19.1.1 A bead slides on a frictionless circular hoop . The mass of the bead $m_{\text{bead}} = 2$ grams, mass of hoop $m_{\text{hoop}} = 1$ kg and radius of hoop $R_{\text{hoop}} = 3$ m. Neglect gravity. The center of the circular hoop is the origin O of a fixed (Newtonian) coordinate system $Oxyz$. The hoop is on the xy plane. The hoop is kept from moving by little angels who let the bead slide by unimpeded. At $t = 0$, the speed of the bead is 4 m/s, it is traveling counter-clockwise (looking down the z-axis), and it is on the $+x$-axis. There are no other external forces applied.

a) At $t = 0$ what is the bead's kinetic energy ?

b) At $t = 0$ what is the bead's linear momentum ?

c) At $t = 0$ what is the bead's angular momentum about the origin?

d) At $t = 0$ what is the bead's acceleration?

e) At $t = 0$ what is the radius of the osculating circle of the bead's path ρ?

f) At $t = 0$ what is the force of the hoop on the bead?

g) At $t = 0$ what is the net force of the angel's hands on the hoop?

h) At $t = 27s$ what is the x-component of the bead's linear momentum?

18.1.1

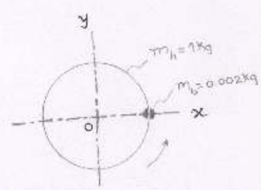
Given mass of bead, $m_b = 2$ gram

mass of the hoop, $m_h = 1$ Kg

radius of the hoop, $r_h = 3$ m

at $t=0$, bead speed, $V_b = 4$ m/s

Neglect gravity



a) Bead's Kinetic energy, $E_K = \frac{1}{2} m_b \cdot V_b^2$ at $t=0$

$\Rightarrow E_K = \frac{1}{2} \times 0.002 \times 4$ J $= 0.016$ J

b) Beads' Linear momentum at $t=0$, $L = mV = 0.008$ N.s

c) Beads' angular momentum at $t=0$, $\underline{H}_{/o} = \underline{r} \times m\underline{v}$

$\Rightarrow \underline{H}_{/o} = 3\hat{\imath} \times m_b 4\hat{\jmath}$ kg m²/s $= 0.024\hat{k}$ N.m.s.

d) Beads' acceleration at $t=0$, $\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$ [given $\dot{\underline{\omega}} = 0$]

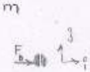
$\Rightarrow \underline{a} = -\omega^2 r_h \hat{\imath}$ as $\underline{v} = \underline{\omega} \times \underline{r}$, $\omega = \frac{4}{3}$ rad/s

$\Rightarrow \underline{a} = -\frac{16}{3}\hat{\imath}$

e) Radius of osculating circle, $\rho = \frac{1}{|\kappa|}$

As $\underline{\kappa} = \frac{d\hat{e}_t}{ds} \Rightarrow |\kappa| = \left|\frac{d\theta \cdot \hat{e}_t}{R d\theta}\right| = \frac{1}{R} \Rightarrow \rho = R = 3$ m

f) Applying L.M.B on bead, $\Sigma \underline{F} = m\underline{a}$

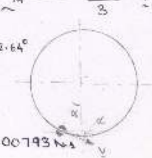


$\Rightarrow \{F_b \hat{\imath} = m\underline{a}\}$; $F_b\hat{\imath} = 0.002 \frac{16}{3}$ N$\hat{\imath}$ $\Rightarrow \{\ \}\cdot\hat{\imath} \Rightarrow F_b = -\frac{0.032}{3}$ N

$\therefore F_b$ is acting towards the center

g) Two forces act on the hoop, one from bead, other from angels' hands(F_A). As acceleration of hoop $= 0$, $\therefore \underline{F}_A = -\frac{0.032}{3}\hat{\imath}$

h) At $t = 27$s, $\theta(t) = \omega . t \Big|_{t=27s} \Rightarrow \theta = 36$ rad. $= 2062.64°$



As 6 cycles correspond to 2160°, $\alpha = 7.35°$

x component of $\underline{L} \Rightarrow \{\underline{L} = m\underline{v}\}\cdot\hat{\imath} \Rightarrow L_x = mv\cos 7.35° = 0.00793$ N.s

18.1.1

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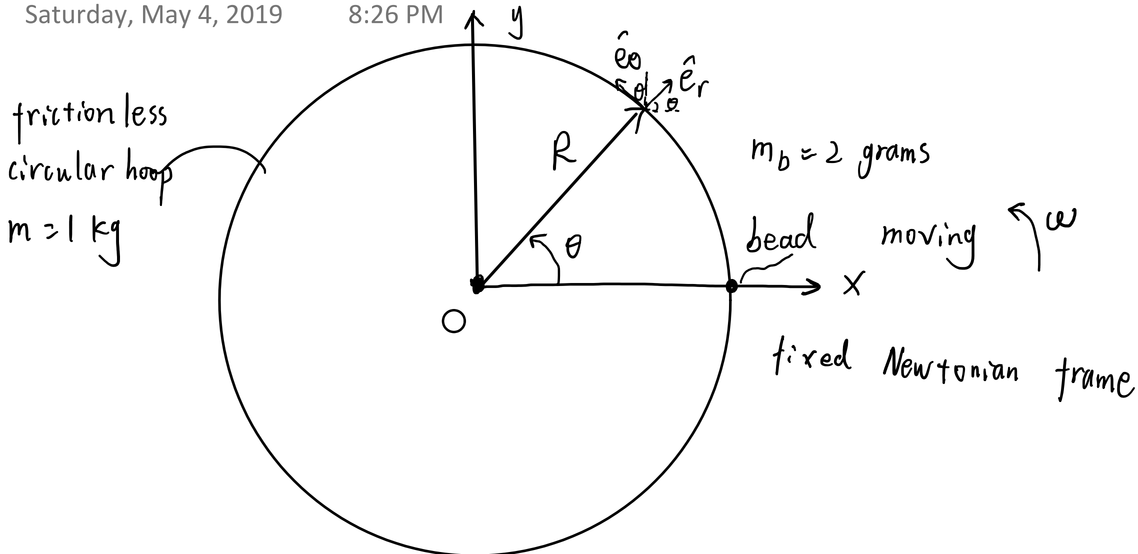


Given: $R = 3m$

$V_b(t=0) = 4\ m/s = V_{b0}$

$t=0$, bead is on the $+x$ axis

At $t=0$

a) Find the bead's kinetic energy

$$E_{k0} = \frac{1}{2} m_b V_b^2(t=0) = \frac{1}{2} \times 2g \times (4m/s)^2 = 0.016\ kg\,m^2/s^2$$

b) Find the bead's linear momentum

$$\vec{L} = m\vec{V}_{b0} = 2g \times 4m/s\ \hat{e}_\theta = 0.008\ \hat{e}_\theta\ kg\,m/s$$

c) Find the bead's angular momentum /O.

$$\vec{H}_{/O} = \vec{r}_{/O} \times m\vec{v} = 3\hat{i} \times 2g \cdot 4\ m/s\ \hat{j} = 0.024\ \hat{k}\ [kg \cdot m/s]$$

d) Find the bead's acceleration

$$\vec{a}_0 = -\frac{V_{b0}^2}{R}\hat{i} = -\frac{(4m/s)^2}{3m}\hat{i} = -5.33\ \hat{i}\ [m/s^2]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

e) Find the radius ρ of the osculating circle of bead's path

$$\rho = \frac{V_{b0}^2}{a_0} = \frac{(4m/s)^2}{5.33\, m/s^2} = 3m = \text{radius of the circle}$$

Circle kisses itself

f) Find the force on the bead

LMB: $\Sigma \vec{F}_0 = m\vec{a}_0 = 2g\ (-5.33\hat{i}\ m/s^2) = -0.0106\,\hat{i}\ [kg\, m/s^2]$

g) Find the net force on the hoop.

LMB: $\Sigma \vec{F}_0' = m\vec{a}_0 = \Sigma \vec{F}_0 = -0.0106\,\hat{i}\ [kg\, m/s]$

h) At $t = 27s$ Find the x-component of the bead's Linear momentum

No friction, and the force on the bead is perpendicular to its velocity. Thus the bead is moving at const speed (acceleration is perpendicular to velocity all the time)

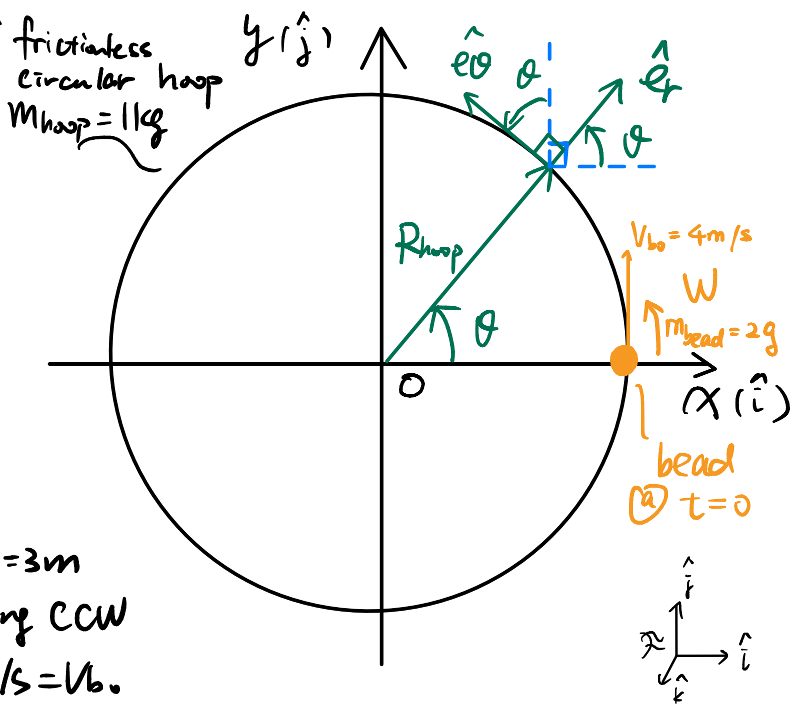
Angular distance $\theta = \frac{V_b t}{R} = \frac{4m/s \cdot 27s}{3m} = 36$ rad

$$x = R \cdot \cos\theta = 3m \cdot \cos 36 = -0.5739\ [m]$$

$$L_x = \vec{L} \cdot \hat{i} = m\vec{V} \cdot \hat{i} = mV\hat{e}_\theta \cdot \hat{i} = mV(-\sin\theta)$$

$$= 0.008\ kg\,m/s \cdot (-0.99) = -0.0079\ kg\,m/s$$

Katherine Cao edited from Shuwen Lu



Given: no gravity
hoop frictionless
$M_{hoop} = 1\,kg$ $R_{hoop} = 3m$
$m_{bead} = 2g$ traveling CCW
$V_{bead}(t=0) = 4m/s = V_{b_0}$
$t=0$ bead on $+x$ axis
fixed newtonian coord $Oxyz$ (z out of page) unit vectors $\hat{i}\ \hat{j}\ \hat{k}$

Find:

$$E_{k_0} = \frac{1}{2} m_{bead} V_{b_0}^2 = \frac{1}{2}(2g)(4m/s)^2 = \boxed{0.016\,[J]}$$

$$\vec{L}_0 = m\vec{V}_{b_0} = mV_{b_0}\hat{j} = (2g)(4m/s)\,\hat{j} = \boxed{0.008\,\hat{j}\,[kg\,m/s]}$$

$$\vec{H}_{/0} = \vec{r}_{/0} \times \underbrace{(m\vec{V}_{b_0})}_{\vec{L}_0} = (R_{hoop}\,\hat{i}) \times (mV_{b_0}\,\hat{j})$$

$$= (3m\,\hat{i}) \times (0.008\,\hat{j}\ kg\,m/s) = \boxed{0.024\,\hat{k}\,[kg\,m^2/s]}$$

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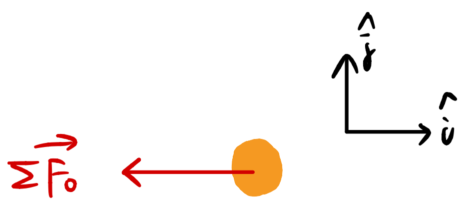
$$\vec{a}_o = -\frac{V_{bo}^2}{R_{hoop}}\,\hat{\imath} = -\frac{(4m/s)^2}{3m}\,\hat{\imath} = \boxed{-5.33\,\hat{\imath}\ [m/s^2]}$$

$$\rho = \frac{V_{bo}^2}{|\vec{a}_o|} = \frac{(4m/s)^2}{5.33m/s^2} = \boxed{3m = R_{hoop}}$$

"backwards" of d) circle kisses itself



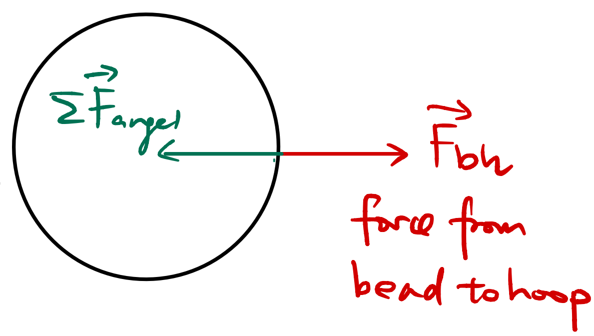
LMB: $\Sigma\vec{F}_o = m\vec{a}_o$

$$= (2g)(-5.33\,\hat{\imath}\ m/s^2)$$

$$\boxed{\Sigma\vec{F}_o = -0.0107\ \hat{\imath}\ [kg\ m/s^2]}$$

$$\Sigma\vec{F}_{hoop} = \Sigma\vec{F}_{angel} + \vec{F}_{bh}$$

$$= m\vec{a}_{hoop} = \vec{0}\ \text{(not moving)}$$



$$\Sigma\vec{F}_{angel} = -\vec{F}_{bh}$$

$$\vec{F}_{bh} = -\Sigma\vec{F}_o \quad \text{(action and reaction forces btw bead \& hoop)}$$

$$\Rightarrow \Sigma\vec{F}_{angel} = -\vec{F}_{bh} = -(-\Sigma\vec{F}_o) = \Sigma\vec{F}_o = \boxed{-0.0107\ \hat{\imath}\ [kg\ m/s^2]}$$

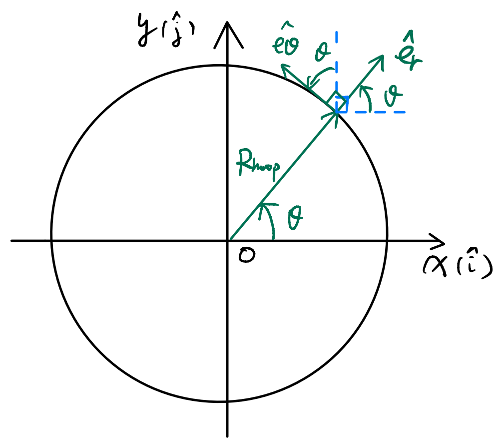
No friction

$\Rightarrow$ force on the bead perpendicular to $\vec{V}_{bead}$ all the time

$\Rightarrow$ constant $V_{bead} = |\vec{V}_{bead}| = 4\,m/s$

Angular distance traveled:

$$\theta = \frac{V_{bead} \cdot t}{R_{hoop}} = \frac{(4\,m/s)(27\,s)}{3\,m} = 36\,rad$$



$$\vec{L} = m\vec{V}_{bead} = mV_{bead}\,\hat{e}_\theta$$

From figure on the right,

$$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\vec{L}_x = (\vec{L}\cdot\hat{i})\,\hat{i}$$

$$= mV_{bead}(\hat{e}_\theta\cdot\hat{i})\,\hat{i}$$

$$= mV_{bead}(-\sin\theta)\,\hat{i}$$

$$= (2g)(4\,m/s)(-\sin(36\,rad))\,\hat{i}$$

$$= \boxed{0.00793\ [kg\ m/s]}$$

19.1.2 A warehouse operator wants to move a crate of weight $W = 100\,\text{lb}$ from a 6 ft high platform to ground level by means of a roller conveyer as shown. The rollers on the inclined plane are well lubricated and thus assumed frictionless; the rollers on the horizontal conveyer are frictional, thus providing an effective friction coefficient μ. Assume the rollers are massless.

