\dot{\mathbf{r}}_G = -\frac{L}{2} \sin \theta \ddot{\theta} \hat{\mathbf{i}} + \frac{L}{2} \cos \theta \ddot{\theta} \hat{\mathbf{j}}$

$\dot{\mathbf{r}}_G = -\frac{L}{2} (\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2) \hat{\mathbf{i}} + \frac{L}{2} (\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2) \hat{\mathbf{j}}$

$\{ \} \cdot \hat{\mathbf{i}} \Rightarrow R_A = -\frac{mL}{2} (\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2)$

$\{ \} \cdot \hat{\mathbf{j}} \Rightarrow R_B = mg + \frac{mL}{2} (\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2)$

Apply A.M.B. about G , $\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G$

$-R_A \frac{L}{2} \sin \theta \hat{\mathbf{k}} + R_B \frac{L}{2} \cos \theta \hat{\mathbf{k}} = I_G \dot{\omega}_{AB} \hat{\mathbf{k}}$

$\left\{ \frac{mL^2}{4} (\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2) + \frac{mL^2}{4} (\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2) + \frac{mgL}{2} \cos \theta \hat{\mathbf{k}} = \frac{mL^2}{12} (\ddot{\theta}) \hat{\mathbf{k}} \right\}$

$\{ \} \cdot \hat{\mathbf{k}} \Rightarrow \frac{mL^2}{4} \ddot{\theta} + \frac{mgL}{2} \cos \theta = -\frac{mL^2}{12} \ddot{\theta}$

$\Rightarrow \ddot{\theta} + \frac{3g}{2L} \cos \theta = 0$

c) $\dot{\omega}_{AB} = -\ddot{\theta} = \frac{3g}{2L} \cos \theta_0$ and $\mathbf{a}_G = \dot{\mathbf{r}}_G = \frac{3g}{4} \cos \theta_0 (\sin \theta_0 \hat{\mathbf{i}} - \cos \theta_0 \hat{\mathbf{j}})$

d) Reactions, $R_A \hat{\mathbf{i}} = \frac{3mg}{4} \sin \theta_0 \cos \theta_0 \hat{\mathbf{i}}$ and $R_B \hat{\mathbf{j}} = mg(1 - \frac{3}{4} \cos^2 \theta_0) \hat{\mathbf{j}}$

e) $\dot{\mathbf{r}}_B = L \sin \theta \dot{\theta} \hat{\mathbf{i}} \Rightarrow \dot{\mathbf{r}}_B = (-L \cos \theta \dot{\theta} - L \sin \theta \ddot{\theta}) \hat{\mathbf{i}} \Rightarrow \dot{\mathbf{r}}_B = \frac{3}{2}g \sin \theta_0 \cos \theta_0 \hat{\mathbf{i}} (\theta = 0)$

f) $\dot{\omega}_{AB} = ?$ $\dot{\mathbf{r}}_A = ?$ at $\theta = \frac{\theta_0}{2}$

Re writing the equation of motion, $\int \ddot{\theta} d\theta = \int_0^{\theta_0} -\frac{3g}{2L} \cos \theta d\theta$

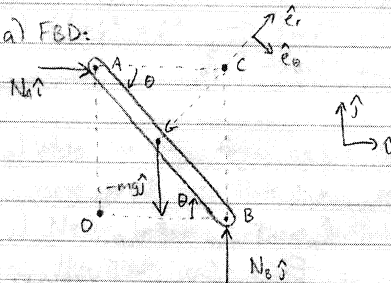
$\therefore \dot{\theta} = -\sqrt{\frac{3g}{L} [\sin \theta_0 - \sin \theta_0/2]}$

$\dot{\mathbf{r}}_A = L \cos \theta \dot{\theta} \hat{\mathbf{i}} \Rightarrow \dot{\mathbf{r}}_A = (L \cos \theta \dot{\theta} - L \sin \theta \ddot{\theta}) \hat{\mathbf{i}} (\text{or}) \dot{\mathbf{r}}_A = -3g \left[\frac{1}{2} \cos^2 \frac{\theta_0}{2} + \sin \frac{\theta_0}{2} (\sin \theta_0 - \sin \frac{\theta_0}{2}) \right] \hat{\mathbf{j}}$

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Johannes Ferry
 HW problems 16.30, 16.39,
 16.41, 16.43, 16.48
 Due 12/4/98
 TAM 2030
 Section 2-1-25
 TA: Rung Long

(6.30) a) FBD:



b) $\theta(t=0) = \theta_0$

$$AMB_{/C}: \sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$$

$$\vec{r}_{G/C} \times (-mg\hat{j}) = \vec{r}_{G/C} \times m\vec{a}_G + I_G \dot{\omega} \hat{k}$$

$$\left(-\frac{L}{2} \cos \theta \hat{i} - \frac{L}{2} \sin \theta \hat{j}\right) \times (-mg\hat{j}) = \left(-\frac{L}{2} \cos \theta \hat{i} - \frac{L}{2} \sin \theta \hat{j}\right) \times m\vec{a}_G - \frac{mL^2}{12} \ddot{\theta} \hat{k}$$

$$\frac{1}{2} mgL \cos \theta \hat{k} = \left(-\frac{L}{2} \cos \theta \hat{i} - \frac{L}{2} \sin \theta \hat{j}\right) \times m\vec{a}_G - \frac{mL^2}{12} \ddot{\theta} \hat{k}$$

Need $\vec{a}_G$: use circular motion of G

$$\begin{aligned} \vec{a}_G &= -\dot{\theta}^2 \frac{L}{2} \hat{e}_r + \ddot{\theta} \frac{L}{2} \hat{e}_\theta = -\dot{\theta}^2 \frac{L}{2} (\cos \theta \hat{i} + \sin \theta \hat{j}) + \ddot{\theta} \frac{L}{2} (\sin \theta \hat{i} - \cos \theta \hat{j}) \\ &= (-\dot{\theta}^2 \frac{L}{2} \cos \theta + \ddot{\theta} \frac{L}{2} \sin \theta) \hat{i} + (-\dot{\theta}^2 \frac{L}{2} \sin \theta - \ddot{\theta} \frac{L}{2} \cos \theta) \hat{j} \end{aligned}$$

