19.2 One-degree-of-freedom 2-D mechanisms

19.2.1 A conservative vibratory system has the following equation of conservation of energy.

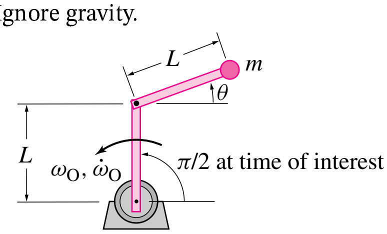
$$m(\ell\dot{\theta})^2 - mg\ell(1-\cos\theta) + K(a\theta)^2 = E_0$$

where E_0 is a constant.

a) Obtain the differential equation of motion of this system by differentiating this energy equation with respect to t.

b) Determine the circular frequency of small oscillations of the system in part (a). HINT: (Let $\sin \cong \theta \cong$ and $\cos\theta \cong 1-\theta^2/2$).

19.2.2 A motor at O turns at rate ω_o whose rate of change is $\dot{\omega}_o$. At the end of a stick connected to this motor is a frictionless hinge attached to a second massless stick. Both sticks have length L. At the end of the second stick is a mass m. For the configuration shown, what is $\ddot{\theta}$? Answer in terms of ω_o, $\dot{\omega}_o$, L, m, θ, and $\dot{\theta}$. Ignore gravity.



Problem 19.2.2

19.2.3 In problem 18.3.4, find

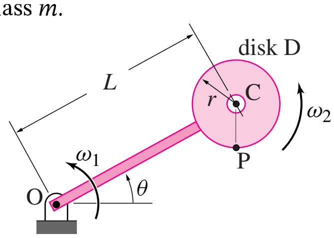
a) the angular momentum about point O, *

b) the rate of change of angular momentum of the disk about point O, *

c) the angular momentum about point C, *

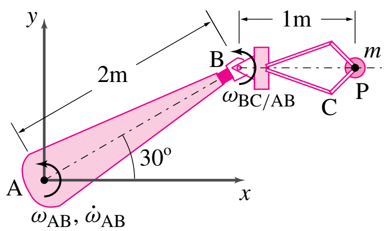
d) the rate of change of angular momentum of the disk about point C. *

Assume the rod is massless and the disk has mass m.



Problem 19.2.3

19.2.4 Robot arm, 2-D . The robot arm AB is rotating about point A with $\omega_{AB} = 5\,\text{rad/s}$ and $\dot{\omega}_{AB} = 2\,\text{rad/s}^2$. Meanwhile the forearm BC is rotating at a constant angular speed *with respect to AB* of $\omega_{BC/AB} = 3\,\text{rad/s}$. Gravity cannot be neglected. At the instant shown, find the *net* force acting on the object P which has mass $m = 1\,\text{kg}$. *



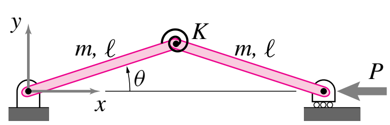
Problem 19.2.4

19.2.5 A crude model for a column, shown in the figure, consists of two identical rods of mass m and length ℓ with hinge connections, a linear torsional spring of stiffness K attached to the center hinge (the spring is relaxed when $\theta = 0$), and a load P applied at the top end.

a) Obtain the exact nonlinear equation of motion.

b) Obtain the squared natural frequency for small motions θ.

c) Check the stability of the straight equilibrium state $\theta = 0$ for all $P \geq 0$ via minimum potential energy. How do these results compare with those from part(b)?



Problem 19.2.5

19.2.6 A thin uniform rod of mass m rests against a frictionless wall and on a frictionless floor. There is gravity.

a) Draw a free-body diagram of the rod.