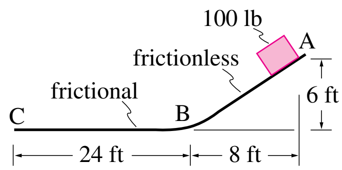
a) What are the kinetic and potential energies(pick a suitable datum) of the crate at the elevated platform, point A?

b) What are the kinetic and potential energies of the crate at the end of the inclined plane just before it moves onto the horizontal conveyor, point B?

c) Using conservation of energy, calculate the maximum speed attained by the crate at point B, assuming it starts moving from rest at point A.

d) If the crate slides a distance x, say, on the horizontal conveyor, what is the energy lost to sliding in terms of μ, W, and x?

e) Calculate the value of the coefficient of friction, μ, such that the crate comes to rest at point C.



Problem 19.2

19.1.2

Weight of the crate, W = 100 lb

a) Kinetic and Potential Energies:

Let BC be the datum

Potential Energy, $E_p^A = mgh$

$m = 45.36\,kg$ $h = 1.83\,m$ $\therefore E_p^A = 45.36 \times 9.81 \times 1.83$

$\Rightarrow \boxed{E_p^A = 814.32\,J}$

As the mass is stationary, Kinetic energy $\boxed{E_K^A = 0\,J}$

b) Potential energy at point B, $\boxed{E_p^B = 0\,J}$ as $h = 0$

As total energy remains constant, $E_{tot}^A = E_{tot}^B$

$E_p^A + E_k^A = E_p^B + E_k^B$

$814.32 + 0 = 0 + \frac{1}{2} m v_B^2$

$\therefore$ Kinetic energy at B, $\boxed{E_K^B = 814.32\,J}$

c) $v_B^2 = \frac{2 E_{tot}^A}{m}$ from conservation of Energy $E_{tot}^A = E_{tot}^B$

$\Rightarrow$ Velocity of the crate at B, $\boxed{v_B = \sqrt{\frac{2 \times 814.32}{45.36}} = 5.99\,\frac{m}{s}}$

d) Energy lost to sliding,

$E_s = \int_0^x F_f \, dx$

where F_f = frictional force, $= \mu N = \mu mg$

$\Rightarrow E_s = \int_0^x \mu mg \, dx \Rightarrow \boxed{E_s = \mu mgx}$

e) Using the principle of work & energy from B to C

$\Sigma U = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

(or) $\frac{1}{2} m v_i^2 + \Sigma U = \frac{1}{2} m v_f^2$

Final velocity, $v_f = 0$

$\Rightarrow \frac{1}{2} m v_B^2 + [-E_s] = 0$

$\Rightarrow \frac{1}{2} m v_B^2 - \mu mg x = 0$

$\Rightarrow \mu = \frac{m v_B^2}{2 m g x}$ (or) $\mu = \frac{v_B^2}{2 g x}$

In the given configuration $x = 24\,ft = 7.3\,m$

$\boxed{\mu = 0.25}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.3 On a wintry evening, a student of mass m starts down the steepest street in town, *Steep Street*, which is of height h and slope θ. At the top of the slope she starts to slide (coefficient of dynamic friction μ).

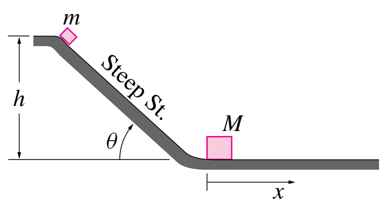
a) What is her initial kinetic and potential energy?

b) What is the energy lost to friction as she slides down the hill?

c) What is her velocity on reaching the bottom of the hill? Ignore air resistance and all cross streets (i.e. assume the hill is of constant slope).

d) If, upon reaching the bottom of the street, she collides with another student of mass M and they embrace, what is their instantaneous mutual velocity just after the embrace?

e) Assuming that friction still acts on the level flats on the bottom, how much time will it take before they come to rest?



Problem 19.3

13.1.3

a) Initial kinetic energy of m

$E_k^A = 0$

Initial Potential energy of m

$E_p^A = mgh$

b) LMB of m

$\{\sum \vec{F} = m\vec{a}\}\cdot\hat{n}$

$N - mg\cos\theta = m\vec{a}\cdot\hat{n} = 0$

∴ Frictional force, $f_f = \mu N = \mu mg\cos\theta$

Energy lost to friction, $E_s = f_f\, d$

$\Rightarrow E_s = \mu mg\cos\theta \frac{h}{\sin\theta}$

∴ Energy lost in sliding, $E_s = \frac{\mu mgh}{\tan\theta}$

c) Using Principle of work and energy,

$\sum U = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$\sum U$ = work done by the forces mg and F_f

$\sum U = +mgh - E_s$

As $v_i = 0$

$\frac{1}{2}mv_f^2 = mgh - \frac{\mu mgh}{\tan\theta} = mgh\left(1-\frac{\mu}{\tan\theta}\right)$

$\Rightarrow v_f = \sqrt{2gh\left(1-\frac{\mu}{\tan\theta}\right)}$

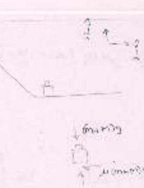
d) If m and M embrace after collision, from Linear momentum balance, we have

$\vec{L} = \vec{L}'$

$mv_f + M(0) = (m+M)v$

$\Rightarrow v = \frac{m}{m+M}\sqrt{2gh\left(1-\frac{\mu}{\tan\theta}\right)}$

c) LMB on (m+M)



$\{\sum \vec{F} = m\vec{a}\}\cdot\hat{i}$

$-\mu(m+M)g = (m+M)a_x$

$\Rightarrow a_x = -\mu g$

Using $v_f = v_i + at$

to $v_f = 0$, $t = \frac{-v_i}{a}$

$\Rightarrow t = \frac{v}{\mu g}$ ∴ $t = \frac{m}{\mu g(m+M)}\sqrt{2gh\left(1-\frac{\mu}{\tan\theta}\right)}$

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19.1.4 Reconsider the system of blocks in problem **??**, this time with equal mass, $m_1 = m_2 = m$. Also, now, both blocks are frictional and sitting on a frictional surface. Assume that both blocks are sliding to the right with the top block moving faster. The coefficient of friction between the two blocks is μ_1. The coefficient of friction between the lower block and the floor is μ_2. It is known that $\mu_1 > 2\mu_2$.

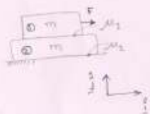
a) Draw free-body diagrams of the blocks together and separately.

b) Write the equations of linear momentum balance for each block.

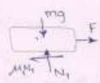
c) Find the acceleration of each block for the case of frictionless blocks. For the frictional blocks, find the acceleration of each block. What happens if $\mu_1 >> \mu_2$?

18.1.4

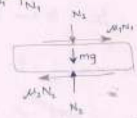
a) FBDs of blocks



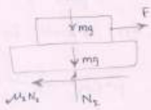
top block ①



Bottom block ②



Together



b) $\{\sum \underline{F}_1 = m_1 \underline{a}_1\} \Rightarrow \underline{F} + \underline{N}_1 - mg\hat{j} - \mu_1 N_1 \hat{i} = m_1 \underline{a}$

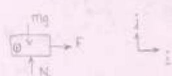
$\{\ \}\cdot\hat{j} \Rightarrow -mg + N_1 = 0 \Rightarrow N = mg$

For block ② $\quad \sum \underline{F} = \dot{\underline{L}} \Rightarrow \underline{N}_1 - mg\hat{j} + \underline{N}_2 + \mu_1 N_1 \hat{i} - \mu_2 N_2 \hat{i} = m_2 \underline{a}$

$\Rightarrow \{\ \}\cdot\hat{i}$

$\Rightarrow \mu_1 N_1 - \mu_2 N_2 = m a_2$

c) Frictionless block



① $\sum \underline{F} = \dot{\underline{L}}$

$\{F\hat{i} + N_1\hat{j} - mg\hat{j} = m\underline{a}\}$

$\{\ \}\cdot\hat{i} \Rightarrow a_x^{①} = \frac{F}{m}$

② LMB on block ②

$\sum \underline{F} = \dot{\underline{L}}$

$\{-N_1\hat{j} - mg\hat{j} + N_2\hat{j} = m\underline{a}\}$

$\{\ \}\cdot\hat{i} \Rightarrow m a_x = 0 \Rightarrow a_x^{②} = 0$

Frictional blocks

① $\sum \underline{F} = \dot{\underline{L}}$

From the FBD and ⓑ $\quad a_x^{①} = \frac{1}{m}(F - \mu_1 N_1)$

for block ②. $\quad a_x^{②} = \frac{1}{m}(\mu_1 N_1 - \mu_2 N_2)$

$N_1 = mg$ and $N_2 = 2mg$

Acceleration of block ②, $a_x^{②} = \frac{1}{m}(\mu_1 mg - \mu_2 2mg)$

$a_x^{②} = \frac{1}{m}(\mu_1 - 2\mu_2) mg$

(or) $a_x^{②} = (\mu_1 - 2\mu_2) g$

If $\mu_1 >> \mu_2$, the relative acceleration between the

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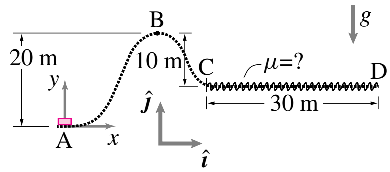
19.1.5 An initially motionless roller-coaster car of mass 20 kg is given a horizontal impulse $\int \vec{F}\, dt = P\hat{\imath}$ at position A, causing it to move along the track, as shown below.

a) Assuming that the track is perfectly frictionless from point A to C and that the car never leaves the track, determine the magnitude of the impulse I so that the car "just makes it" over the hill at B.

b) In the ensuing motion, assume that the horizontal track C-D is frictional and determine the coefficient of friction required to bring the car to a stop at D.

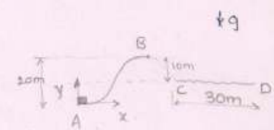


Problem 19.5

18.1.5

Mass of roller-coaster car, $m = 20\,\text{Kg}$

Impulse, $\int \underline{F}\, dt = P\underline{i}$



a) Find impulse, I, so that the car "just makes it" over the hill at B

Using the linear impulse - momentum relationship, at A

$$P\underline{i} = \int \underline{F}\, dt = \underline{L}^{+} - \underline{L}^{-} = m(\underline{v}^{+} - \underline{v}^{-}) \quad [\text{given } \underline{v}^{-} = \underline{0}]$$

$$\Rightarrow |P\underline{i}| = I = mv^{+}$$

track from A through C is frictionless, hence, using energy balance,

$$E^{A}_{tot} = E^{B}_{tot}$$

$$\frac{1}{2} m v^{+2} = mgh \Rightarrow v^{+} = \sqrt{2gh}$$

$$\therefore I = m\sqrt{2gh} \Rightarrow \boxed{I = 20\sqrt{2 \cdot 9.81 \cdot 20} = 396.18\ \text{N·s}}$$

b) Using the principle of work & energy

$$\Sigma U = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \qquad v_f = 0$$

$$-\mu N \cdot d = -\frac{1}{2} m v_i^2$$

m
μN
N

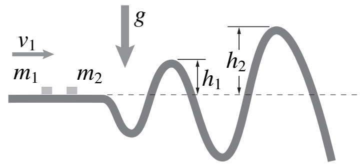
$$\mu = \frac{m v_i^2}{N d}$$, v_i is obtained from energy balance, between B and C

$$E^{B}_{tot} = E^{C}_{tot} \Rightarrow mgh = mg\frac{h}{2} + \frac{1}{2} m v_i^2 \Rightarrow v_i = \sqrt{gh}$$

$$\therefore \mu = \frac{mgh}{Nd}$$ ∴ Required $\boxed{\mu \text{ to stop Car at D} = 0.67}$

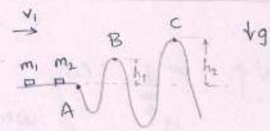
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.6 Masses $m_1 = 1\,\text{kg}$ and $m_2\,\text{kg}$ move on the frictionless varied terrain shown. Initially, m_1 has speed $v_1 = 12\,\text{m/s}$ and m_2 is at rest. The two masses collide on the flat section. The coefficient of restitution in the collision is $e = 0.5$. Find the speed of m_2 at the top of the first hill of elevation $h_1 = 1\,\text{m}$. Does m_2 make it over the second hill of elevation $h_2 = 4\,\text{m}$?



Problem 19.6

18.1.6



Speed of $m_1 = V_1^- = 12\ m/s$

Speed of m_2, $V_2^- = 0$.

Coefficient of restitution, $e = 0.5$

From definition,
$$e = \frac{V_2^+ - V_1^+}{V_1^- - V_2^-}$$

$$V_2^+ - V_1^+ = 0.5\,(12 - 0)\tfrac{m}{s} = 6\ m/s$$

$$m_1 V_1^- + m_2 V_2^- = m_1 V_1^+ + m_2 V_2^+ \qquad \text{(Conservation of Momentum)}$$

$$12\tfrac{m}{s} m_1 + 0\tfrac{m}{s} = m_1 V_1^+ + m_2 (V_1^+ + 6\tfrac{m}{s})$$

$$\Rightarrow V_1^+ = \frac{12 m_1 - 6 m_2}{(m_1 + m_2)}\ \tfrac{m}{s}$$

If $m_1 = 1\,\text{kg}$ and $m_2 = 2\,\text{kg}$, $V_1^+ = 0$ & $V_2^+ = 6\ m/s$

Using energy balance, $E^A_{tot} = E^B_{tot} = E^C_{tot}$

$$\tfrac{1}{2} m_2 V_A^2 + 0 = m_2 g h_1 + \tfrac{1}{2} m_2 V_B^2$$

$$\Rightarrow V_B^2 = V_A^2 - 2gh_1$$

$$\Rightarrow V_B = \sqrt{V_2^{+2} - 2gh_1} \Rightarrow V_B = 4.05\ m/s$$

Consider $E^A_{tot} = E^C_{tot}$

$$\tfrac{1}{2} m_2 V_2^{+2} = m_2 g h_2 + \tfrac{1}{2} m_2 V_C^2 \Rightarrow V_C^2 = V_2^{+2} - 2gh_2$$

$$V_C^2 = [36 - 78.48]\left(\tfrac{m}{s}\right)^2 \Rightarrow V_C^2 < 0$$

$\therefore$ m_2 can not reach the second hill of elevation 'C'.

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19.1.7 A **Scotch yoke** is a device for converting rotary motion into linear motion. In this case it is used to make a horizontal platform go up and down. The motion of the platform is $y = A\sin(\omega t)$ with constant ω. A dead bug with mass m is standing on the platform. Her feet have no glue on them. There is gravity.