Plug $\vec{a}_G$ into AMB:

$$\left\{ \frac{1}{2} mgL \cos \theta \hat{k} = -\left(\frac{mL^2}{4} (\ddot{\theta} \sin \theta \cos \theta + \dot{\theta}^2 \cos^2 \theta) + \frac{mL^2}{4} (-\dot{\theta}^2 \sin \theta \cos \theta + \ddot{\theta} \sin^2 \theta) \right) \hat{k} - \frac{mL^2}{12} \ddot{\theta} \hat{k} \right\}$$

$$\left\{ \hat{i} \cdot \hat{k} \rightarrow \frac{1}{2} mgL \cos \theta = -\frac{mL^2}{4} \ddot{\theta} (\cos^2 \theta + \sin^2 \theta) - \frac{mL^2}{12} \ddot{\theta} \right\}$$

$$6g \cos \theta = -3L\ddot{\theta} - L\ddot{\theta} = -4L\ddot{\theta}$$

$$\ddot{\theta} = -\frac{3g}{4L} \cos \theta$$

c) From part (b): $\dot{\theta}(t=0) = \frac{-3g}{4L} \cos \theta_0$ (released from rest)

$$\vec{a}_G(t=0) = (-\dot{\theta}_0^2 \frac{L}{2} \cos \theta_0 + \ddot{\theta}_0 \frac{L}{2} \sin \theta_0) \hat{i} + (-\dot{\theta}_0^2 \frac{L}{2} \sin \theta_0 - \ddot{\theta}_0 \frac{L}{2} \cos \theta_0) \hat{j}$$

$$\vec{a}_G(t=0) = -\frac{3g}{4} \cos \theta_0 (\sin \theta_0 \hat{i} - \cos \theta_0 \hat{j})$$

d) LMB: $\sum \vec{F} = m\vec{a}_G$

$$\{ N_A \hat{i} + (N_B - mg) \hat{j} = m\vec{a}_G \}$$

$$\{ \hat{i} \cdot \hat{i} \rightarrow N_A = \frac{mL}{2} (-\dot{\theta}^2 \cos \theta + \ddot{\theta} \sin \theta) \}$$

$$\{ \hat{j} \cdot \hat{j} \rightarrow N_B - mg = \frac{mL}{2} (-\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta) \}$$

$$\{ N_B = m \left(\frac{L}{2} (-\dot{\theta}^2 \sin \theta - \ddot{\theta} \cos \theta) + g \right) \}$$

$$\begin{aligned} \text{e) } \vec{a}_B &= \vec{a}_G + \vec{a}_{G/B} = (-\dot{\theta}^2 \frac{L}{2} \cos \theta + \ddot{\theta} \frac{L}{2} \sin \theta) \hat{i} + (-\dot{\theta}^2 \frac{L}{2} \sin \theta - \ddot{\theta} \frac{L}{2} \cos \theta) \hat{j} - \dot{\theta}^2 \frac{L}{2} \hat{e}_\theta + \ddot{\theta} \frac{L}{2} \hat{e}_r \\ &= (-\dot{\theta}^2 \frac{L}{2} \cos \theta + \ddot{\theta} \frac{L}{2} \sin \theta) \hat{i} + (-\dot{\theta}^2 \frac{L}{2} \sin \theta - \ddot{\theta} \frac{L}{2} \cos \theta) \hat{j} - \dot{\theta}^2 \frac{L}{2} (\sin \theta \hat{i} - \cos \theta \hat{j}) + \ddot{\theta} \frac{L}{2} (\cos \theta \hat{i} + \sin \theta \hat{j}) \\ &= \left[-\dot{\theta}^2 \frac{L}{2} (\cos \theta + \sin \theta) + \ddot{\theta} \frac{L}{2} (\sin \theta + \cos \theta) \right] \hat{i} + \left[-\dot{\theta}^2 \frac{L}{2} (\sin \theta + \cos \theta) + \ddot{\theta} \frac{L}{2} (\cos \theta + \sin \theta) \right] \hat{j} \end{aligned}$$



f) Point A moves in circles around Point G

$$\vec{V}_A = -\omega \frac{L}{2} \hat{e}_r = -\omega \frac{L}{2} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\vec{V}_A = \vec{V}_B \text{ (rigid object)}$$

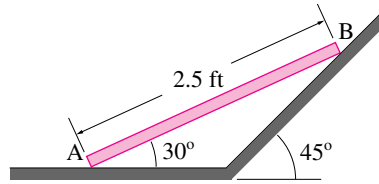
$$|\vec{a}_A| = |\vec{a}_B|$$

19.2.7 Bar leans on a crooked wall.

A uniform 3 lbm bar leans on a wall and floor and is let go from rest. Gravity pulls it down.

- Draw a Free-Body Diagram of the bar.
- Kinematics: find the velocity and acceleration of point B in terms of the velocity and acceleration of point A
- Using equations of motion, find the acceleration of point A.

- d) Do the reaction forces at A and B add up to the weight of the bar? Why or why not? (You do not need to solve for the reaction forces in order to answer this part.)



Problem 19.7

19.2.7

a) Free body diagram:

b) Velocity and acceleration of point B?

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$; Velocity of point B: $\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A}$

$\vec{r}_A = (l \cos \theta - l \sin \theta)(-\hat{i}) \Rightarrow \dot{\vec{r}}_A = (l \sin \theta \dot{\theta} + l \cos \theta \ddot{\theta}) \hat{i}$

$\vec{r}_{B/A} = l \cos \theta \hat{i} + l \sin \theta \hat{j} \Rightarrow \dot{\vec{r}}_{B/A} = -l \sin \theta \dot{\theta} \hat{i} + l \cos \theta \dot{\theta} \hat{j}$

Acceleration of point B: $\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A}$

$\ddot{\vec{r}}_A = l(\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2 + \cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2) \hat{i}$

$\ddot{\vec{r}}_{B/A} = (-l \sin \theta \ddot{\theta} - l \cos \theta \dot{\theta}^2) \hat{i} + (l \cos \theta \ddot{\theta} - l \sin \theta \dot{\theta}^2) \hat{j}$

$\ddot{\vec{r}}_B = +l[\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2](\hat{i} + \hat{j})$

c) Using AMB about I: $\sum \vec{M}_I = \dot{\vec{H}}_I$

$\Rightarrow \vec{r}_{G/I} \times mg(-\hat{j}) = \vec{I} \dot{\omega} + \vec{r}_{G/I} \times m \vec{a}_G$

$\vec{r}_{G/I} = \frac{l}{2} \cos \theta \hat{i} + (l \cos \theta + \frac{l}{2} \sin \theta)(-\hat{j})$

$\vec{a}_G = \dot{\omega} \times \vec{r}_{G/I} + \omega \times (\omega \times \vec{r}_{G/I}) = \dot{\omega} \times \vec{r}_{G/I} - \omega^2 \vec{r}_{G/I}$

$\Rightarrow -\frac{mgl}{2} \cos \theta \hat{k} = (-\dot{\theta}) \frac{1}{12} m l^2 \hat{k} + \left[(-\dot{\theta}) \left\{ \frac{1}{2} \cos \theta \hat{k} + (-\dot{\theta}) \left[-l \cos \theta - \frac{l}{2} \sin \theta \right] \hat{k} \right\} m \right]$

$\Rightarrow \ddot{\theta} = \frac{3g \cos \theta}{l(2 + 6 \cos^2 \theta + 3 \sin^2 \theta)}$

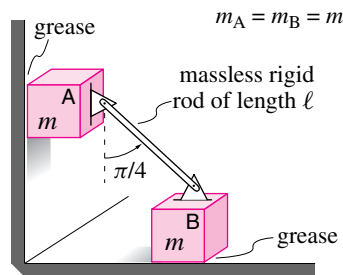
As the bar starts from rest, $\dot{\theta} = 0$

$\therefore \ddot{\vec{r}}_A = -l(\sin \theta_0 + \cos \theta_0) \frac{3g \cos \theta_0}{l(2 + 6 \cos^2 \theta_0 + 3 \sin^2 \theta_0)} \hat{i} = -\frac{3g}{2} \frac{\sqrt{3} + 3}{3\sqrt{3} + 13} \hat{i}$

d) Reaction forces at A and B do not add up to the weight of the bar as non zero angular acceleration of the bar would modify them.