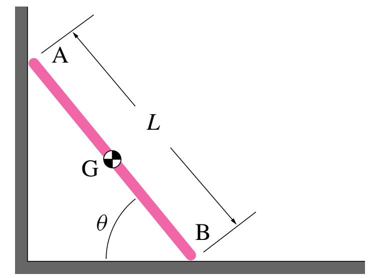
b) The rod is released from rest at $\theta = \theta_0 \neq 0$. Write the equation of motion of the rod. *

c) Using the equation of motion, find the initial angular acceleration, $\dot{\boldsymbol{\omega}}_{AB}$, and the acceleration of the center of mass, $\vec{\boldsymbol{a}}_G$, of the rod. *

d) Find the reactions on the rod at points A and B. *

e) Find the acceleration of point B. *

f) When $\theta = \frac{\theta_0}{2}$, find $\vec{\boldsymbol{\omega}}_{AB}$ and the acceleration of point A. *



Problem 19.2.6

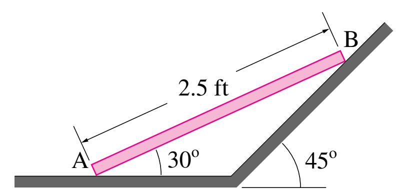
19.2.7 Bar leans on a crooked wall. A uniform 3 lbm bar leans on a wall and floor and is let go from rest. Gravity pulls it down.

a) Draw a Free-Body Diagram of the bar.

b) Kinematics: find the velocity and acceleration of point B in terms of the velocity and acceleration of point A

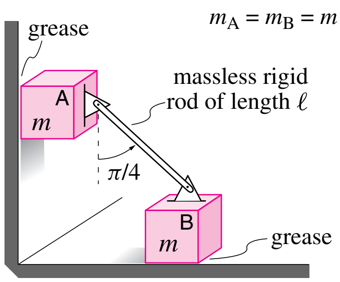
c) Using equations of motion, find the acceleration of point A.

d) Do the reaction forces at A and B add up to the weight of the bar? Why or why not? (You do not need to solve for the reaction forces in order to answer this part.)



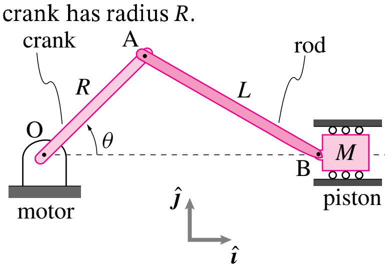
Problem 19.2.7

19.2.8 Two blocks, each with mass m, slide without friction on the wall and floor shown. They are attached with a rigid massless rod of length ℓ that is pinned at both ends. The system is released from rest when the rod makes an angle of 45°. What is the acceleration of the block B immediately after the system is released?



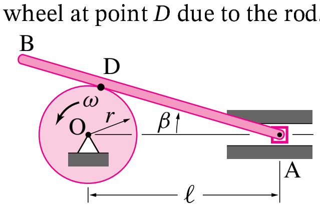
Problem 19.2.8

19.2.9 Slider Crank. 2-D . No gravity. Refer to the figure in problem 18.4.2. What is the tension in the massless rod AB (with length L) when the slider crank is in the position with $\theta = 0$ (piston is at maximum extent)? Assume the crankshaft has constant angular velocity ω, that the connecting rod AB is massless, that the cylinder walls are frictionless, that there is no gas pressure in the cylinder, that the piston has mass M and the crank has radius R.



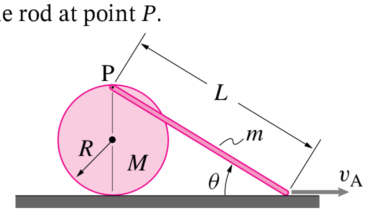
Problem 19.2.9

19.2.10 In problem 18.4.8, find the force on the wheel at point D due to the rod.



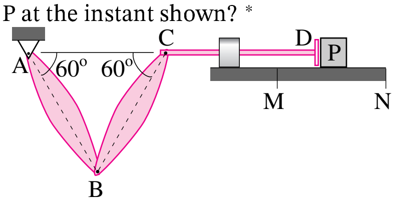
Problem 19.2.10

19.2.11 In problem 18.4.9, find the force on the rod at point P.



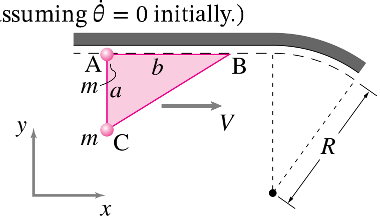
Problem 19.2.11

19.2.12 Slider-Crank mechanism. The slider crank mechanism shown is used to push a 2 lbm block P. Arm AB and BC are each 0.5 ft long. Given that arm AB rotates counterclockwise at a constant 2 revolutions per second, what is the force on P at the instant shown? *