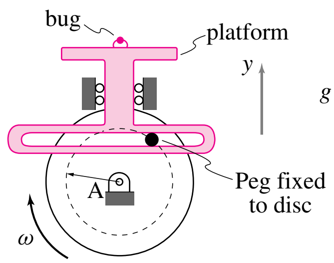
a) Draw a free-body diagram of the bug.

b) What is the acceleration of the platform and, hence, the bug?

c) Write the equation of linear momentum balance for the bug.

d) What condition must be met so that the bug does not bounce the platform?

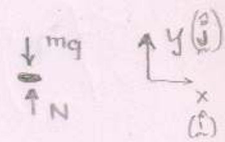
e) What is the maximum spin rate ω that can be used if the bug is not to bounce off the platform?



Problem 19.7

18.1.7 Bug on a Scotch Yoke

(a)



(b) Motion of the platform, $y = A\sin\omega t$

Acceleration of the platform, $\frac{d^2y}{dt^2} = \ddot{y} = -A\omega^2\sin\omega t$

(c) L.M.B for the bug.

$$\left\{\sum \underset{\sim}{F} = m\underset{\sim}{a}\right\}$$

$$\{\ \}\cdot\underset{\sim}{j} \Rightarrow N - mg = m\ddot{y}$$

(d) Bug looses contact with the platform if the downward acceleration is greater than 'g'. (or) if $N=0$.

$$-\ddot{y} > g$$

For the bug not to bounce, $-\ddot{y} < g$

(e) Spin rate Limit

From (b) and (d) $\quad -[-A\omega^2\sin\omega t] < g$

$\therefore$ Maximum Spin rate, $\boxed{\omega < \sqrt{\frac{g}{A}}}$

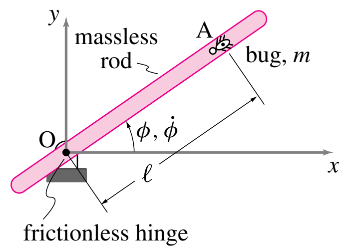
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.8 A bug of mass m walks straightforwards with speed v_A and rate of change of speed $\dot{v}_A$ on a straight light (assumed to be massless) stick. The stick is hinged at the origin so that it is always horizontal but is free to rotate about the z axis. Assume that the distance the bug is from the origin, ℓ, the angle the stick makes with the x axis, ϕ, and its rate of change, $\dot{\phi}$, are known at the instant of interest. Ignore gravity. Answer the following questions in terms of ϕ, $\dot{\phi}$, ℓ, m, v_A, and $\dot{v}_A$.

a) What is $\ddot{\phi}$?

b) What is the force exerted by the rod on the support?

c) What is the acceleration of the bug?



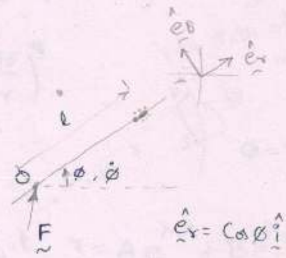
Problem 19.8

18.1.8

Given $\phi, \dot{\phi}, l, m, V_A$ and $\dot{V}_A$

a) $\ddot{\phi}$ is the angular acceleration.

$\ddot{\phi} = \frac{d\dot{\phi}}{dt}$ (or $\ddot{\phi} = \frac{d^2\phi}{dt^2}$



$\hat{e}_r = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$

$\hat{e}_\theta = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$

b) L.M.B for the stick, $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$\underset{\sim}{F} = m\{\cancel{\underset{\sim}{a}_0}^{\,0} + -\dot{\phi}^2 l\,\hat{\underset{\sim}{e}}_r + \ddot{\underset{\sim}{\phi}} \times l\,\hat{\underset{\sim}{e}}_r + \dot{\underset{\sim}{V}}_A\,\hat{\underset{\sim}{e}}_r + 2\dot{\phi}\hat{k} \times \underset{\sim}{V}_A\hat{e}_r\}$

$\Rightarrow \underset{\sim}{F} = m\{-\dot{\phi}^2 l\,\hat{\underset{\sim}{e}}_r + \ddot{\phi} l\,\hat{\underset{\sim}{e}}_\theta + \dot{V}_A\hat{\underset{\sim}{e}}_r + 2\dot{\phi}V_A\,\hat{e}_\theta\}$

$\underset{\sim}{F} = m\{(\dot{V}_A - \dot{\phi}^2 l)\,\hat{\underset{\sim}{e}}_r + (\ddot{\phi} l + 2\dot{\phi}V_A)\,\hat{e}_\theta\}$

Force exerted by stick on the support $= -F$

$-F$

$\therefore$ Force on support $= m\{(\dot{\phi}^2 l - \dot{V}_A)(\cos\phi\,\hat{\underset{\sim}{i}} + \sin\phi\,\hat{\underset{\sim}{j}})$

$- (\ddot{\phi} l + 2\dot{\phi}V_A)(-\sin\phi\,\hat{\underset{\sim}{i}} + \cos\phi\,\hat{\underset{\sim}{j}})\}$

(c) Acceleration of the bug

$\underset{\sim}{a} = \cancel{\underset{\sim}{a}_0}^{\,0} - \dot{\phi}^2 l\,\hat{\underset{\sim}{e}}_r + \ddot{\phi} l\,\hat{\underset{\sim}{e}}_\theta + \dot{V}_A\hat{\underset{\sim}{e}}_r + 2\dot{\phi}V_A\,\hat{\underset{\sim}{e}}_\theta$

$\underset{\sim}{a} = (\dot{V}_A - \dot{\phi}^2 l)\,\hat{\underset{\sim}{e}}_r + (\ddot{\phi} l + 2\dot{\phi}V_A)\,\hat{\underset{\sim}{e}}_\theta$

(or)

$\underset{\sim}{a} = (\dot{V}_A - \dot{\phi}^2 l)(\cos\phi\,\hat{\underset{\sim}{i}} + \sin\phi\,\hat{\underset{\sim}{j}}) + (\ddot{\phi} l + 2\dot{\phi}V_A)(-\sin\phi\,\hat{\underset{\sim}{i}} + \cos\phi\,\hat{\underset{\sim}{j}})$ //

19.1.9 A new kind of gun. Assume $\omega = \omega_0$ is a constant for the rod in the figure. Assume the mass is free to slide. At $t = 0$, the rod is aligned with the x-axis and the bead is one foot from the origin and has no radial velocity ($dR/dt = 0$).

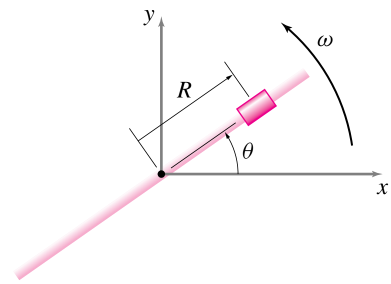
a) Find a differential equation for $R(t)$.

b) Turn this equation into a differential equation for $R(\theta)$.

c) How far will the bead have moved after one revolution of the rod? How far after two?

d) What is the speed of the bead after one revolution of the rod (use $\omega_0 = 2\pi$ rad/s = 1 rev/sec)?

e) How much kinetic energy does the bead have after one revolution and where did it come from?



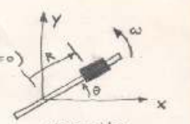
Problem 19.9

Answer:
(a) $\ddot{R}(t) - \omega_0^2 R(t) = 0$.
(b) $\frac{d^2R(\theta)}{d\theta^2} - R(\theta) = 0$.
(c) $R(\theta = 2\pi) = 267.7\,\text{ft}$, $R(\theta = 4\pi) = 1.43 \times 10^5\,\text{ft}$.
(d) $v = 1682.3\,\text{ft/s}$
(e) $E_K = (2.83 \times 10^6) \times (mass)(\text{ft/s})^2$.

\# 6.38

FBD

$\omega = \omega_0$ constant
bead has mass m
$R = 1\,ft = r(t=0)$
$\dot{r}(t=0) = 0$
no friction



A

LMB, $N\hat{e}_\theta = m\underline{a}$ no gravity

$N\hat{e}_\theta = m[(\ddot{r} - r\omega^2)\hat{e}_r + (2\dot{r}\omega + r\dot{\omega})\hat{e}_\theta]$

dotting with $\hat{e}_r$

$0 = m(\ddot{r} - r\omega^2)$ $\boxed{\ddot{R}(t) - R(t)\omega_0^2 = 0}$

B

we can rewrite $\frac{d^2R(t)}{dt^2} = \frac{d}{d\theta}\left(\frac{dR(\theta)}{d\theta}\frac{d\theta}{dt}\right)\frac{d\theta}{dt} = \frac{d^2R(\theta)}{d\theta^2}\omega_0^2$

so $\frac{d^2R(\theta)}{d\theta^2}\omega_0^2 - R(\theta)\omega_0^2 = 0$ or

$\boxed{\frac{d^2R(\theta)}{d\theta^2} - R(\theta) = 0}$ solve this equation for Part C

C

from ODE's we know $R(\theta) = Ae^{\theta} + Be^{-\theta}$

$R(\theta=0) = R(t=0) = 1\,ft \Rightarrow A + B = 1\,ft$

$\left.\frac{dR}{d\theta}\right|_{\theta=0} = 0 \Rightarrow A - B = 0$

so $A = \tfrac{1}{2}\,ft = B$

$R(\theta) = \cosh(\theta)\,ft$ or $R(\theta) = (\tfrac{1}{2}ft)(e^{\theta} + e^{-\theta})$ if your calculator doesn't do cosh!

$\boxed{\begin{aligned} R(\theta = 2\pi) &= 267.7\,ft \\ R(\theta = 4\pi) &= 1.43 \times 10^5\,ft \end{aligned}}$

D

$\underline{v}_{B/\mathcal{F}} = \underline{v}_{B/\mathcal{B}} + \underline{\omega}_{\mathcal{B}/\mathcal{F}} \times \underline{r}_{B/\mathcal{B}}$ velocity of bead in Newtonian Frame, speed is magnitude of $\underline{v}_{\mathcal{F}}$

$\underline{v}_{B/\mathcal{B}}$ = velocity of bead along rod $\frac{d(R(\theta))}{dt} = \sinh(\theta)\dot{\theta}$ (ft)

but $\dot{\theta} = \omega_0 = 2\pi$ rad/sec so $\underline{v}_B = 1682$ ft/sec $\hat{e}_r$

$\underline{r}_B = R(\theta = 2\pi) = 267.7$ ft $\hat{e}_r$ from above

$\underline{\omega}_B = 2\pi$ rad/sec $\hat{k}$

$\underline{v}_{B/\mathcal{F}} = 1682$ ft/sec $\hat{e}_r$ + 1682 ft/sec $\hat{e}_\theta$

$\boxed{v_{B/\mathcal{F}} = 2378.7\text{ ft/sec}}$

E

$E_K = \frac{1}{2}m(v_{B/\mathcal{F}})^2 = \frac{1}{2}(mass)(2378.7\text{ ft/sec})^2$

$\boxed{E_K = 2.83 \times 10^6\,(\text{ft/sec})^2\,(mass)}$

In order to keep the rod spinning at a constant speed ω_0, there must be a motor attached to it. The torque (moment) required to maintain constant angular velocity ω_0 is increasing exponentially. I calculate the moment required at any angle θ to be

$M(\theta) = (mass)(ft^2)\,2\cosh(\theta)\sinh(\theta)\,\omega_0^2$ Hence, motor provides E_K.

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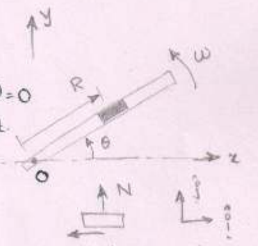
19.1.10 The new gun gets old and rusty. Reconsider the bead on a rod in problem 19.1.9. This time, friction cannot be neglected. The friction coefficient is μ. At the instant of interest the bead with mass m has radius R_0 with rate of change $\dot{R}_0$. The angle θ is zero and ω is a constant. Neglect gravity.

a) What is $\ddot{R}$ at this instant? Give your answer in terms of any or all of R_0, $\dot{R}_0$, ω, m, μ, $\hat{\boldsymbol{e}}_\theta$, $\hat{\boldsymbol{e}}_R$, $\hat{\boldsymbol{\imath}}$, and $\hat{\boldsymbol{\jmath}}$.

b) After a very long time it is observed that the angle ϕ between the path of the bead and the rod/trough is nearly constant. What, in terms of μ, is this ϕ?

Answer:
(a) $\ddot{R} = R_0\omega^2 - 2\mu\dot{R}_0\omega$.
(b) $\phi = \sin^{-1}\left[\frac{1}{\sqrt{(\sqrt{1+\mu^2}-\mu)^2+1}}\right]$.

18.1.10 a)



At the instant of interest, $r = R_0$; $\dot{r} = \dot{R}_0$, $\theta = 0$ and ω = Constant.

Applying LMB on the bead.

$$\Sigma \underline{F} = m\underline{a}$$

$$\Rightarrow \left\{ N\hat{\underline{j}} - \mu N \hat{\underline{i}} = m\left[\underline{a}_0 + -\dot{\theta}^2 r\hat{\underline{i}} + \underline{\ddot{\theta}} \times r\hat{\underline{i}} + \ddot{r}\hat{\underline{i}} + 2\underline{\dot{\theta}} \times \dot{r}\hat{\underline{i}}\right]\right\}$$

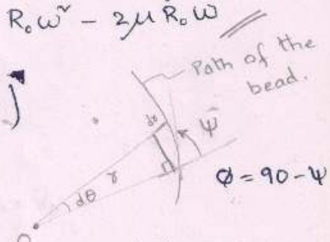
$$\{\}\cdot\hat{\underline{j}} \Rightarrow N = 2m\dot{r}\dot{\theta}$$

$$\{\}\cdot\hat{\underline{i}} \Rightarrow -\mu N = -m\dot{\theta}^2 r + m\ddot{r}$$

$\Rightarrow$ at the instant of interest $\ddot{r} = R_0\omega^2 - 2\mu\dot{R}_0\omega$

b) From the figure, $\cot\phi \approx \left(\frac{r\,d\theta}{dr}\right)$

$$\Rightarrow \tan\psi = \frac{r}{dr/d\theta}$$



$\phi = 90 - \psi$

Solving the differential equation, we get $r = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

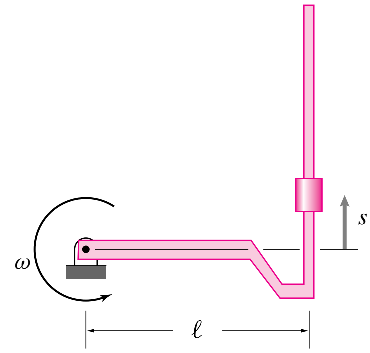
where $\lambda_{1,2} = -\mu \pm \sqrt{1-\mu^2}\,\omega$

After a very long time, $r = Ae^{(-\mu + \sqrt{1-\mu^2})\theta}$

$$\therefore \tan\psi = \frac{1}{-\mu + \sqrt{1-\mu^2}} \quad (\text{or}) \quad \psi = \sin^{-1}\left[\frac{1}{\sqrt{1+(-\mu+\sqrt{1-\mu^2})^2}}\right]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

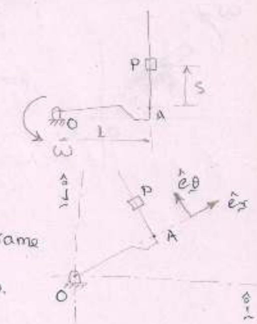
19.1.11 A newer kind of gun. As an attempt to make an improvement on the 'new gun' demonstrated in problem 19.1.9, a person adds a length ℓ to the shaft on which the bead slides. Assume there is no friction between the bead (mass m) and the wire. Assume the bead starts at $s = 0$ with $\dot{s} = v_0$. The rigid rod, on which the bead slides, rotates at a constant rate $\omega = \omega_0$. Find $s(t)$ in terms of ℓ, m, v_0, and ω_0.