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

19.2.8 Two blocks, each with mass m , slide without friction on the wall and floor shown. They are attached with a rigid massless rod of length ℓ that is pinned at both ends. The system is released from rest when the rod makes an angle of 45° . What is the acceleration of the block B immediately after the system is released?



Problem 19.8

18.2.8

Acceleration of Block B: ?

$$\vec{r}_{G/I} = \vec{r}_A + \vec{r}_{G/A}$$

$$= -l \sin \theta \hat{i} + \frac{1}{2} l \sin \theta \hat{i} - \frac{1}{2} l \cos \theta \hat{j}$$

$$\Rightarrow \vec{r}_{G/I} = -\frac{1}{2} l \sin \theta \hat{i} - \frac{1}{2} l \cos \theta \hat{j}$$

Applying AMB about I: $\Sigma \vec{M}_I = \dot{\vec{H}}_I$

$$\Rightarrow \vec{r}_{A/I} \times m \vec{g} (-\hat{j}) = \vec{r}_G \times m \vec{a}_G = 2m \left(\frac{1}{2} \right) \ddot{\theta} \hat{k} + \vec{r}_G \times m [\ddot{\theta} \hat{k} \times \vec{r}_G - \dot{\theta}^2 \vec{r}_G]$$

$$\Rightarrow \left\{ -l \sin \theta \hat{i} \times m g (-\hat{j}) = \frac{m l}{2} \ddot{\theta} \hat{k} + m \ddot{\theta} \left[\left(-\frac{1}{2} l \sin \theta \right) \hat{i} + \left(-\frac{1}{2} l \cos \theta \right) \hat{j} \right] \times \hat{k} \right\}$$

$$\left\{ \right\} \cdot \hat{k} \Rightarrow m g l \sin \theta = \frac{m l}{2} (1 + \frac{1}{2}) \ddot{\theta} = \frac{3}{4} m l \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = \frac{4g}{3l} \sin \theta$$

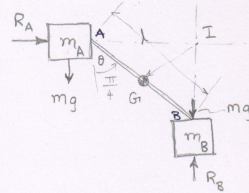
$$\vec{r}_B = l \sin \theta \hat{i}$$

$$\dot{\vec{r}}_B = l \cos \theta \dot{\theta} \hat{i}$$

$$\ddot{\vec{r}}_B = (-l \sin \theta \ddot{\theta} + l \cos \theta \dot{\theta}^2) \hat{i}$$

Immediately after release, $\theta = \frac{\pi}{4}$; $\dot{\theta} = 0$

$$\Rightarrow \ddot{\vec{r}}_B = \frac{4g}{3} \sin \theta \cos \theta = \frac{2g}{3}$$



18.2.9

Tension in AB = ?

$$\vec{r}_B = (R \cos \theta + L \cos \phi) \hat{i}$$

$$\dot{\vec{r}}_B = (-R \sin \theta \dot{\theta} - L \sin \phi \dot{\phi}) \hat{i}$$

$$\ddot{\vec{r}}_B = \left\{ -R [\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2] - L [\sin \phi \ddot{\phi} + \cos \phi \dot{\phi}^2] \right\} \hat{i}$$

When piston is at its maximum extent, $\theta = \phi = 0 \Rightarrow \dot{\phi} = \frac{R}{L} \dot{\theta}$

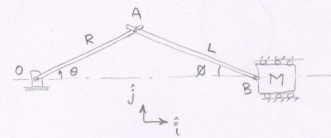
$$\Rightarrow \ddot{\vec{r}}_B = \left\{ -R \ddot{\theta} - L \frac{R}{L} \ddot{\theta} \right\} \hat{i} = -R \ddot{\theta} \left(1 + \frac{R}{L} \right) \hat{i}$$

Apply LMB on mass: $\Sigma \vec{F} = m \vec{a}$

$$\Rightarrow \left\{ -T \hat{i} + R \hat{j} = m \ddot{\vec{r}}_B = -m R \ddot{\theta} \left(1 + \frac{R}{L} \right) \hat{i} \right\}$$

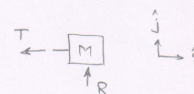
$$\left\{ \right\} \cdot \hat{i} \Rightarrow -T = -m R \ddot{\theta} \left(1 + \frac{R}{L} \right)$$

$$\therefore \text{Tension in the rod AB, } T = m(L+R) \ddot{\theta} \left(\frac{R}{L} \right)$$



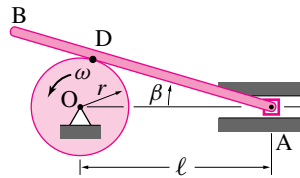
$$R \sin \theta = L \sin \phi$$

$$\cos \phi \dot{\phi} = \frac{R}{L} \cos \theta \dot{\theta}$$



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19.2.10 In problem ??, find the force on the wheel at point D due to the rod.



Problem 19.10

18.2.10

Force on the wheel at 'D' due to the rod = ?

To find ω_{AB} , $\mathbf{v}_A = \mathbf{v}_D + \mathbf{v}_{A/D}$

$$\mathbf{v}_D = \omega \hat{k} \times \mathbf{r}_D = r\omega \hat{e}_\theta$$

$$\mathbf{v}_{A/D} = \omega_{AB} \hat{k} \times \mathbf{r}_{A/D} = \omega_{AB} \hat{k} \times \sqrt{l^2 - r^2} (-\hat{e}_\theta)$$

$$\left\{ \begin{array}{l} \mathbf{v}_A = -\omega r \cos\beta \hat{i} + \omega r \sin\beta \hat{j} + \omega_{AB} \sqrt{l^2 - r^2} [\sin\beta \hat{i} + \cos\beta \hat{j}] \end{array} \right\}$$

$$\left\{ \right\} \cdot \hat{j} \Rightarrow \omega_{AB} = \frac{-\omega r^2}{(l^2 - r^2)} \Rightarrow \mathbf{v}_{A/D} = \frac{-\omega r l}{\sqrt{l^2 - r^2}} \hat{i}$$