Problem 19.2.12

19.2.13 The large masses m at C and A were supported by the light triangular plate ABC, whose corners follow the guide. B enters the curved guide with velocity V. Neglecting gravity, find the vertical reactions (in the y direction) at A and B. (Hint: for the rigid planar body ABC, find $\ddot{y}_A$ and $\ddot{y}_B$ in terms of a_y^{cm} and $\ddot{\theta}$, assuming $\dot{\theta} = 0$ initially.)



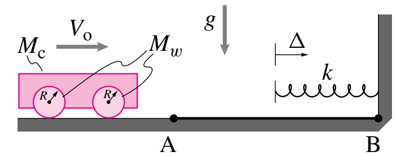
Problem 19.2.13

19.2.14 An idealized model for a car comprises a rigid chassis of mass M_c and four identical rigid disks (wheels) of mass M_w and radius R, as shown in the figure. Initially in motion with speed v_0, the car is momentarily brought to rest by compressing an initially uncompressed spring of stiffness k. Assume no frictional losses.

a) Assuming no wheel slip, determine the compression Δ of the spring required to stop the car. *

b) While the car is in contact with the spring but still moving forward, in which direction is the *tangential* force on any one of the wheels (due to contact with the ground) pointing? Why? (Illustrate with a sketch). *

c) Repeat part (a) assuming now that the ground is perfectly frictionless from points A to B shown in the figure. *

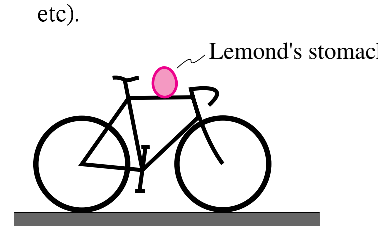


Problem 19.2.14

19.2.15 Assume Greg Lemond's riding "tuck" was so good that you can neglect air resistance when you think about him and his bike. Further, you can regard his and the bike's combined mass (all 70 kg)) as concentrated at a point in his stomach somewhere. Greg's left foot has just fallen off the pedal so he is only pedaling with his right foot, which at the moment in question is at its lowest point in the motion. You note that, relative to the ground the right foot is only going 3/4 as fast as the bike (since it is going backwards relative to the bike), though you can't make out all the radii of his frictionless gears and rigid round wheels. Greg, ever in touch with his body, tells you he is pushing back on the pedal with a force of 70 N. You would like to know Greg's acceleration.

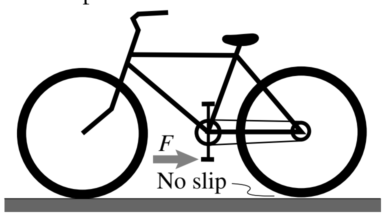
a) In your first misconceived experiment you set up a 70 kg bicycle in your laboratory and balance it with strings that cause no fore or aft forces. You tie a string to the vertically down right pedal and pull back with a force of 70 N. What acceleration do you measure?

b) What is Greg's actual acceleration? [Hint: Greg's massless leg is pushing forward on his body with a force of 70 N] You can neglect the mass of the wheels and other transmission parts (chain, crank, etc).



Problem 19.2.15

19.2.16 Which way does the bike accelerate? A bicycle with all frictionless bearings is standing still on level ground. A horizontal force F is applied on one of the pedals as shown. There is no slip between the wheels and the ground. The bicycle is gently balanced from falling over sideways. It is heavy enough so that both wheels stay on the ground. Does the bicycle accelerate forward, backward, or not at all? Make any reasonable assumptions about the dimensions and mass. Justify your answer as clearly as you can, clearly enough to convince a person similar to yourself but who has not seen the experiment performed.



Problem 19.2.16