Problem 19.11

18.1.11

FBD of the bead



Let the body fixed coordinate frame be at 'A' with $\hat{e}_r$ & $\hat{e}_\theta$ as shown.

Acceleration of the bead:

$$\underset{\sim}{a}_P = \underset{\sim}{a}_A + \underset{\sim}{a}_{P/A}$$

Given $\omega = \text{Constant} = \omega_0$, we have

$$\underset{\sim}{a}_A = -\omega_0^2 \, \ell \, \hat{e}_r$$

$$\underset{\sim}{a}_{P/A} = +\ddot{s}\,\hat{e}_\theta - \omega_0^2 s\,\hat{e}_\theta + 2\omega_0 \dot{s}\,(-\hat{e}_r)$$

L.M.B for bead: $\Sigma \underline{F} = m\underset{\sim}{a}_P$

$$N\hat{e}_r = -(m\ell\omega_0^2 + 2m\omega_0\dot{s})\,\hat{e}_r + (m\ddot{s} - ms\omega_0^2)\,\hat{e}_\theta$$

$$\{\ \}\cdot\hat{e}_\theta \Rightarrow \quad 0 = m\ddot{s} - ms\omega_0^2$$

$$\Rightarrow \quad \ddot{s} = \omega_0^2 s$$

$$\Rightarrow \quad s = Ae^{\omega_0 t} + Be^{-\omega_0 t}$$

Given that at $t=0$, $s=0$ and $\dot{s} = V_0$

$$\Rightarrow A = -B \quad \& \quad V_0 = \omega_0 A - \omega_0 B \quad \Rightarrow B = \frac{-V_0}{2\omega_0}$$

$$\therefore \quad s = \frac{V_0}{2\omega_0} e^{\omega_0 t} - \frac{V_0}{2\omega_0} e^{-\omega_0 t}$$

$$(\text{or}) \quad s = \frac{V_0}{\omega_0} \sinh \omega_0 t$$

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19.1.12 Slippery bead on straight rotating stick. A long stick with mass m_s rotates at a constant rate and a bead, modeled as a point mass, slides on the stick. The stick rotates at constant rate $\vec{\omega} = \omega\hat{\boldsymbol{k}}$. Neglect gravity. The bead has mass m_b. Initially the mass is at a distance R_0 from the hinge point on the stick and has no radial velocity ($\dot{R} = 0$). The initial angle of the stick is $\theta = 0$, measured counterclockwise from the positive x axis.

a) What is the torque, as a function of the net angle the stick has rotated, θ, required in order to keep the stick rotating at a constant rate ?

b) What is the path of the bead in the xy-plane? (Draw an accurate picture showing about one half of one revolution.)

c) How long should the stick be if the bead is to fly in the negative x-direction when it gets to the end of the stick?

d) **Add friction.** How does the speed of the bead over one revolution depend on μ, the coefficient of friction between the bead and the wire? Make a plot of $|\vec{\boldsymbol{v}}_{\text{one revolution}}|$ versus μ. [Note, you have been working with $\mu = 0$ in the problems above.]

TAM 203 · Summer 96 · HW8 Solutions ·
TA: Erich Schneider · July 21, 1996

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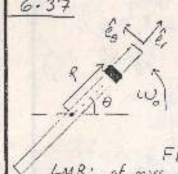

At $t=0$,
$\theta = 0$ and $\dot{R} = 0$
$\underset{\sim}{\omega} = \omega_0 \hat{k} = \text{const}$

a) Find a differential eqn for $R(t)$.

FBD:

LMB: of mass

$$\sum \underset{\sim}{F} = N(t)\hat{e}_\theta = m\left((-R\dot{\theta}^2 + \ddot{R})\hat{e}_r - (R\underbrace{\ddot{\theta}}_{=0 \text{ b/c } \omega = \dot{\theta} = \text{const}} + 2\dot{R}\dot{\theta})\hat{e}_\theta\right)$$

dot LMB w/ $\hat{e}_r, \hat{e}_\theta$, get 2 scalar eqns:

$\hat{e}_r \Rightarrow \quad \ddot{R} - R\dot{\theta}^2 = 0 \xRightarrow{\dot{\theta} = \text{const} = \omega_0} \boxed{\ddot{R} - \omega_0^2 R = 0}$

$\hat{e}_\theta \Rightarrow \quad 2\dot{R}\dot{\theta} = \dfrac{N(t)}{m}$

b) Turn the above into a differential eqn for $R(\theta)$.

$$\frac{d^2R}{dt^2} - \omega_0^2 R = 0 \Rightarrow \frac{d}{dt}\left(\frac{dR}{d\theta}\underbrace{\frac{d\theta}{dt}}_{=\omega_0}\right) - \omega_0^2 R = 0$$

$$\Rightarrow \omega_0 \frac{d^2R}{dt\,d\theta} - \omega_0^2 R = 0 \Rightarrow \omega_0 \frac{d}{d\theta}\left(\frac{dR}{d\theta}\frac{d\theta}{dt}\right) - \omega_0^2 R = 0$$

$$\Rightarrow \omega_0^2 \frac{d^2R}{d\theta^2} - \omega_0^2 R = 0 \Rightarrow \boxed{\frac{d^2R}{d\theta^2} - R = 0}$$

c) what is $R(\theta = 2\pi)$ & $R(\theta = 4\pi)$?

Solve the above dif. eq. for $R(\theta)$:

soln of form $R = Ae^{\theta} + Be^{-\theta}$

$\Rightarrow R(\theta = 0) = 1\text{ft} \Rightarrow 1\text{ft} = A + B \quad *$

to use second IC, $\frac{dR}{d\theta}(\theta = 0) = 0$, differentiate:

$\frac{dR}{d\theta} = Ae^{\theta} - Be^{-\theta}$

$\Rightarrow \quad 0 = A - B \quad **$

From $*$ and $**$ we find $A = B = 0.5\text{ft}$

thus $R(\theta) = (0.5\text{ft})(e^{\theta} + e^{-\theta})$

Putting in $\theta = 2\pi$ rad & 4π rad to the above gives:

$$\boxed{\begin{aligned} R(\theta = 2\pi \text{ rad}) &= 268\text{ ft} \\ R(\theta = 4\pi \text{ rad}) &= 143{,}000\text{ ft !} \end{aligned}}$$

d. What is $|\underset{\sim}{v}|$ after 1 rev.? ($\omega_0 = 2\pi$ rad/s)

$\Rightarrow |\underset{\sim}{v}| = |R\dot{\theta}\hat{e}_\theta + \dot{R}\hat{e}_r|$

continued

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where $\dot{R} = \frac{dR}{dt} = \frac{dR}{d\theta}\frac{d\theta}{dt} = \omega\frac{dR}{d\theta}$

at $\theta = 2\pi$ rad, $\omega\frac{dR}{d\theta} = (2\pi \tfrac{1}{s})(0.5\text{ft})(e^{2\pi} - e^{-2\pi}) = 1682\text{ ft/s}$

thus $|\underset{\sim}{v}| = |(268\text{ft})(2\pi\text{ rad/s})\hat{e}_\theta + 1682\text{ ft/s}\ \hat{e}_r| =$

$\boxed{|\underset{\sim}{v}| = 2380\text{ ft/s}}$

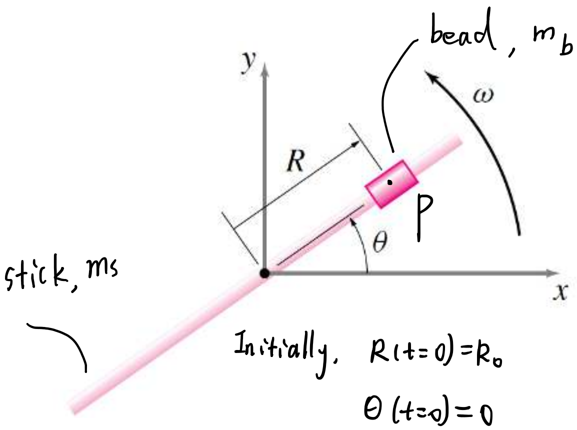
e. $KE = \frac{1}{2}m|\underset{\sim}{v}|^2 = (\text{mass})(2.83E6)\ \text{ft}^2/\text{s}^2$

The angular speed of the disk would not normally stay constant - try it yourself in lab by standing on the disk, spinning around and putting your arms out (increasing your moment of inertia). You will slow down. Thus, there has to be a motor of some sort attached to the rod that applies the power that keeps ω const (and thus accelerates the mass).

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.1.12

Saturday, April 27, 2019 6:22 PM



$\vec{\omega} = \omega \hat{k}$

Initially, $R(t=0) = R_0$

$\theta(t=0) = 0$

$\dot{R}(t=0) = 0$

a) Find the torque required to keep the stick rotating as const rate ω (in terms of θ)

AMB/o :

$$\vec{M}_{/O} = \dot{\vec{H}}_{/O}$$

$$\vec{H}_{/O} = I^c \vec{\omega} + \vec{r}_{P/O} \times m \vec{v}_{P/O}$$

$$= I^c \vec{\omega} + \vec{r}_{P/O} \times m (\vec{\omega} \times \vec{r}_{P/O})$$

$$= I^c \vec{\omega} + m r_{P/O}^2 \vec{\omega}$$

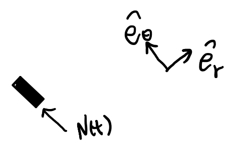
$$= [I^c + m|\vec{R}(t)|^2] \vec{\omega}$$

Since we need to take derivative of $\vec{H}_{/O}$ to get $\vec{M}_{/O}$ in terms of θ.

We need to find $R(\theta)$.

Firstly we find $R(t)$ in ode.

FBD:



LMB: $\sum \vec{F} = m\vec{a}$ in polar coordinates:

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_r + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{e}_\theta$$

$$\Rightarrow \{ \vec{N}(t) = m(\ddot{R} - R\dot{\theta}^2)\hat{e}_r + m(R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{e}_\theta \}$$

$$\{ \} \cdot \hat{e}_r \Rightarrow \quad 0 = \ddot{R} - R\dot{\theta}^2 = \ddot{R} - R\omega^2 \quad (*)$$

Then turn the above into an ode for $R(\theta)$

$$\frac{dR}{dt} = \frac{dR}{d\theta}\frac{d\theta}{dt} = \omega\frac{dR}{d\theta}$$

$$\frac{d^2R}{dt^2} = \frac{d}{dt}\left(\omega_0\frac{dR}{d\theta}\right) = \omega_0\frac{d}{d\theta}\left(\frac{dR}{d\theta}\right)\cdot\frac{d\theta}{dt} = \omega^2\frac{d^2R}{d\theta^2}$$

plug into $(*)$ we get

$$\frac{d^2R}{d\theta^2} - R = 0$$

Soln: $R(\theta) = Ae^{\theta} + Be^{-\theta}$

plus initial conditions: $R(\theta=0) = R_0$

$\dot{R}(\theta=0) = 0$

$$\Rightarrow \begin{cases} A + B = R_0 \\ A - B = 0 \end{cases} \Rightarrow \quad A = B = \frac{R_0}{2}$$

$$\therefore \quad R(\theta) = \frac{R_0}{2}(e^{\theta} + e^{-\theta}) \qquad \frac{dR}{d\theta} = \frac{R_0}{2}(e^{\theta} - e^{-\theta})$$

Thus $\vec{M}_{/0} = \dot{\vec{H}}_{/0} = m2R\dot{R}\omega = 2m\omega^2 R\frac{dR}{d\theta}$

$$= 2m\omega^2\frac{R_0}{2}(e^{\theta} + e^{-\theta})\cdot\frac{R_0}{2}(e^{\theta} - e^{-\theta})$$

$$= \frac{m\omega^2R_0^2}{2}(e^{2\theta} - e^{-2\theta})$$

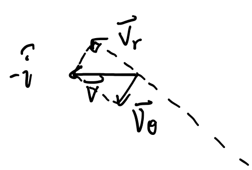
b) Find path in x-y coordinates in $\frac{1}{2}$ revolution

$$\begin{cases} x = R(\theta)\cdot\cos\theta \\ y = R(\theta)\cdot\sin\theta \end{cases} \qquad 0 \le \theta \le \pi .$$

Code & path see attached

c) Find the length of the stick l if the bead is to fly in $-x$ direction when it gets to the end

$\vec{v}$ is in $-\hat{\imath}$ direction



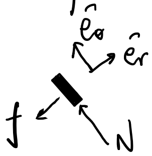
Thus $\theta_f = 2k\pi + \pi \qquad k = 0, 1, 2 \cdots$

$= (2k+1)\pi$

$\therefore \quad l = R(\theta_f) = \dfrac{m\,\omega^2 R_0}{2}\left(e^{(2k+1)\pi} + e^{-(2k+1)\pi}\right)$

d) Same setup as before but with friction. coefficient is μ.
Find speed $v = |\vec{v}|$ in terms of μ

FBD



LMB: $\Sigma \vec{F} = \dot{\vec{L}} \Rightarrow$

$$\left\{ N\hat{e}_\theta - \mu N \hat{e}_r = m\left((-R\dot\theta^2 + \ddot{R})\hat{e}_r + (R\ddot\theta + 2\dot{R}\dot\theta)\hat{e}_\theta\right)\right\}$$

$\{\}\cdot\hat{e}_r \Rightarrow \quad -\dfrac{\mu N}{m} = \ddot{R} - R\dot\theta^2 \qquad ①$

$\{\}\cdot\hat{e}_\theta \Rightarrow \quad \dfrac{N}{m} = 2\dot{R}\dot\theta \qquad ②$

plug ② into ① to eliminate N:

$\ddot{R} - R\dot\theta = -\mu(2\dot{R}\dot\theta)$

$\dot\theta = \omega \Rightarrow \ddot{R} - \omega R = -2\mu\omega\dot{R} \quad *$

Also we have from part A:

$\Rightarrow \quad \dot{R} = \omega \frac{dR}{d\theta}$

$\quad \ddot{R} = \omega^2 \frac{d^2R}{d\theta^2}$

plug into $*$ $\quad \frac{d^2R}{d\theta^2} - R = -2\mu \frac{dR}{d\theta}$

$\therefore \quad R = A e^{c_1\theta} + B e^{c_2\theta}$

where c_1 , c_2 satisfy the characteristic equation

$x^2 + 2\mu x - 1 = 0$

$c_1 = -\mu + \sqrt{\mu^2+1} \qquad c_2 = -\mu - \sqrt{\mu^2+1}$

with initial conditions, $R(\theta=0) = R_0$

$\dot{R}(\theta=0) = 0$

$\Rightarrow \begin{cases} A + B = R_0 \\ Ac_1 + Bc_2 = 0 \end{cases} \Rightarrow \begin{cases} A = -\frac{c_2}{c_1-c_2} R_0 \\ B = \frac{c_1}{c_1-c_2} R_0 \end{cases}$

$\therefore \quad \frac{dR}{d\theta} = \frac{c_1c_2}{c_1-c_2} R_0 (e^{c_2\theta} - e^{-c_1\theta})$