To find $\dot{\omega}_{AB}$, consider $\mathbf{a}_A = \mathbf{a}_D + \mathbf{a}_{A/D}$

$$\mathbf{a}_D = \dot{\omega} \hat{k} \times \mathbf{r}_D + \omega \times (\omega \times \mathbf{r}_D) \Rightarrow \mathbf{a}_D = -r\omega^2 \hat{e}_r$$

$$\mathbf{a}_{A/D} = \dot{\omega}_{AB} \hat{k} \times \mathbf{r}_{A/D} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{A/D})$$

$$\left\{ \begin{array}{l} \mathbf{a}_A = \mathbf{a}_D + \mathbf{a}_{A/D} = -r\omega^2 (\sin\beta \hat{i} + \cos\beta \hat{j}) + \sqrt{l^2 - r^2} [\dot{\omega}_{AB} (\cos\beta \hat{j} + \sin\beta \hat{i}) - \omega_{AB}^2 (\cos\beta \hat{i} - \sin\beta \hat{j})] \end{array} \right\}$$

$$\left\{ \right\} \cdot \hat{j} \Rightarrow 0 = -\omega^2 r \cos\beta + \sqrt{l^2 - r^2} \dot{\omega}_{AB} \cos\beta + \sqrt{l^2 - r^2} \omega_{AB}^2 \sin\beta$$

$$\Rightarrow \dot{\omega}_{AB} = \frac{r l^2 (1 - 2r^2)}{(l^2 - r^2)^{5/2}} \omega^2 \quad \text{and} \quad \mathbf{a}_A = \frac{-l r^4 \omega^2}{(l^2 - r^2)^2} \hat{i}$$

$$\mathbf{a}_G = \mathbf{a}_D + \mathbf{a}_{G/D}$$

$$= -r\omega^2 \hat{e}_r + \dot{\omega}_{AB} \hat{k} \times (\omega_{AB} \times \mathbf{r}_{G/D}) + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{G/D})$$

$$= (\dot{\omega}_{AB} |\mathbf{r}_{G/D}| - r\omega^2) \hat{e}_x - \omega_{AB}^2 |\mathbf{r}_{G/D}| \hat{e}_\theta$$

Applying L.M.B on rod: $\Sigma \mathbf{F} = m \mathbf{a}_G$

$$\Rightarrow \left\{ \begin{array}{l} F_x \hat{i} + F_y \hat{j} - mg \hat{j} + N \hat{j} = m a_{Gx} \hat{i} + m a_{Gy} \hat{j} \end{array} \right\}$$

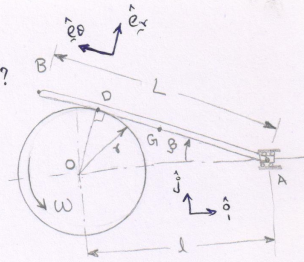
$$\left\{ \right\} \cdot \hat{i} \Rightarrow F_x = -m r \omega^2 \left\{ \frac{L(l^2 - r^2)^2 - l r^2 (L - 2\sqrt{l^2 - r^2})}{2l(l^2 - r^2)^{5/2}} \right\}$$

$$\left\{ \right\} \cdot \hat{j} \Rightarrow N = mg - F_y - m r \omega^2 \frac{L}{2l}$$

Applying A.M.B about 'D': $\Sigma \mathbf{M}_D = \dot{\mathbf{H}}_D$

$$\Rightarrow \left\{ \begin{array}{l} \mathbf{r}_{G/D} \times mg(-\hat{j}) + \mathbf{r}_{A/D} \times N \hat{j} = \mathbf{I}_G \dot{\omega}_{AB} \hat{k} + \mathbf{r}_{G/D} \times m \mathbf{a}_G \end{array} \right\}$$

$$\left\{ \right\} \cdot \hat{k} \Rightarrow F_y = \left[\frac{L}{2} \sqrt{l^2 - r^2} (mg - m r \omega^2 \frac{L}{2l}) - r \left(\frac{L}{2} - \sqrt{l^2 - r^2} \right) F_x + \frac{m r \omega^2 L (1 - 2r^2)}{12(l^2 - r^2)^{5/2}} \right] / \left[\sqrt{l^2 - r^2} (L - \sqrt{l^2 - r^2}) \right]$$

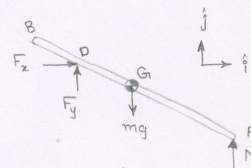
 $\therefore$ Force on wheel, $\mathbf{F} = -F_x \hat{i} - F_y \hat{j}$ 

$$\hat{e}_r = \sin\beta \hat{i} + \cos\beta \hat{j}$$

$$\hat{e}_\theta = -\cos\beta \hat{i} + \sin\beta \hat{j}$$

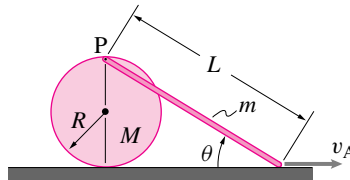
$$\sin\beta = \frac{r}{l}, \cos\beta = \frac{\sqrt{l^2 - r^2}}{l}$$

$$\mathbf{r}_{G/D} = \left(\sqrt{l^2 - r^2} - \frac{L}{2} \right) (-\hat{e}_\theta)$$



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19.2.11 In problem ??, find the force on the rod at point P.



Problem 19.11

18.2.11:

Force on rod at point 'P' = ?

$$\vec{V}_A = \vec{V}_A \hat{i} = \vec{V}_P + \vec{V}_{A/P}$$

$$\vec{V}_P = \omega_D \hat{k} \times 2R \hat{j}$$

$$\vec{V}_{A/P} = \omega_r \hat{k} \times -L \hat{\lambda}$$

$$\Rightarrow \left\{ \vec{V}_A = (-2R\omega_D + \omega_r L \sin\theta) \hat{i} + \omega_r L \cos\theta \hat{j} \right\}$$

$$\left\{ \right\} \cdot \hat{j} \Rightarrow \omega_r = 0 \text{ and } V_A = -2R\omega_D //$$

$$\vec{a}_A = \vec{a}_P + \vec{a}_{A/P}$$

$$\vec{a}_P = \dot{\omega}_D (-R \hat{i}) \times 2R \hat{j} + \omega_D \times (\omega_D \times 2R \hat{j}) = 2R\dot{\omega}_D \hat{i} - 2R\omega_D^2 \hat{j}$$

$$\vec{a}_{A/P} = \dot{\omega}_r \hat{k} \times L(-\hat{\lambda}) + \omega_r^2 \hat{k} \times L \hat{\lambda} = L\dot{\omega}_r \sin\theta \hat{i} + L\dot{\omega}_r \cos\theta \hat{j}$$

$$\Rightarrow \vec{a}_A = (2R\dot{\omega}_D + L\dot{\omega}_r \sin\theta) \hat{i} + \left(-\frac{V_A}{2R} + L\dot{\omega}_r \cos\theta \right) \hat{j} = 0$$

$$\Rightarrow \dot{\omega}_r = \frac{V_A}{2RL \cos\theta} \text{ and } \dot{\omega}_D = -\frac{V_A}{4R} \tan\theta ; \vec{a}_P = \frac{-V_A}{2R} \tan\theta \hat{i} - \frac{V_A}{2R} \hat{j}$$

$$\vec{a}_G = \vec{a}_P + \vec{a}_{G/P}$$

$$\vec{a}_{G/P} = \dot{\omega}_r \times \vec{r}_{G/P} + \omega_r \times (\omega_r \times \vec{r}_{G/P}) = \frac{V_A}{4R} \hat{j} + \frac{V_A}{4R} \tan\theta \hat{i}$$

$$\Rightarrow \vec{a}_G = \frac{-V_A}{4R} \tan\theta \hat{i} - \frac{V_A}{4R} \hat{j}$$

Applying L.M.B on rod: $\sum \vec{F} = m \vec{a}_G$

$$\Rightarrow \left\{ F_x \hat{i} + F_y \hat{j} - mg \hat{j} + N \hat{j} = -\frac{mV_A}{4R} \tan\theta \hat{i} - \frac{mV_A}{4R} \hat{j} \right\}$$

$$\left\{ \right\} \cdot \hat{i} \Rightarrow F_x = -\frac{mV_A}{4R} \tan\theta // \text{ and } \left\{ \right\} \cdot \hat{j} \Rightarrow N = -F_y + mg - \frac{mV_A}{4R}$$