$\vec{v} = R\dot{\theta}\hat{e}_\theta + \dot{R}\hat{e}_r$

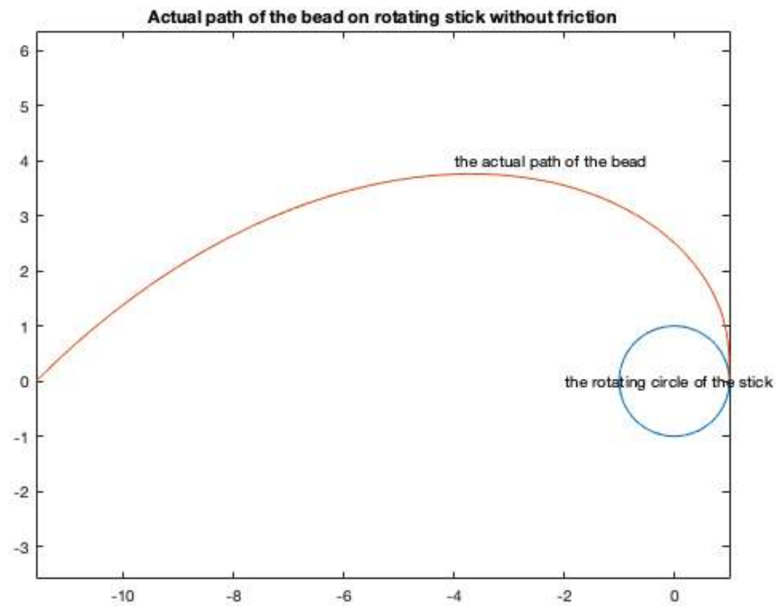
$\therefore \quad |\vec{v}| = \sqrt{R^2\dot{\theta}^2 + \dot{R}^2} = \sqrt{R^2\omega^2 + \dot{R}^2} = \omega\sqrt{R^2 + (\frac{dR}{d\theta})^2}$

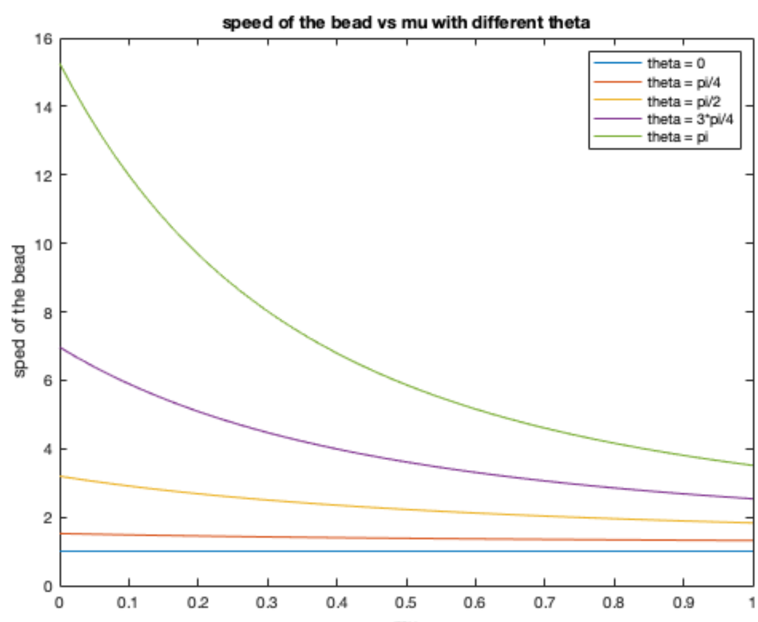
$\boxed{\text{Code \& plots see attached}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
% Problem 18.1.12 bead on straight rotating stick
clc;
clear;
close all;
% Parameters
mb = 1;       % mass of the bead
ms = 2;       % mass of the stick
R0 = 1;       % initially the bead is at a distance R0 from the origin
omega = 1;    % const rotating angular velocity
% the circular path of the initial point of the stick
angle = 0:pi/80:2*pi;
circlex = R0*cos(angle);
circley = R0*sin(angle);
plot(circlex,circley);
text(-2,0,'the rotating circle of the stick');
hold on
axis equal
% Path of the bead
theta = 0:pi/80:pi;     % half of the revolution
R = R0/2*(exp(theta)+exp(-theta));     % R in terms of theta
% in x-y coordinates
x = R.*cos(theta);
y = R.*sin(theta);
plot(x,y)
text(-4,4,'the actual path of the bead');
title('Actual path of the bead on rotating stick without friction');

% speed vs mu with friction
mu = 0:0.02:1;
c1 = -mu + sqrt(mu.^2+1);
c2 = -mu - sqrt(mu.^2+1);
A = - R0 * c2./(c1-c2);
B =   R0 * c1./(c1-c2);
figure
for theta = 0:pi/4:pi     % make theta different for each plot
    R = A.*exp(c1*theta) + B.*exp(c2*theta);
    dRdtheta = R0.*c1.*c2/(c1-c2).*(exp(c2*theta) - exp(c1*theta));
    v = omega*sqrt(R.^2+dRdtheta.^2);
    plot(mu,v)
    hold on
end
title('speed of the bead vs mu with different theta');
xlabel('mu');
ylabel('sped of the bead');
legend('theta = 0','theta = pi/4','theta = pi/2','theta =
 3*pi/4','theta = pi');
```


Actual path of the bead on rotating stick without friction
the actual path of the bead
the rotating circle of the stick
6
5
4
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0


speed of the bead vs mu with different theta
theta = 0
theta = pi/4
theta = pi/2
theta = 3*pi/4
theta = pi
sped of the bead
mu
0
2
4
6
8
10
12
14
16
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

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#67 Solution Michelle deSouza

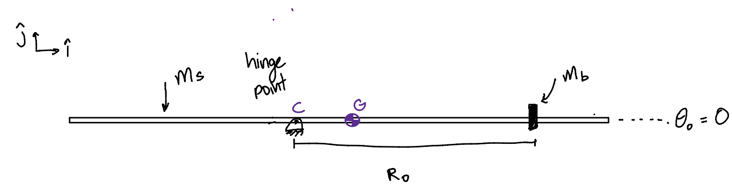
19.1.12 Slippery bead on straight rotating stick. A long stick with mass m_s rotates at a constant rate and a bead, modeled as a point mass, slides on the stick. The stick rotates at constant rate $\vec{\omega} = \omega\hat{k}$. Neglect gravity. The bead has mass m_b. Initially the mass is at a distance R_0 from the hinge point on the stick and has no radial velocity ($\dot{R} = 0$). The initial angle of the stick is $\theta = 0$, measured counterclockwise from the positive x axis.

a) What is the torque, as a function of the net angle the stick has rotated, θ, required in order to keep the stick rotating at a constant rate ?

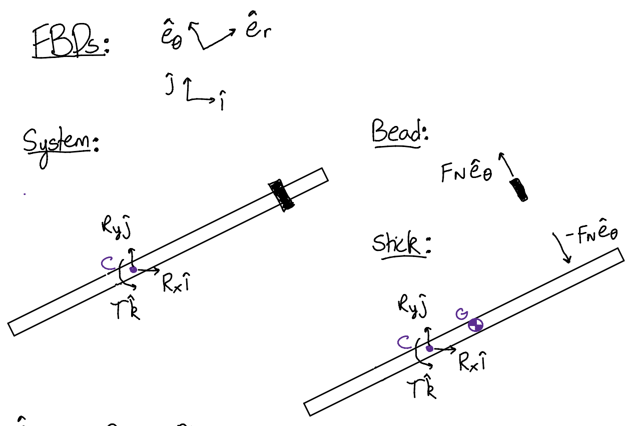
b) What is the path of the bead in the xy-plane? (Draw an accurate picture showing about one half of one revolution.)

c) How long should the stick be if the bead is to fly in the negative x-direction when it gets to the end of the stick?

d) **Add friction.** How does the speed of the bead over one revolution depend on μ, the coefficient of friction between the bead and the wire? Make a plot of $|\vec{v}_{\text{one revolution}}|$ versus μ. [Note, you have been working with $\mu = 0$ in the problems above.]



FBDs:



$\hat{e}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$

$\hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$ $\qquad \theta = \omega t + \cancel{\theta_o}\,0$

What is $r(\theta)$ for the bead?

Bead LMB: $\sum\vec{F} = F_N\hat{e}_\theta = m_b\vec{a}_b$

$\vec{a}_b = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta \qquad \dot{\theta} = \omega,\ \ddot{\theta} = 0$

$= (\ddot{r} - r\omega^2)\hat{e}_r + 2\dot{r}\omega\hat{e}_\theta$

$F_N\hat{e}_\theta = m_b[(\ddot{r} - r\omega^2)\hat{e}_r + 2\dot{r}\omega\hat{e}_\theta]$

$\{\}\cdot\hat{e}_\theta \quad F_N = 2m_b\dot{r}\omega$

$\{\}\cdot\hat{e}_r \quad 0 = m_b(\ddot{r} - r\omega^2)$

$0 = \ddot{r} - r\omega^2 \longleftarrow$ ODE for $r(t)$, use chain rule to get $r(\theta)$

$0 = \omega^2\frac{d^2r}{d\theta^2} - \omega^2 r \qquad \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega\frac{dr}{d\theta}$

$0 = \frac{d^2r}{d\theta^2} - r \qquad \frac{d^2r}{dt^2} = \frac{d}{dt}\left(\omega\frac{dr}{d\theta}\right) = \cancel{\dot{\omega}}\frac{dr}{d\theta} + \omega\frac{d}{d\theta}\left(\omega\frac{dr}{d\theta}\right) = \omega^2\frac{d^2r}{d\theta^2}$

Assume solution of the form $r = Ae^{\lambda\theta}$

$0 = \lambda^2\cancel{Ae^{\lambda\theta}} - \cancel{Ae^{\lambda\theta}}$

$\lambda^2 = 1 \rightarrow \lambda = \pm 1$ (compare with frictional case below)

$r = Ae^{\theta} + Be^{-\theta}$ Use I.C.s to find A, B

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

① $r(0) = R_0 = A + B$

② $\frac{dr}{d\theta}(0) = 0 = A - B$

$A = B = \frac{R_0}{2}$

$\boxed{\vec{r}_{b/c} = \frac{R_0}{2}(e^{\theta} + e^{-\theta})\,\hat{e}_r}$

Given: ① $r(t)|_{t=0} = R_0 \rightarrow r(\theta)|_{\theta=0} = R_0$

② $\dot{r}(t)|_{t=0} = 0 \rightarrow \cancel{\omega}\,\frac{dr(\theta)}{d\theta}\Big|_{\theta=0} = 0$

③ $\theta(0)|_{t=0} = 0$ plug into ①,②

a) Torque required to maintain constant ω

Stick AMB: $\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$

$$\vec{r}_b \times -F_N\hat{e}_\theta + T\hat{k} = I^G\ddot{\theta}\hat{k} + \underbrace{\vec{r}_{G/c} \times M_s\vec{a}_G}$$

$$r_{G/c}\hat{e}_r \times M_s(-r_{G/c}\dot{\theta}^2\hat{e}_r + r_{G/c}\cancel{\ddot{\theta}}^{\,0}\hat{e}_\theta)$$

$$= r_{G/c}\hat{e}_r \times -M_s r_{G/c}\omega^2\hat{e}_r$$

$$= \vec{0}$$

$$T\hat{k} = \vec{r}_{b/c} \times F_N\hat{e}_\theta \;*$$

$$\vec{r} = \frac{R_0}{2}(e^{\theta} + e^{-\theta})\hat{e}_r$$

$$F_N = 2m_b\omega\dot{r} = 2m_b\omega^2\frac{dr}{d\theta} = \cancel{2}m_b\omega^2\frac{R_0}{\cancel{2}}(e^{\theta} - e^{-\theta})$$

$$T\hat{k} = \frac{R_0}{2}(e^{\theta} + e^{-\theta})\hat{e}_r \times m_b R_0\omega^2(e^{\theta} - e^{-\theta})\hat{e}_\theta$$

$$\{\}\cdot\hat{k} \quad T = \frac{1}{2}m_b R_0^2\omega^2(e^{2\theta} - e^{-2\theta})$$

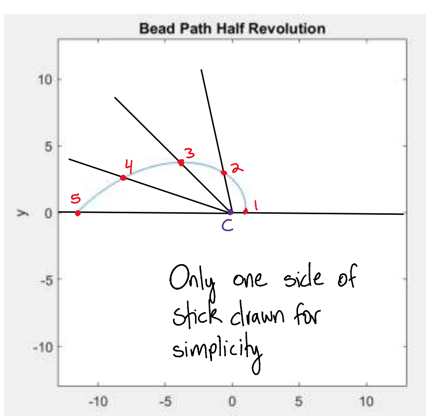
b) Path in x-y plane

$$r = \frac{R_0}{2}(e^{\theta} + e^{-\theta})$$

$x = r\cos\theta \qquad y = r\sin\theta$

plot for half a rotation $\rightarrow \theta = [0, \pi]$

See Matlab

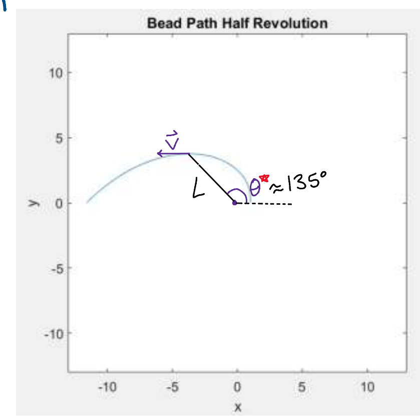


Bead rotates with the stick but also moves radially outward (Would require a centripetal force to keep circular)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

c) Stick length for bead to fly off in $-\hat{\imath}$ direction

- To move in $-\hat{\imath}$, $\vec{v}\cdot\hat{\jmath} = 0 \longrightarrow$ Find θ^* to make this true
- To fly off stick, stick length must be equal to $r(\theta^*)$.
- Can see from part b) graph that θ^* occurs at point 3



$$\vec{v} = \overbrace{\dot{r}}^{\omega \frac{dr}{d\theta}}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \qquad \hat{e}_r = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$$

$$= \omega\frac{R_0}{2}(e^{\theta} - e^{-\theta})\hat{e}_r + \omega\frac{R_0}{2}(e^{\theta} + e^{-\theta})\hat{e}_\theta \qquad \hat{e}_\theta = -\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}$$

$$\vec{v}\cdot\hat{\jmath} = \omega\frac{R_0}{2}\underbrace{\left[\sin\theta(e^{\theta} - e^{-\theta}) + \cos\theta(e^{\theta} + e^{-\theta})\right]}_{=0}$$

Find zero using Matlab $\rightarrow \theta = 2.35$ rad $(\sim 135°)$ matches graph ✓

$\theta = 2.35 + 2\pi n$ for n=integer (once per rev)

$L = \frac{R_0}{2}\left(e^{2.35+2\pi n} + e^{-2.35-2\pi n}\right)$ For $R_0 = 1$ m, $n = 0$, $L = 5.3$ m, matches graph ✓

d) Add friction, find $\vec{v}$ vs. μ after one revolution

FBD:

$F_f = \mu F_N$

$\vec{v} = \dot{r}\hat{e}_r + r\omega\hat{e}_\theta$ solve bead LMB for $r, \dot{r}$

<u>Bead LMB</u>: $\Sigma\vec{F} = F_N\hat{e}_\theta - \mu F_N\hat{e}_r = m_b\left[(\ddot{r} - r\omega^2)\hat{e}_r + 2\dot{r}\omega\hat{e}_\theta\right]$

$\{\}\cdot\hat{e}_\theta \quad F_N = 2m_b\dot{r}\omega$

$\{\}\cdot\hat{e}_r \quad -\mu F_N = m_b(\ddot{r} - r\omega^2)$

$$-\mu(2\cancel{m_b}\dot{r}\omega) = \cancel{m_b}(\ddot{r} - r\omega^2)$$

$$0 = \ddot{r} + 2\mu\omega\dot{r} - r\omega^2 \longleftarrow \text{Looking for speed over 1 revolution, use chain rule for } r(\theta)$$

$$\dot{r} = \frac{dr}{dt} = \omega\frac{dr}{d\theta} \qquad \ddot{r} = \frac{d^2r}{dt^2} = \omega^2\frac{d^2r}{d\theta^2}$$

$$0 = \frac{d^2r}{d\theta^2}\omega^2 + 2\mu\omega^2\frac{dr}{d\theta} - r\omega^2$$

$$0 = \frac{d^2r}{d\theta^2} + 2\mu\frac{dr}{d\theta} - r$$

Assume solution of the form $r = Ae^{\lambda\theta}$

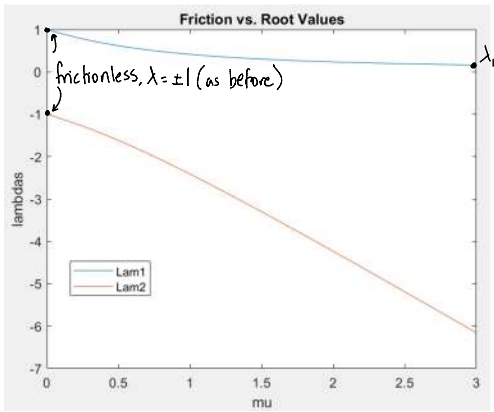
$$0 = \lambda^2\cancel{Ae^{\lambda\theta}} + 2\mu\lambda\cancel{Ae^{\lambda\theta}} - \cancel{Ae^{\lambda\theta}}$$

$$0 = \lambda^2 + 2\mu\lambda - 1$$

$$\lambda = \frac{-2\mu \pm \sqrt{4\mu^2 + 4}}{2} = \underline{-\mu \pm \sqrt{\mu^2 + 1}} \quad \text{call this } \lambda_1, \lambda_2$$

$$r = Ae^{\lambda_1\theta} + Be^{\lambda_2\theta}$$

<u>Note</u>: If $\mu = 0$, λ reduces to ± 1, like solution above



$\lambda_1 \approx 0.1$ (slow exponential decay)

– $\lambda_1 = -\mu + \sqrt{\mu^2+1}$ $A = \frac{\mu + \sqrt{\mu^2+1}}{2\sqrt{\mu^2+1}} R_0$ (found below)

As μ increases, $\lambda_1 \to 0$, $Ae^{\lambda_1\theta} \to A \to R_0$

– $\lambda_2 = -\mu - \sqrt{\mu^2+1}$

As μ increases, $\lambda_2 \to -\infty$, $Be^{\lambda_2\theta} \to 0$

$$r = Ae^{\lambda_1\theta} + Be^{\lambda_2\theta}$$

Same I.C.s as frictionless case

① $r(0) = R_0 = A + B$

② $\frac{dr}{d\theta}(0) = 0 = \lambda_1 A + \lambda_2 B$

$$0 = \lambda_1 A + \lambda_2 (R_0 - A)$$

$A' = \frac{-\lambda_2}{\lambda_1 - \lambda_2} = \frac{\mu + \sqrt{\mu^2+1}}{2\sqrt{\mu^2+1}}$ $\quad A = A'R_0$ $\quad$ Result scales with R_0

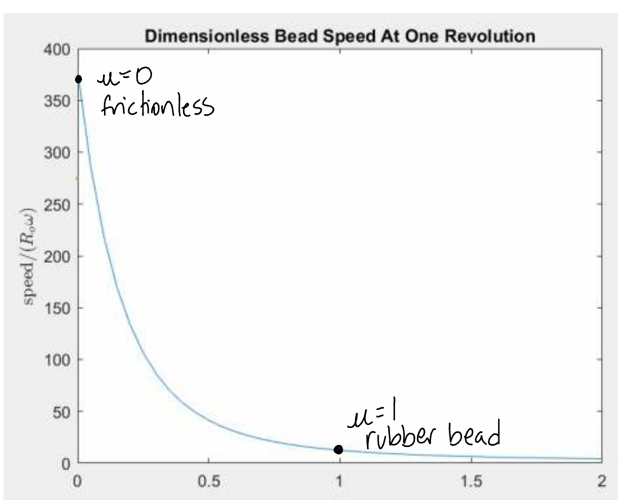
$B' = \frac{-\mu + \sqrt{\mu^2+1}}{2\sqrt{\mu^2+1}}$ $\quad B = B'R_0$ $\quad$ Define A' and B' independent of R_0

$$r = R_0 (A'e^{\lambda_1\theta} + B'e^{\lambda_2\theta})$$

$$\dot{r} = \omega R_0 (\lambda_1 A' e^{\lambda_1\theta} + \lambda_2 B' e^{\lambda_2\theta})$$

$$\vec{v} = \dot{r}\hat{e}_r + r\omega\hat{e}_\theta$$

$$\frac{\vec{v}}{R_0\omega} = [A'e^{\lambda_1\theta} + B'e^{\lambda_2\theta}]\hat{e}_r + [\lambda_1 A' e^{\lambda_1\theta} + \lambda_2 B' e^{\lambda_2\theta}]$$ ⟵ dimensionless, use in Matlab



As μ increases, speed decreases

Frictionless: $v = 379 R_0\omega$

Rubber bead: $v = 12.5 R_0\omega$

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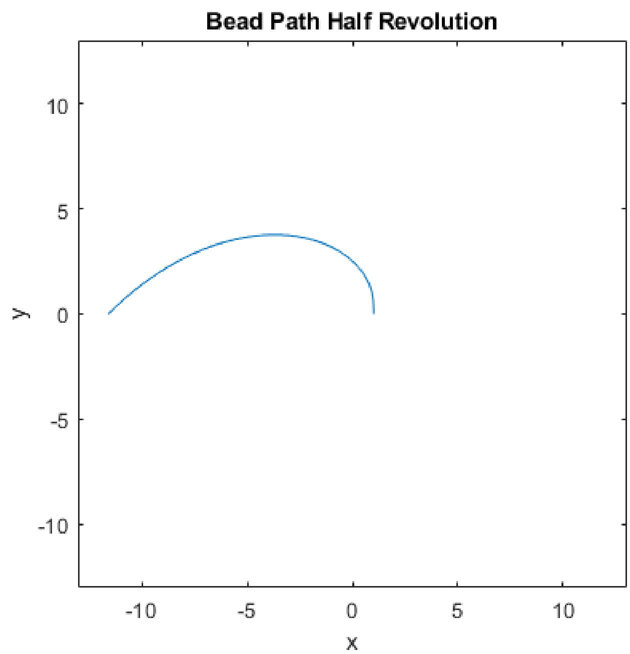
Table of Contents


```
% MAE 2030
% HW 12 #67 (19.1.12)
% Michelle deSouza
clear variables; close all; clc;

Ro = 1; %hinge is at the origin
omega = 0.1;
```

b) bead path over 1/2 revolution

```
th = linspace(0,pi,201); % half a circle
r = 0.5*Ro*(exp(th) + exp(-th)); %derived formula for r
%to Cartesian
x = r.*cos(th);
y = r.*sin(th);

plot(x,y)
axis equal
title("Bead Path Half Revolution")
xlabel("x")
ylabel("y")
axis([-13 13 -13 13])
```



c) Find length for bead to fly in -i direction

```
syms x; %correct angle
vdotj = sin(x)*(exp(x)-exp(-x))+cos(x)*(exp(x)+exp(-x));
assume(x,'positive')
theta = double(solve(vdotj));
```

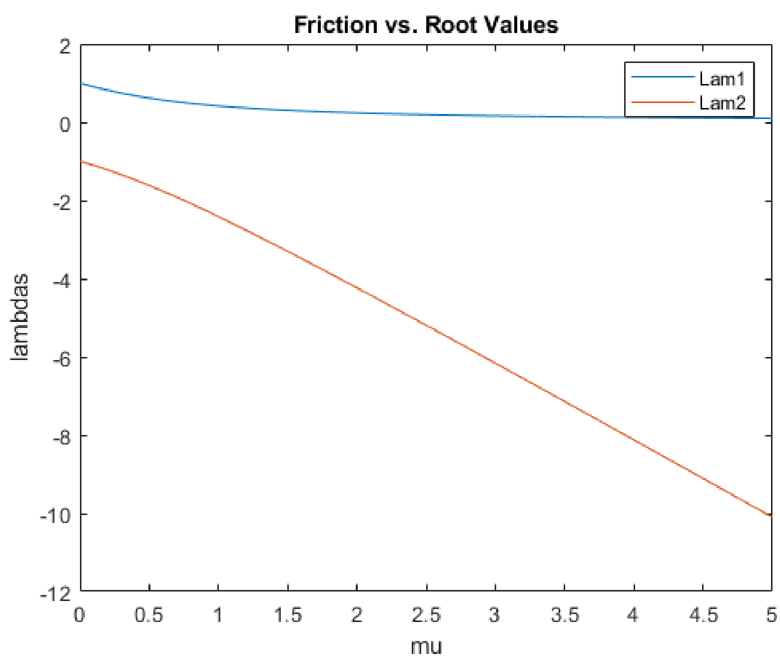
```
Warning: Unable to solve symbolically. Returning a numeric solution
 using <a
href="matlab:web(fullfile(docroot, 'symbolic/
vpasolve.html'))">vpasolve</a>.
```

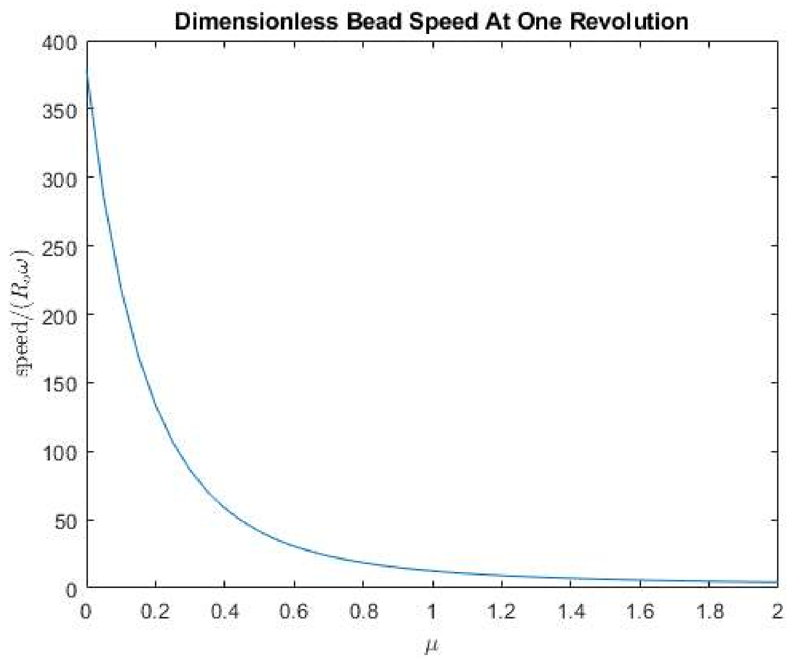
d) Add friction - speed vs. mu after one revolution

```
th = 2*pi; % one circle
mu = linspace(0,5,101);

%roots of characteristic equation
lam1 = -mu + sqrt(mu.^2+1);
lam2 = -mu - sqrt(mu.^2+1);
figure
plot(mu,lam1,mu,lam2)
title("Friction vs. Root Values")
```


Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
xlabel("mu")
ylabel("lambdas")
legend("Lam1", "Lam2")
%coefficients
Ap = -lam2./(lam1-lam2); %Ap = A/Ro
Bp = 1-Ap; %Bp = B/Ro
%r, rdot
rp = Ap.*exp(lam1*th) + Bp.*exp(lam2*th); %rp = r/Ro
rpdot = Ap.*lam1.*exp(lam1.*th) + Bp.*lam2.*exp(lam2.*th); %rpdot =
 rdot/(Ro*omega)
%find dimensionless speed and plot
vp= [rpdot; rp];
vpmag = sqrt(vp(1,:).^2 + vp(2,:).^2);

figure
plot(mu, vpmag)
xlabel('\mu')
xlim([0 2])
ylabel('speed/$(R_{o}\omega)$', 'Interpreter', 'latex')
title("Dimensionless Bead Speed At One Revolution")
hold off
```





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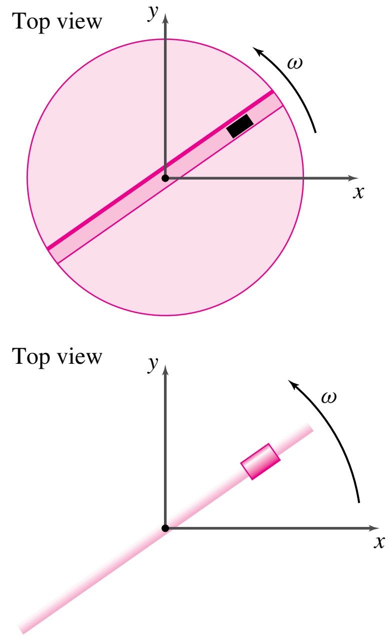
19.1.13 Mass on a lightly greased slotted turntable or spinning uniform rod. Assume that the rod/turntable in the figure is massless and also free to rotate. Assume that at $t = 0$, the angular velocity of the rod/turntable is 1 rad/s, that the radius of the bead is one meter, and that the radial velocity of the bead, dR/dt, is zero. The bead is free to slide on the rod. Where is the bead at $t = 5$ sec?



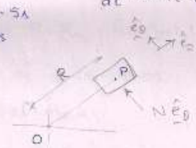
Problem 19.13

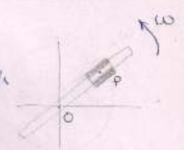
18.1.13

At $t = 0s$, $R = 1\,m$, $\frac{dR}{dt} = 0\,m/s$ & $\omega = 1\,rad/s$

Find R at $t = 5s$

FBD of mass





Acceleration of mass:

$$\underset{\sim}{a}_P = \underset{\sim}{a}_O + \underset{\sim}{a}_{P/O}$$

$$= 0 + -R\omega^2 \hat{e}_r + \dot{\omega} R \hat{e}_\theta + \ddot{R}\hat{e}_r + 2\omega\dot{R}\hat{e}_\theta$$

$$\underset{\sim}{a}_P = (\ddot{R} - R\omega^2)\hat{e}_r + (\dot{\omega}R + 2\omega\dot{R})\hat{e}_\theta$$

L.M.B for mass: $\sum \underset{\sim}{F} = m\underset{\sim}{a}_P$

$$\left\{ N\hat{e}_\theta = m(\ddot{R} - R\omega^2)\hat{e}_r + m(\dot{\omega}R + 2\omega\dot{R})\hat{e}_\theta \right\}$$

$\{\ \}\cdot\hat{e}_r \Rightarrow 0 = m(\ddot{R} - R\omega^2)$

$\Rightarrow \ddot{R} = R\omega^2$ is the equation of motion

$$(D^2 - \omega^2)R = 0$$

$\Rightarrow R = Ae^{\omega t} + Be^{-\omega t} \Rightarrow \dot{R} = A\omega e^{\omega t} - B\omega e^{-\omega t}$

At $t = 0$, $R = 1$ & $\dot{R} = 0 \Rightarrow A = B$ & $1 = A + B \Rightarrow A = B = \frac{1}{2}$

$$R = \frac{1}{2}e^{\omega t} + \frac{1}{2}e^{-\omega t} \quad \text{(or)} \quad R = \cosh \omega t$$

Given: $\omega = 1\,rad/s$

$\therefore$ After 5 sec, $R = \cosh(1 \times 5)$

$\Rightarrow R = 74.209\,m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

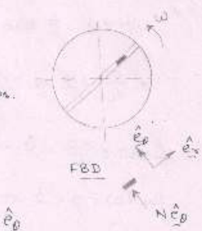
19.1.14 A bead slides in a frictionless slot in a turntable. The turntable spins a constant rate ω. The slot is straight and goes through the center of the turntable. If the bead is at radius R_o with $\dot{R} = 0$ at $t = 0$, what are the components of the acceleration vector in the directions normal and tangential to the path of the bead after one revolution? Neglect gravity.