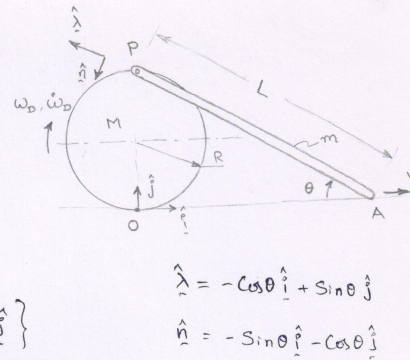
Applying A.M.B about G: $\sum \vec{M}_G = \vec{H}_G$

$$\Rightarrow \left\{ \frac{L}{2} \hat{\lambda} \times (F_x \hat{i} + F_y \hat{j}) - \frac{L}{2} \hat{\lambda} \times N \hat{j} = I_G \dot{\omega}_r \hat{k} \right\}$$

$$\left\{ \right\} \cdot \hat{k} \Rightarrow -F_x \sin\theta - F_y \cos\theta + N \cos\theta = \frac{mV_A}{12R \cos\theta}$$

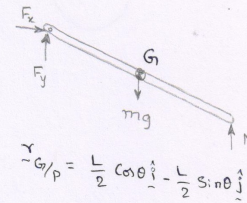
Upon simplification, we get $F_y = \frac{mg}{2} - \frac{mV_A}{8R} + \frac{mV_A}{24R} (2 \tan^2\theta - 1)$

$\therefore$ Force on rod at P, $= F_x \hat{i} + F_y \hat{j}$



$$\hat{\lambda} = -\cos\theta \hat{i} + \sin\theta \hat{j}$$

$$\hat{n} = -\sin\theta \hat{i} - \cos\theta \hat{j}$$



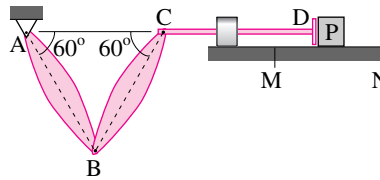
$$\vec{r}_{G/P} = \frac{L}{2} \cos\theta \hat{i} - \frac{L}{2} \sin\theta \hat{j}$$

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19.2.12

Slider-Crank

mechanism. The slider crank mechanism shown is used to push a 2 lbm block P. Arm AB and BC are each 0.5 ft long. Given that arm AB rotates counterclockwise at a constant 2 revolutions per second, what is the force on P at the instant shown?



Problem 19.12

Answer:

There are many solution methods.
 $-8\pi^2$ poundals to the left. (Note: 1 poundal = $1 \text{ ft} \cdot \text{lbm} \cdot \text{s}^{-2}$.)

18.2.12

Slider Crank Mechanism

Mass of block, $m = 2 \text{ lbm}$ Arm length, $AB = BC = 0.5 \text{ ft}$ Angular velocity of link AB, $\omega_{AB} = 2 \text{ rev/s (ccw)}$

What is the force on P?

Link CD is rigid constrained to move in $\pm \hat{j}$ direction. $\therefore \underline{v}_c = \underline{v}_D$ and $\underline{a}_c = \underline{a}_D$ Angular velocity of link BC, $\underline{\omega}_{BC} = ?$

$$\underline{v}_D = \underline{v}_c = \underline{v}_B + \underline{v}_{c/B}$$

$$\Rightarrow \left\{ \underline{v}_c = \underline{v}_c \hat{j} = \omega_{AB} \hat{k} \times [1 \sin 60^\circ \hat{i} + 1 \cos 60^\circ \hat{j}] + \omega_{c/B} \hat{k} \times [-1 \sin 60^\circ \hat{i} + 1 \cos 60^\circ \hat{j}] \right\}$$

$$\left\{ \right\} \cdot \hat{i} \quad \underline{v}_c \hat{j} = \omega_{AB} 1 \sin 60^\circ \hat{j} - \omega_{AB} 1 \cos 60^\circ \hat{i} - \omega_{c/B} [1 \sin 60^\circ \hat{j} + 1 \cos 60^\circ \hat{i}]$$

$$\Rightarrow 0 = -\omega_{AB} 1 \cos 60^\circ - \omega_{c/B} 1 \cos 60^\circ \Rightarrow \omega_{c/B} = -\omega_{AB} \quad \therefore \underline{\omega}_{BC} = -2 \frac{\text{rev}}{\text{s}} \hat{k}$$

Acceleration of D = ?

$$\underline{a}_D = \underline{a}_c = \underline{a}_B + \underline{a}_{c/B}$$

$$\Rightarrow \underline{a}_c = \underline{a}_c \hat{j} = -\omega_{AB}^2 [1 \sin 60^\circ \hat{i} + 1 \cos 60^\circ \hat{j}] + \underline{\dot{\omega}}_{BC} \hat{k} \times [-1 \sin 60^\circ \hat{i} + 1 \cos 60^\circ \hat{j}] -$$

$$\Rightarrow \left\{ \underline{a}_c \right\} = \left[-\omega_{AB}^2 1 \sin 60^\circ + \omega_{BC}^2 1 \sin 60^\circ - \dot{\omega}_{BC} 1 \cos 60^\circ \right] \hat{i} + \left[-\omega_{AB}^2 1 \cos 60^\circ - \omega_{BC}^2 1 \cos 60^\circ - \dot{\omega}_{BC} 1 \sin 60^\circ \right] \hat{j}$$

$$\left\{ \right\} \cdot \hat{i} \Rightarrow \dot{\omega}_{BC} = 0 \quad \text{and} \quad \left\{ \right\} \cdot \hat{j} \Rightarrow \underline{a}_c = -(\omega_{AB}^2 + \omega_{BC}^2) 1 \cos 60^\circ$$

$$\therefore \underline{a}_c = \underline{a}_D = -2 \cdot \omega_{AB}^2 1 \cos 60^\circ \hat{j} \quad (\text{or}) \quad \underline{a}_D = -1 \omega_{AB}^2 \hat{j}$$

Given that $\omega_{AB} = 2 \text{ rev/s} = 2 \cdot 2\pi \text{ rad/s}$

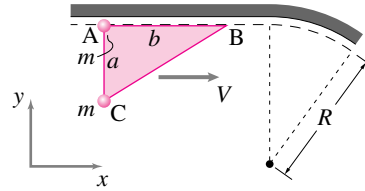
$$\therefore \text{Force on P at the instant shown, } \underline{F} = m \underline{a}_D = 2(1 \text{ lbm})(0.5 \text{ ft})(4\pi^2 \frac{\text{rad}^2}{\text{s}^2}) \hat{j}$$

$$\therefore \underline{F} = -16\pi^2 \hat{j} \text{ poundals}$$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

19.2.13 The large masses m at C and A were supported by the light triangular plate ABC , whose corners follow the guide. B enters the curved guide with velocity V . Neglecting gravity, find the vertical reactions (in the y direction) at A and B . (Hint: for the rigid planar body ABC , find $\ddot{y}_A$ and $\ddot{y}_B$ in terms of $\ddot{a}_y^{cm}$ and

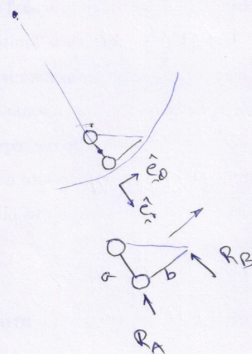
$\ddot{\theta}$, assuming $\dot{\theta} = 0$ initially.)