18.1.14

Given that ω = constant.

Bead radius, $R = R_o$ & $\dot{R} = 0$ at $t = 0$s.

Components of acceleration.



Acceleration of mass;

$\underline{a} = (\ddot{R} - R\omega^2)\hat{e}_r + (\dot{\omega}R + 2\omega\dot{R})\hat{e}_\theta$

L.M.B for mass: $\Sigma \underline{F} = m\underline{a}$

$\left\{ N\hat{e}_\theta = (m\ddot{R} - mR\omega^2)\hat{e}_r + (mR\dot{\omega} + 2m\omega\dot{R})\hat{e}_\theta \right\}$

$\{\ \} \cdot \hat{e}_r \Rightarrow \ddot{R} - \omega^2 R = 0$

$\Rightarrow R = A\cosh\omega t + B\sinh\omega t$

Using the initial conditions, $B = 0$ & $A = R_o$

$\therefore R = R_o\cosh\omega t$

$\Rightarrow \dot{R} = R_o\omega\sinh\omega t$ and $\ddot{R} = R_o\omega^2\cosh\omega t$

Normal acceleration after one revolution, $a_r = (\ddot{R} - R\omega^2)\hat{e}_r$

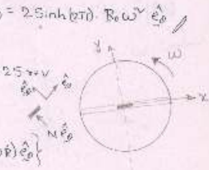
$\Rightarrow \underline{a}_r = \underline{0}$ //

Tangential acceleration after one revolution, $a_\theta = 2\omega\dot{R}\hat{e}_\theta$

$\Rightarrow \underline{a}_\theta = 2\omega R_o\omega \sinh(2\pi)\hat{e}_\theta \Rightarrow \underline{a}_\theta = 2\sinh(2\pi) R_o\omega^2 \hat{e}_\theta$ //

18.1.15

Find the force on the bead at $\theta = 1.125$ rev



L.M.B for mass $\Sigma \underline{F} = m\underline{a}$

$\left\{ N\hat{e}_\theta = (m\ddot{R} - mR\omega^2)\hat{e}_r + (mR\dot{\omega} + 2m\omega\dot{R})\hat{e}_\theta \right\}$

$\{\ \} \cdot \hat{e}_\theta \Rightarrow N = 2m\omega\dot{R}$

To find $\dot{R}$ at $\theta = 1.125$ rev

$\{\ \} \cdot \hat{e}_r \Rightarrow \ddot{R} - \omega^2 R = 0 \Rightarrow R = A\cosh\omega t + B\sinh\omega t$

Given that at $t = 0$; $R = 0$m and $\dot{R} = 0.5$ m/s

$\Rightarrow A = 0$ & $0.5 = B \times \omega(1) \Rightarrow B = \frac{0.5}{3}$ m

$\Rightarrow R = \frac{0.5}{3}\sinh\omega t \Rightarrow \dot{R} = \frac{0.5}{3} 3\cosh\omega t$

When $\theta = 1.125$ rev $\Rightarrow \dot{R} = 0.5\cosh\omega t$

$\omega t = 1.125 \times 2\pi$ rad

$\dot{R} = 0.5 \times \cosh(1.125 \times 2\pi) = 293.62$ m/s

$N\hat{e}_\theta = 2 \times m \times \omega \times \dot{R}\ \hat{e}_\theta$

$= 2 \cdot 0.001(\text{kg}) \cdot 3(\text{rad/s}) \cdot 293.62(\text{m/s})\hat{e}_\theta$

$N\hat{e}_\theta = 1.762(\text{N})\hat{e}_\theta$

$\hat{e}_\theta = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$

$\phi = 0.125$ rev $= \frac{1}{8} \times 2\pi = \pi/4$ rad

$\hat{e}_\theta = -\sin\frac{\pi}{4}\hat{\imath} + \cos\frac{\pi}{4}\hat{\jmath} \Rightarrow \hat{e}_\theta = -\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$

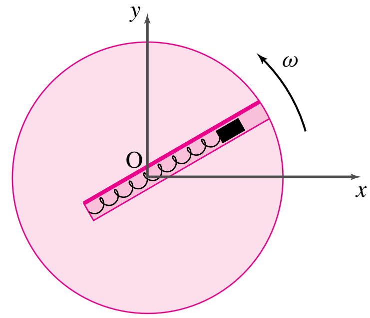
$\therefore$ Force in $\hat{\imath}, \hat{\jmath}$ is $\underline{N} = \{-1.246\hat{\imath} + 1.246\hat{\jmath}\}$ (N)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.16 Bead on springy leash in a slot on a turntable. The bead in the figure is held by a spring that is relaxed when the bead is at the origin. The constant of the spring is k. The turntable speed is controlled by a strong stiff motor.

a) Assume $\omega = 0$ for all time. What are possible motions of the bead?

b) Assume $\omega = \omega_0$ is a constant. What are possible motions of the bead? Notice there are two cases depending on the value of ω_0. What is going on here?

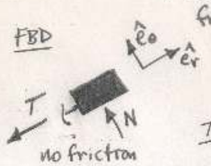


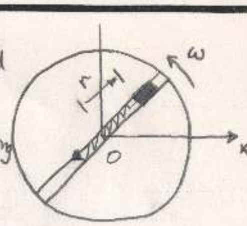
Problem 19.16: Mass in a slot.

\# 6.42

bead has mass, m

r distance of bead from the origin





force of spring

$\underset{\sim}{T} = -kr\,\hat{e}_r$

LMB $\quad -kr\,\hat{e}_r + N\hat{e}_\theta = m\left[(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta\right]$

$\boxed{A}$ if $\omega = 0$ then $\dot{\theta} = 0$, $\ddot{\theta} = 0$ dotting LMB with $\hat{e}_r$

$m\ddot{r} + kr = 0$ or $\ddot{r} + \frac{k}{m}r = 0$

motion of bead is oscillatory $\lambda = \sqrt{\frac{k}{m}}$

$r(t) = A\cos\lambda t + B\sin\lambda t$ simple harmonic oscillator!

$\boxed{B}$ if $\omega = \omega_0$ is a constant dotting LMB with $\hat{e}_r$

$-kr = m\ddot{r} - mr\omega_0^2$ or $\ddot{r} + \left(\frac{k}{m} - \omega_0^2\right)r = 0$

suppose $\lambda = \frac{k}{m} - \omega_0^2$, λ either greater than, less than, or equal to 0

case 1 $\quad \lambda > 0 \quad \omega_0 < \sqrt{\frac{k}{m}}$

motion is oscillatory, $\ddot{r} + \lambda r = 0$

case 2 $\quad \lambda < 0 \quad \omega_0 > \sqrt{\frac{k}{m}}$

radial position of bead increases exponentially $\ddot{r} - |\lambda| r = 0$

case 3 $\quad \lambda = 0 \quad \omega = \sqrt{\frac{k}{m}}$

radial position of bead doesn't change

for $\omega_0 < \sqrt{\frac{k}{m}}$ spring is strong enough to supply centripetal acceleration of bead (k is large enough).

for $\omega_0 > \sqrt{\frac{k}{m}}$ spring is not strong enough to supply centripetal acceleration of bead (k is small, spring is wimpy).

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

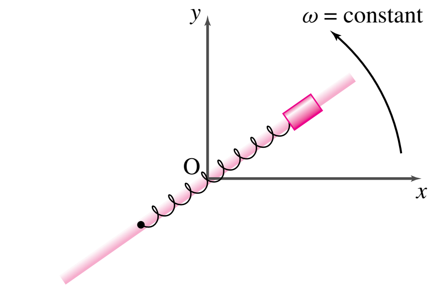
19.1.17 A small bead with mass m slides without friction on a rigid rod which rotates about the z axis with constant ω (maintained by a stiff motor not shown in the figure). The bead is also attached to a spring with constant K, the other end of which is attached to the rod. The spring is relaxed when the bead is at the center position. Assume the bead is pulled to a distance d from the center of the rod and then released with an initial $\dot{R} = 0$. If needed, you may assume that $K > m\omega^2$.

a) Derive the equation of motion for the position of the bead $R(t)$.

b) How is the motion affected by large versus small values of K?

c) What is the magnitude of the force of the rod on the bead as the bead passes through the center position? Neglect gravity. Answer in terms of m, ω, d, K.

d) Write an expression for the Coriolis acceleration. Give an example of a situation in which this acceleration is important and explain why it arises.



Problem 19.17

18.1.17

Spring Constant = K N/m

(a) Let the mass be pulled to a distance R from the center of rod.

FBD

L.M.B for the mass: $\sum \underline{F} = m\underline{a}$

$$\left\{ KR(-\hat{e}_r) + N\hat{e}_\theta = m(\ddot{R} - R\omega^2)\hat{e}_r + m\,2\omega\dot{R}\,\hat{e}_\theta \right\}$$

$\{\,\}\cdot\hat{e}_r \Rightarrow m\ddot{R} - mR\omega^2 = -KR$

$\Rightarrow m\ddot{R} + (K - m\omega^2)R = 0$

$\Rightarrow \ddot{R} + \left(\frac{K}{m} - \omega^2\right)R = 0$

(b) If $K > m\omega^2$, $R = Ae^{i\sqrt{\frac{K}{m}-\omega^2}\,t} + Be^{-i\sqrt{\frac{K}{m}-\omega^2}\,t}$

At $t=0$, $R = d$ and $\dot{R} = 0$

$\Rightarrow A + B = d$

$Ai\sqrt{\frac{K}{m}-\omega^2} - Bi\sqrt{\frac{K}{m}-\omega^2} = 0 \Rightarrow A = B \Rightarrow A = \frac{d}{2} = B$

$\therefore R = \frac{d}{2}\,2\cos\sqrt{\frac{K}{m}-\omega^2}\,t$

$\Rightarrow R = d\cos\left[\left(\sqrt{\frac{K}{m}-\omega^2}\right)t\right]$

For large values of K, bead oscillates about the center of the rod.

However, if $K < m\omega^2$, $R = d\cosh\left[\sqrt{\frac{K}{m}-\omega^2}\,t\right]$, indicating that bead's position increases exponentially.

(c) From (a) and (b), for $K > m\omega^2$, Force of rod on bead as it passes through center position is

$N\hat{e}_\theta = 2m\omega\dot{R}\,\hat{e}_\theta$

$\Rightarrow N = 2m\omega d\sqrt{\frac{K}{m}-\omega^2}$ as $t = \frac{\pi}{2\sqrt{\frac{K}{m}-\omega^2}}$

d) Coriolis acceleration $\underline{a}_{cor}$

$\underline{r}_A = \underline{r}_{O'} + \underline{r}_{A/O'}$

Let A' be glued to A in x'oy' frame

Velocity of A in OXY frame is

$\underline{v}_A = \underline{v}_{O'} + \frac{d}{dt}(\underline{r}_{A/O'})$

$\underline{v}_A = \underline{v}_{O'} + \dot{\underline{r}}_{A/O'} + \underline{\omega} \times \underline{r}_{A/O'}$

Acceleration of A in OXY frame is

$\underline{a}_A = \dot{\underline{v}}_{O'} + \frac{d}{dt}(\underline{v}_{A/O'}) + \dot{\underline{\omega}} \times \underline{r}_{A/O'} + \underline{\omega} \times \frac{d}{dt}(\underline{r}_{A/O'})$

$= \underline{a}_{O'} + \dot{\underline{v}}_{A/O'} + \underline{\omega} \times \underline{v}_{A/O'} + \dot{\underline{\omega}} \times \underline{r}_{A/O'} + \underline{\omega} \times \left[\dot{\underline{r}}_{A/O'} + \underline{\omega} \times \underline{r}_{A/O'}\right]$

$= \underline{a}_{A'/O} + \dot{\underline{v}}_{A/O'} + \underbrace{\underline{\omega} \times \underline{v}_{A/O'} + \underline{\omega} \times \dot{\underline{r}}_{A/O'}}_{\underline{a}_{cor}}$

$\underline{a}_A = \underline{a}_{A'/O} + \underline{a}_{A/O'} + \underline{a}_{cor}$

where $\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}_{A/O'}$

Coriolis acceleration has two sources. One originates from the change in the direction of the radial velocity $\underline{v}_{A/O'}$ and the other from the change in the position of the tangential velocity. This acceleration plays major role in estimating the position of ballistic missiles, projectiles, rockets and aircrafts moving in the 'rotating' atmosphere due to earth's rotation.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

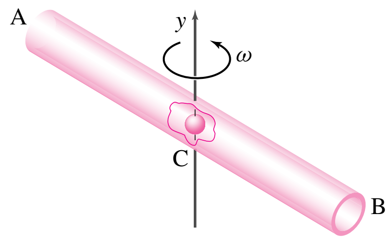
19.1.18 A small ball of mass m_b = 500 grams may slide in a slender tube of length l = 1.2 m. and of mass m_t = 1.5 kg. The tube rotates freely about a vertical axis passing through its center C. (Hint: Treat the tube as a slender rod.)

a) If the angular velocity of the tube is $\omega = 8\,\mathrm{rad/s}$ as the ball passes through C with speed relative to the tube, v_0, calculate the angular velocity of the tube just before the ball leaves the tube.

b) Calculate the angular velocity of the tube just after the ball leaves the tube.

c) If the speed of the ball as it passes $C$is $v = 1.8\,\mathrm{m/s}$, determine the transverse and radial components of velocity of the ball as it leaves the tube.

d) After the ball leaves the tube, what constant torque must be applied to the tube (about its axis of rotation) to bring it to rest in 10 s?



Problem 19.18

Answer:
(a) $\omega^- = 4\,\mathrm{rad/s}$, where the minus sign '-' means 'just before leaving'.
(b) $\omega^+ = 4\,\mathrm{rad/s}$, where the plus sign '+' means 'just after leaving'.
(c) $v_r = 3.841\,\mathrm{m/s}$, $v_t = 2.4\,\mathrm{m/s}$.
(d) torque=0.072 N·m

18.1.18

Given: ball mass, $m_b = 0.5\,\mathrm{Kg}$

mass of the tube, $m_t = 1.5\,\mathrm{Kg}$

Length of the tube, $l = 1.2\,\mathrm{m}$

a) Angular velocity of the tube, $\omega^- = 8\,\mathrm{rad/s}$

Angular velocity of the tube just before the ball leaves the tube be ω^+

Conservation of angular momentum applied when ball is at C & B:

$\Rightarrow \quad \underline{H}_{/o}^- = \underline{H}_{/o}^+$

$\Rightarrow \left\{ m_t \frac{l^2}{12} \omega^- \hat{\underline{k}} = \frac{m_t l^2}{12} \omega^+ \hat{\underline{k}} + \underline{r} \times m_b \underline{v} \right\}$

$\{\}\cdot\hat{\underline{k}} \Rightarrow \quad \omega^+ = \dfrac{m_t \frac{l^2}{12}}{m_t \frac{l^2}{12} + m_b \left(\frac{l}{2}\right)^2} \omega^- = \dfrac{1}{1 + \frac{3m_b}{m_t}} \omega^- = 4\,\mathrm{rad/s}.$

b) As there is no torque acting on the tube during the balls' motion, angular velocity does not change even after the ball leaves the tube.

c) Speed of the ball as it passes 'C' is $V = 1.8\,\mathrm{m/s}$.