Problem 19.13

18-2.13

Find Reaction at A & B .



$$\begin{aligned} \underline{a}_{cm} &= \underline{\dot{\omega}} \times R \underline{\hat{e}}_r + \underline{\omega} \times (\underline{\omega} \times R \underline{\hat{e}}_r) \\ &= \ddot{\theta} R \underline{\hat{e}}_\theta - \dot{\theta}^2 R \underline{\hat{e}}_r \end{aligned}$$

$$\sum \underline{F} = m \underline{a}_{cm}$$

$$\left\{ R_A (-\underline{\hat{e}}_r) + R_B (-\underline{\hat{e}}_r) = m (\ddot{\theta} R \underline{\hat{e}}_\theta - \dot{\theta}^2 R \underline{\hat{e}}_r) \right\}$$

$$\left\{ \cdot \underline{\hat{e}}_\theta \Rightarrow R_A (-\underline{\hat{e}}_r \cdot \underline{\hat{e}}_\theta) + R_B (-\underline{\hat{e}}_r \cdot \underline{\hat{e}}_\theta) = m \ddot{\theta} R (\underline{\hat{e}}_\theta \cdot \underline{\hat{e}}_\theta) \right.$$

$$\underline{\hat{e}}_{rA} = \cos \theta_A \underline{\hat{j}} + \sin \theta_A \underline{\hat{i}}$$

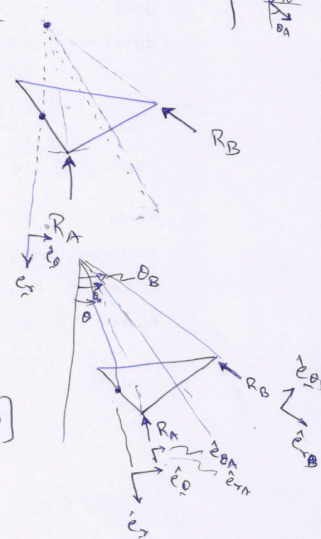
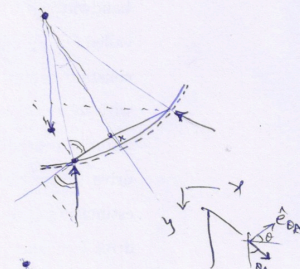
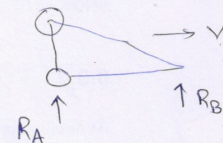
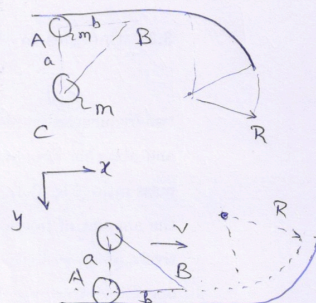
$$\underline{\hat{e}}_{rB} = \cos \theta_B \underline{\hat{j}} + \sin \theta_B \underline{\hat{i}}$$

$$\underline{\hat{e}}_\theta = \cos \theta \underline{\hat{j}} - \sin \theta \underline{\hat{i}}$$

$$\Rightarrow \left\{ \cdot \underline{\hat{e}}_\theta = R_A \left[-\left[\sin \theta_A \cos \theta - \sin \theta \cos \theta_A \right] \right] + R_B \left[-\sin \theta_B \cos \theta + \cos \theta_B \sin \theta \right] \right.$$

$$= m R \ddot{\theta}$$

$$\Rightarrow R_A [\sin(\theta - \theta_A)] + R_B [\sin(\theta - \theta_B)] = m R \ddot{\theta}$$



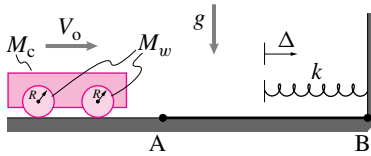
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19.2.14 An idealized model for a car comprises a rigid chassis of mass M_c and four identical rigid disks (wheels) of mass M_w and radius R , as shown in the figure. Initially in motion with speed v_0 , the car is momentarily brought to rest by compressing an initially uncompressed spring of stiffness k . Assume no frictional losses.

- Assuming no wheel slip, determine the compression Δ of the spring required to stop the car.
- While the car is in contact with the spring but still moving forward, in which direction is the *tangential* force on any one of the wheels (due to contact with the ground)

pointing? Why? (Illustrate with a sketch).

- Repeat part (a) assuming now that the ground is perfectly frictionless from points A to B shown in the figure.



Problem 19.14

Answer:

$$(a) \Delta = \sqrt{\frac{(M_c + 6M_w)v_0^2}{k}}$$

(b) Tangential forces point toward the wall!

$$(c) \Delta = \sqrt{\frac{(M_c + 4M_w)v_0^2}{k}}$$

19.2.14

a) Compression required to stop the car.

No slip and A to B is not frictionless

Using energy balance: $E_i = E_f$

Initial energy, $E_i = K.E$

$$= \frac{1}{2} M_c v_0^2 + 4 \left(\frac{1}{2} I \omega^2 \right) + \frac{1}{2} M_w v_0^2$$

$$= \frac{1}{2} M_c v_0^2 + 2 \left(\frac{1}{2} M_w R^2 \right) \left(\frac{v_0}{R} \right)^2 + 2 M_w v_0^2$$

$$= \frac{1}{2} M_c v_0^2 + 3 M_w v_0^2$$

Final energy, $E_f = P.E$

$$= \frac{1}{2} k \Delta^2$$

$\therefore E_i = E_f \Rightarrow \frac{1}{2} k \Delta^2 = \frac{1}{2} M_c v_0^2 + 3 M_w v_0^2$

$$\Rightarrow \Delta = \frac{v_0}{\sqrt{k}} \sqrt{M_c + 6 M_w} = \sqrt{\frac{(M_c + 6 M_w) v_0^2}{k}}$$

b) While the car is in contact with the spring but still moving forward, the tangential force at the point of contact of wheels with the ground remain directed towards the wall. Even though the $\uparrow$ force on the wheel is in the opposite direction to its motion, the direction of the tangential force at the point of contact with the ground is in the direction of wheels' motion / heading.

c) If AB is frictionless, $E_i = \frac{1}{2} M_c v_0^2 + 4 \left(\frac{1}{2} M_w v_0^2 \right)$

$$E_f = E_i \Rightarrow \frac{1}{2} k \Delta^2 = \frac{1}{2} (M_c + 4 M_w) v_0^2$$

$\therefore$ Deflection of the spring required to bring the car to rest is, $\Delta = \sqrt{\frac{(M_c + 4 M_w) v_0^2}{k}}$

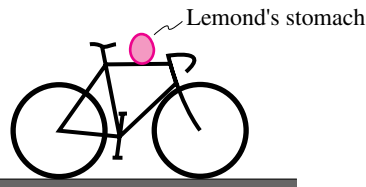
Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

19.2.15 Assume Greg Lemond's riding "tuck" was so good that you can neglect air resistance when you think about him and his bike. Further, you can regard his and the bike's combined mass (all 70 kg) as concentrated at a point in his stomach somewhere. Greg's left foot has just fallen off the pedal so he is only pedaling with his right foot, which at the moment in question is at its lowest point in the motion. You note that, relative to the ground the right foot is only going $3/4$ as fast as the bike (since it is going backwards relative to the bike), though you can't make out all the radii of his frictionless gears and rigid round wheels. Greg, ever in touch with his body, tells you he is pushing back on the pedal with a force of 70 N. You would like to know Greg's acceleration.

- a) In your first misconceived experiment you set up a 70 kg bicy-

cle in your laboratory and balance it with strings that cause no fore or aft forces. You tie a string to the vertically down right pedal and pull back with a force of 70 N. What acceleration do you measure?

- b) What is Greg's actual acceleration? [Hint: Greg's massless leg is pushing forward on his body with a force of 70 N] You can neglect the mass of the wheels and other transmission parts (chain, crank, etc).



Problem 19.15

16.39 (A)

Assume the wheels are massless and roll without slip.

If you pull the right pedal with $\vec{F}_P = -70 \text{ N } \hat{i}$, the FBD is shown on the left.

Since the wheels are massless, there is no friction on the front and a forward friction force on the back wheel.

The friction force $F_A \hat{i}$ on the rear wheel is static friction which does no work to the system. So we can use energy method.

Let the velocity of the bike be $\vec{V}_{\text{bike}} = V_b \hat{i}$

The velocity of the pedal is then $\vec{V}_{\text{pedal}} = \frac{3}{4} V_b \hat{i}$

$\therefore$ Power of $\vec{F}_P$

$$P_P = \vec{F}_P \cdot \vec{V}_{\text{pedal}} = -\frac{3}{4} V_b \times 70 = -52.5 V_b$$

(V_b in terms of "m/s")

Since $\vec{F}_P$ is the only force that does work to this system,

$$P_P = \dot{E}_K \quad \text{where } \dot{E}_K \text{ is rate of change of the kinetic energy of the system}$$

In this case

$$E_K = \frac{1}{2} m V_b^2 \Rightarrow \dot{E}_K = m V_b \dot{V}_b$$

$$\Rightarrow -52.5 V_b = 70 V_b \dot{V}_b \Rightarrow \dot{V}_b = -0.75 \text{ m/s}^2$$

$\therefore$ The acceleration of the bike is $\boxed{\vec{a}_{\text{bike}} = \dot{V}_b \hat{i} = -0.75 \text{ m/s}^2 \hat{i}}$

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(B) If Greg is riding on the bike, he pushes the pedal with $\vec{F}_p = -70 \text{ N } \hat{i}$ and he also receive the reaction force from the pedal of $70 \text{ N } \hat{i}$.

Velocity of the pedal is $\vec{V}_{\text{pedal}} = \frac{3}{4} V_b \hat{i}$

velocity of gr Greg is $\vec{V}_{\text{Greg}} = V_b \hat{i}$

Similary the total power to the system (Greg and bike) is

$$P = \vec{F}_p \cdot \vec{V}_{\text{pedal}} - \vec{F}_p \cdot \vec{V}_{\text{Greg}}$$

$$= -70 \text{ N } \hat{i} \cdot \left(\frac{3}{4} V_b \hat{i} - V_b \hat{i} \right) = 17.5 V_b \hat{i}$$

(V_b in terms of m/s)

$$\therefore P = \dot{E}_k = m V_b \dot{V}_b$$

$$\Rightarrow 17.5 V_b = 70 V_b \dot{V}_b \Rightarrow \dot{V}_b = 0.25 \text{ m/s}^2$$

$\therefore$ Greg's accelerative is

$$\boxed{\begin{aligned} \vec{a}_{\text{Greg}} &= \dot{\vec{V}}_{\text{Greg}} = \dot{V}_b \hat{i} \\ &= 0.25 \text{ m/s}^2 \hat{i} \end{aligned}}$$

This result means Greg ~~Lemond~~ accelerates forward by pushing the pedal backwards.

19.2.16 Which way does the bike accelerate? A bicycle with all frictionless bearings is standing still on level ground. A horizontal force F is applied on one of the pedals as shown. There is no slip between the wheels and the ground. The bicycle is gently balanced from falling over sideways. It is heavy enough so that both wheels stay on the ground. Does the bicycle accelerate forward, backward, or not at all? Make any reasonable assumptions about the dimensions and mass.

Justify your answer as clearly as you can, clearly enough to convince a person similar to yourself but who has not seen the experiment performed.



Problem 19.16

BIKE PEDAL-FORCE SOLUTION: WHICH WAY DOES THE BIKE GO?

A Ruina, April 20, 2009

Question. You stand next to a normal bike on flat ground. You delicately balance the bike with the steering straight. Your friend stands next to the bike and pushes backwards (to the left on the figure) on the bottom pedal. Which way does the bike accelerate?

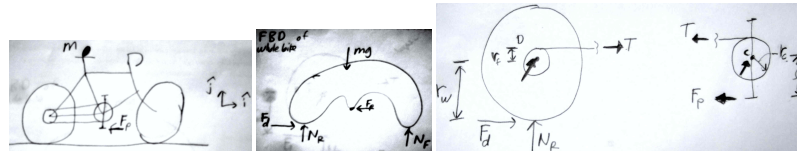
Assumptions. You apply negligible force. Your friend applies $-F_p \hat{i}$ to the pedal only. The rear wheel rolls. The only negligible mass is of the bike as a whole, m . All the bearings have negligible friction. The gearing of the bike is normal. The tension in the bottom part of the chain is negligible (not needed, but simplifies calculation). The key dimensions are

ℓ = the crank length. The crank is the piece from the middle of the bottom bracket to the pedal.

r_c = the radius of the chain wheel. The chain wheel is the gear that is in the front of the bike. It chain wheel is rigidly attached to the crank and rotates with the crank.

r_f = radius of the free-wheel sprocket. This is also called the cluster. It is the gear at the back of the bike. When there is tension on the upper chain, and the rear ratchet is thus engaged, the rear sprocket rotates with the rear wheel.

r_w = radius of the rear wheel.



Correct Answers.

- (1) The bike accelerates to the left. Try it. It always goes backwards. The next answer is different and is also correct.
- (2) The answer depends on the bike. Despite (1) above, the answer actually depends on what bike you try it on. For any bike you have ever seen or can buy, in any gear, answer (1) is correct. However, if the gearing is sufficiently low (how low? read solution below) then the bike will go forwards.

Reasoning.

A. The bike starts with no kinetic energy. It ends with some kinetic energy. Therefore some force had to do some positive work. The only force that has non-zero dot product with the velocity of the point it is touching is $-F_p \hat{i}$. Therefore *the pedal has to go backwards*. Does this mean the bike goes forwards or backwards? When a bike goes forwards the bottom pedal goes backwards relative to the bike. But relative to the ground it goes forwards. Otherwise people wouldn't ride bikes. The whole idea of a bike is that you go faster with a given leg motion than you do when you walk or run. So the bottom pedal goes the same direction as the bike. Since the pedal goes backwards, so does the bike.

B. Same as above, but let's calculate the pedal velocity and see if it really is in the same direction as the bike. Let's define ω_w and ω_c as the counterclockwise angular velocities of the rear wheel and crank,

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2

respectively. Assume bike and pedal have absolute velocity $v_b \hat{i}$ and $v_p \hat{i}$, then

$$\begin{aligned}
 (1) \quad v_p &= v_b + v_{p/b} \\
 (2) \quad &= v_b + \omega_c \ell \\
 (3) \quad &= v_b + \omega_w \frac{r_f}{r_c} \ell \\
 (4) \quad &= v_b + (-v_b/r_w) \frac{r_f}{r_c} \ell \\
 (5) \quad &= v_b \left(1 - \frac{r_f \ell}{r_c r_w} \right).
 \end{aligned}$$

So the pedal goes forward when the bike goes forward so long as $\frac{r_f \ell}{r_c r_w} < 1$. Thus the bike goes back when the pedal goes back with the same inequality. Such bikes all go back when you push back on the pedal. On the other hand, The bike would go forwards when you push back on the pedal if the bike has a ‘very low gear’. You could get such low gears from having some combination of a big rear sprocket radius r_f , a big crank length ℓ , a small front chain ring radius r_c and a small rear wheel radius r_w .

C. Here’s a different approach. Given F_p moment balance about C for the crank and chain wheel tells us the chain tension

$$T = \frac{\ell}{r_c} F_p.$$

Moment balance about the rear axle, for the rear wheel and freewheel gear tell us that the drive force F_d is

$$F_d = \frac{r_f}{r_w} T = \frac{r_f}{r_w} \underbrace{\left(\frac{\ell}{r_c} F_p \right)}_T.$$

Now look at linear momentum for the whole bike:

$$\begin{aligned}
 (6) \quad -F_p + F_d &= ma_b \\
 (7) \quad -F_p + \underbrace{\frac{r_f}{r_w} \frac{\ell}{r_c} F_p}_{F_d} &= ma_b \\
 (8) \quad \Rightarrow a_b &= -\frac{F_p}{m} \left(1 - \frac{r_f \ell}{r_w r_c} \right).
 \end{aligned}$$

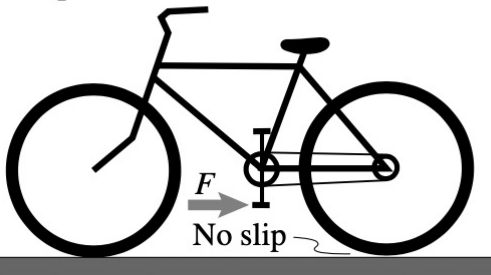
So, now by different reasoning, if $\frac{r_f \ell}{r_c r_w} < 1$ then a push back of F_p causes a negative acceleration a_b of the bike. That is, if $\frac{r_f \ell}{r_c r_w} < 1$ (which it is for all real bikes) then the bike accelerates backwards when you push backwards on the pedal.

MAE 2030 HW14 Problem 76 Solution

19.2.16

Dopei Yu

19.2.16 Which way does the bike accelerate? A bicycle with all frictionless bearings is standing still on level ground. A horizontal force F is applied on one of the pedals as shown. There is no slip between the wheels and the ground. The bicycle is gently balanced from falling over sideways. It is heavy enough so that both wheels stay on the ground. Does the bicycle accelerate forward, backward, or not at all? Make any reasonable assumptions about the dimensions and mass. Justify your answer as clearly as you can, clearly enough to convince a person similar to yourself but who has not seen the experiment performed.



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Problem 19.2.16

Given: horizontal force F applied at pedal
both wheels on ground
no slip
balanced

Find: direction of acceleration of bicycle

Solution: multiple FBDs

Assumptions: • pedal gears are massless

• bicycle wheels are massless

• only upper half of chain is in tension (bottom half slack)

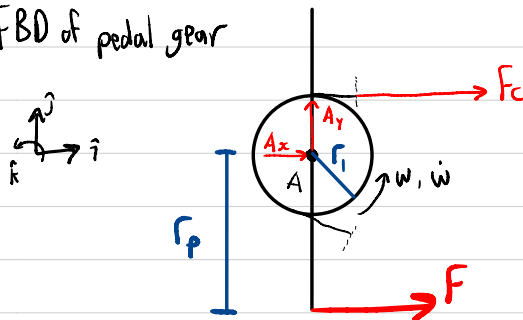
• wheels and pedals are uniform

• front and back pedal gears are same radius

• bicycle is rigid (all pts. have same $\vec{a}$)

• bicycle has mass m

FBD of pedal gear



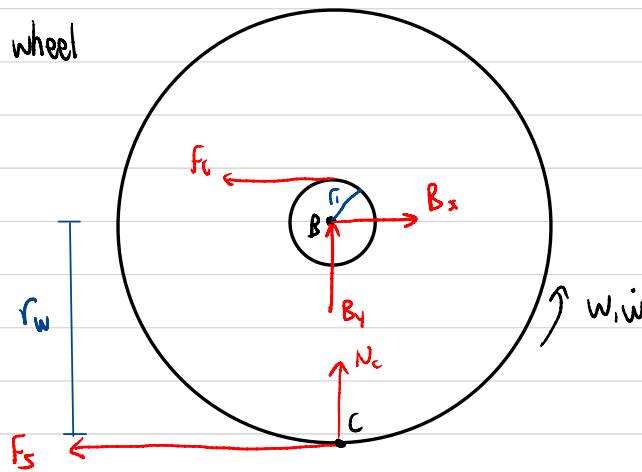
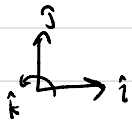
Assuming massless pedal:

$$\sum M_{A/B} : F r_p - F_c r_1 = 0$$

$$\textcircled{1} F_c = \frac{r_p}{r_1} F \quad \text{and since } r_p > r_1, F_c > F$$

* Assume only top half of chain is in tension (bottom half is slack) *

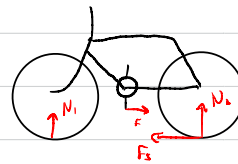
FBD of rear wheel



$$\text{AMB}_{/B} : F_c r_i - F_s r_w = 0 \quad \text{plug in ①}$$

$$F_s = \frac{r_i}{r_w} F_c = \frac{r_i}{r_w} \frac{r_p}{r_i} F = \frac{r_p}{r_w} F \quad \text{②}$$

LMB of entire bike in x-direction:



$$\sum F_x = F - F_s = m a_G$$

$$F - \frac{r_p}{r_w} F = m a_G \rightarrow a_G = \left(1 - \frac{r_p}{r_w}\right) \frac{F}{m}$$

since $r_p < r_w$, $1 - \frac{r_p}{r_w} > 0$, and thus a_G is positive.

Also, since both wheels are in contact with ground, $\vec{a}_G = a_G \hat{i}$.

Since a_G is positive, the bicycle accelerates to the right.