Energy balance when ball is at C and B:

$\frac{1}{2} m_b v^{-2} + \frac{1}{2} I \omega^{-2} = \frac{1}{2} m_b v^{+2} + \frac{1}{2} I \omega^{+2} \qquad I = \frac{m_t l^2}{12}$

$\Rightarrow \quad v^+ = \left(\dfrac{m_b v^{-2} + I(\omega^{-2} - \omega^{+2})}{m_b} \right)^{1/2} = 4.53\,\mathrm{m/s}$

As $v^+ = \left| \omega^+ \times \frac{l}{2} \hat{\underline{e}}_\theta + v_r \hat{\underline{e}}_r \right|$, $V_t = 2.40\,\mathrm{m/s}$ and $V_r = 3.84\,\mathrm{m/s}$

d) Apply A.M.B about O: $\sum \underline{M}_{/o} = \dot{\underline{H}}_{/o}$ (or) $\left\{ T \hat{\underline{k}} = \frac{m_t l^2}{12} \dot{\omega} \hat{\underline{k}} \right\}$

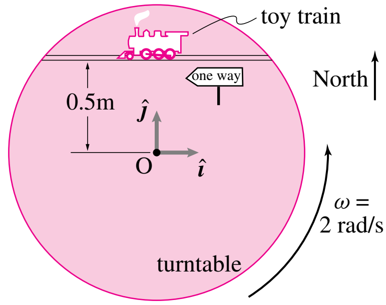
As $\omega_f = \omega_i + \dot{\omega} t$, for $\omega_f = 0$ & $t = 10\,\mathrm{s}$, $\dot{\omega} = \frac{-\omega_i}{t}$ (or) $\dot{\omega} = -0.4\,\mathrm{rad/s^2}$

$\{\}\cdot\hat{\underline{k}}$

$\Rightarrow \quad T = -0.072\,\mathrm{Nm}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.19 Toy train car on a turn-around. The 0.1 kg toy train's speedometer reads a constant $1 m/s$ when, heading west, it passes due north of point O. The train is on a level straight track which is mounted on a spinning turntable whose center is 'pinned' to the ground. The turntable spins at the constant rate of 2 rad/s. What is the force of the turntable on the train? (Don't worry about the z-component of the force.)



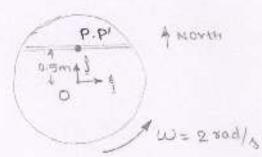
Problem 19.19

Answer:
$\vec{F} = -0.6\,\mathrm{N}\,\hat{j}$.

mass of the toy train, $m = 0.1$ Kg

Velocity, $\underset{\sim}{v} = -1\hat{i}$ m/s

Spin rate, $\underset{\sim}{\omega} = 2\hat{k}$ rad/s



Applying LMB on toy train, $\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$\underset{\sim}{a} = (-r\omega^2\hat{j} + \cancel{(\dot{\underset{\sim}{\omega}} \times \underset{\sim}{r})}^{0}) + (\cancel{\underset{\sim}{a}_{P'/O}}^{0}) + 2\underset{\sim}{\omega} \times \underset{\sim}{v})$

$\Rightarrow \underset{\sim}{a} = -2\hat{j} - 2(2\hat{k} \times -1\hat{i}) = -2\hat{j} - 4\hat{j} = -6\hat{j}$ m/s²

$\therefore \underset{\sim}{N} = m\underset{\sim}{a} \Rightarrow \underset{\sim}{N} = -0.6\hat{j}$ N

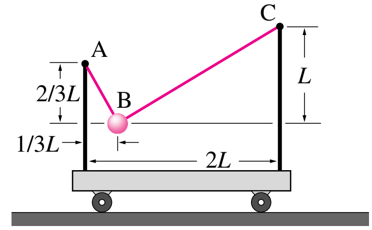
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.20 Due to forces not shown, the cart moves to the right with constant acceleration a_x. The ball B has mass m_B. At time $t = 0$, the string AB is cut. Find

a) the tension in string BC before cutting,

b) the absolute acceleration of the mass at the instant of cutting,

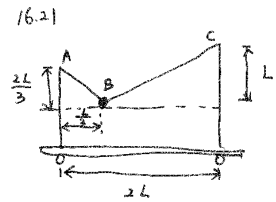
c) the tension in string BC at the instant of cutting.



Problem 19.20

Answer:
(a) $T_{BC} = m\frac{\sqrt{34}}{13}(2a_x + g)$.
(b) $\vec{a} = \frac{1}{34}[(25a_x + 15g)\hat{\imath} + (15a_x - 25g)\hat{\jmath}]$
(c) $T_{BC} = m\frac{1}{\sqrt{34}}(5a_x + 3g)$.

16.21



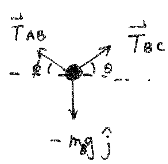
Cart moving forward with $\vec{a} = a_x\hat{\imath}$

At time $t=0$, cut AB.

i) Before cutting, tension in BC ?

The ball B moves with the cart, $\vec{a}_B = a_x\hat{\imath}$

FBD :



$\vec{T}_{BC} = T_{BC}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$

$\vec{T}_{AB} = T_{AB}(-\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath})$

LMB : $\{\vec{T}_{AB} + \vec{T}_{BC} - m_B g\hat{\jmath} = m_B a_x\hat{\imath}\}$

$\{\}\cdot(\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath})$

$\Rightarrow \quad T_{BC}(\cos\theta\sin\phi + \sin\theta\cos\phi) = m_B g\cos\phi + m_B a_x\sin\phi$

$\therefore \quad \cos\theta = \dfrac{\frac{5}{3}L}{\sqrt{(\frac{5}{3}L)^2 + L^2}} = \dfrac{5}{\sqrt{34}}, \quad \sin\theta = \dfrac{3}{\sqrt{34}}$

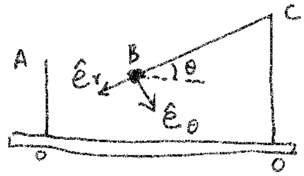
$\cos\phi = \dfrac{1}{\sqrt{5}}, \quad \sin\phi = \dfrac{2}{\sqrt{5}}$

$\Rightarrow \boxed{T_{BC} = \frac{\sqrt{34}}{13} m_B(g + 2a_x)}$

$\cancel{Be}$ before cutting AB

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

After cutting AB, the tension in BC is not that before cutting.



Since BC is inextensible, ball B can only swing about C relative to the cart.

$$\therefore \vec{a}_B = \vec{a}_{cart} + \vec{a}_{B/cart}$$

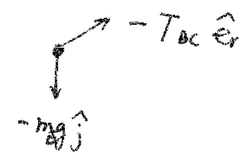
$$= a_x \hat{i} + \ddot{\theta} d \hat{e}_\theta - d\dot{\theta}^2 \hat{e}_r$$

where $d = |\vec{r}_{B/C}|$: distance between B and C.

Just after cutting, $\dot{\theta} = 0$

$$\therefore \vec{a}_B = a_x \hat{i} + \ddot{\theta} d \hat{e}_\theta$$

FBD



LMB: $-T_{BC}\hat{e}_r - mg\hat{j} = m\vec{a}_B$

$$\Rightarrow \{-T_{BC}\hat{e}_r - mg\hat{j} = m a_x \hat{i} + m\ddot{\theta} d \hat{e}_\theta\}$$

$\{\}\cdot\hat{e}_r \Rightarrow$ $-T_{BC} = mg(\hat{j}\cdot\hat{e}_r) + m a_x(\hat{i}\cdot\hat{e}_r)$

$\{\}\cdot\hat{e}_\theta \Rightarrow$ $m\ddot{\theta} d = -mg(\hat{j}\cdot\hat{e}_\theta) - m a_x(\hat{i}\cdot\hat{e}_\theta)$

$\hat{j}\cdot\hat{e}_r = -\sin\theta$, $\hat{i}\cdot\hat{e}_r = -\cos\theta$, $\hat{j}\cdot\hat{e}_\theta = -\cos\theta$, $\hat{i}\cdot\hat{e}_\theta = \sin\theta$

$$\Rightarrow T_{BC} = mg\sin\theta + m a_x\cos\theta = \frac{m}{\sqrt{34}}(3g + 5a_x)$$

and $m\ddot{\theta} d = mg\cos\theta - m a_x \sin\theta = \frac{m}{\sqrt{34}}(5g - 3a_x)$

$\therefore$ Absolute acceleration of B: $\cancel{\vec{a}_B = a_x} \hat{i}$

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$$\vec{a}_B = a_x\hat{\imath} + \ddot{\theta} d\, \hat{e}_\theta$$

$$= a_x\hat{\imath} + \ddot{\theta} d \sin\theta\, \hat{\imath} + \ddot{\theta} d\, (-\cos\theta)\hat{\jmath}$$

$$= a_x\hat{\imath} + \frac{1}{\sqrt{34}}(5g - 3a_x)\frac{3}{\sqrt{34}}\hat{\imath} + \frac{1}{\sqrt{34}}(5g-3a_x)(-\frac{5}{\sqrt{34}})\hat{\jmath}$$

$$= \frac{1}{34}(25a_x + 15g)\hat{\imath} + \frac{1}{34}(15a_x - 25g)\hat{\jmath}$$

$$\therefore \boxed{\vec{a}_B = \frac{1}{34}(25a_x + 15g)\hat{\imath} + \frac{1}{34}(15a_x - 25g)\hat{\jmath}}$$

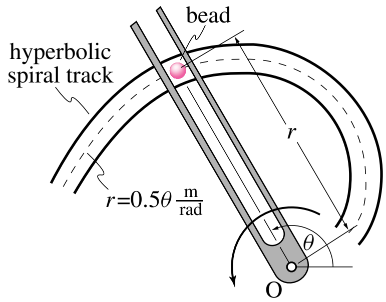
Tension in BC after cutting:

$$\boxed{T_{BC} = \frac{m}{\sqrt{34}}(3g + 5a_x)}$$

19.1.21 The forked arm mechanism pushes the bead of mass 1 kg along a frictionless hyperbolic spiral track given by $r = 0.5\theta(\text{m}/\text{rad})$. The arm rotates about its pivot point at O with constant angular acceleration $\ddot{\theta} = 1\,\text{rad}/\text{s}^2$ driven by a motor (not shown). The arm starts from rest at $\theta = 0^\circ$.

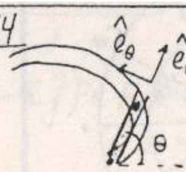
a) Determine the radial and transverse components of the acceleration of the bead after 2 s have elapsed from the start of its motion.

b) Determine the magnitude of the net force on the mass at the same instant in time.



Problem 19.21

6.44



Spiral track: particle position given by
$r = 0.5\theta$ [m/rad]
$\ddot{\theta} = 1$ rad/s²

a) determine $\underset{\sim}{a}\cdot\hat{e}_r$ & $\underset{\sim}{a}\cdot\hat{e}_\theta$

continued

6.44 continued

p.4

if $\ddot{\theta} = 1\,\text{rad}/\text{s}^2$, then:

$\dot{\theta} = \int \ddot{\theta}\, dt = t$ rad/s

$\theta = \int \dot{\theta}\, dt = t^2/2$ rad.

Thus, since $R = 0.5\theta$ (m/rad)

$\dot{R} = 0.5\dot{\theta}$ (m/s·rad) and $\ddot{R} = 0.5\ddot{\theta}$ (m/s²·rad)

So, at $t = 2$sec, we have

$\ddot{\theta} = 1\,\text{rad}/\text{s}^2$, $\dot{\theta} = 2$ rad/s, $\theta = 2$ rad,

$\ddot{R} = 0.5\,\text{m}/\text{s}^2$, $\dot{R} = 1$ m/s, $R = 1$ m

Then $\underset{\sim}{a} = (-R\dot{\theta}^2 + \ddot{R})\hat{e}_r + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$

$\underset{\sim}{a} = [(-4 + 0.5)\hat{e}_r + (4 + 1)\hat{e}_\theta]\,\text{m}/\text{s}^2$

$\boxed{\underset{\sim}{a}\cdot\hat{e}_r = -3.5\,\text{m}/\text{s}^2, \quad \underset{\sim}{a}\cdot\hat{e}_\theta = 5\,\text{m}/\text{s}^2}$

b) Net force: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

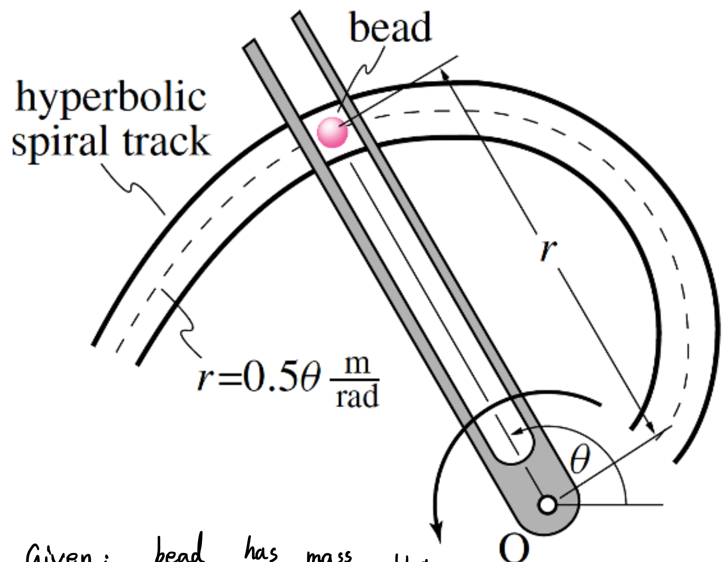
magnitude: $|\Sigma \underset{\sim}{F}| = m|\underset{\sim}{a}| = 1\text{kg}\left((-3.5\,\text{m}/\text{s}^2)^2 + (5\,\text{m}/\text{s}^2)^2\right)^{1/2}$

$\Rightarrow \boxed{|\Sigma \underset{\sim}{F}| = 6.10\ \text{N}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.1.21

Saturday, May 4, 2019 7:50 PM



Given: bead has mass 1kg

hyperbolic spiral track is frictionless

$r = 0.5\,\theta$ [m/rad]

rotates about O with const angular acceleration $\ddot{\theta} = 1$[rad/s²]

Starts from rest at $\theta = 0°$.

a) Find $\vec{a} \cdot \hat{e}_r$ & $\vec{a} \cdot \hat{e}_\theta$ of the bead

$\dot{\theta} = \int_0^t \ddot{\theta}\, dt = t$ [rad/s²]

$\ddot{\theta} = \int_0^t \dot{\theta}\, dt = \frac{1}{2} t^2$ [rad/s²]

Since $R = 0.5\,\theta \,\frac{m}{rad}$

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Take derivative on both sides

$$\dot{R} = 0.5\,\dot{\theta}\,\frac{m}{rad}$$

$$\ddot{R} = 0.5\,\ddot{\theta}\,\frac{m}{rad}$$

At $t = 2s$, we have

$\ddot{\theta} = 1 rad/s^2$ $\dot{\theta} = 2 rad/s$ $\theta = 2 rad$

$\ddot{R} = 0.5\ m/s^2$ $\dot{R} = 1 m/s$ $R = 1m$

Thus $\vec{a} = (-R\dot{\theta}^2 + \ddot{R})\hat{e}_r + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$

$= -3.5\ \hat{e}_r + 5\ \hat{e}_\theta\ [m/s^2]$

$\therefore\ \vec{a}\cdot\hat{e}_r = -3.5\ [m/s^2]$

$\vec{a}\cdot\hat{e}_\theta = 5\ [m/s^2]$

b) Find the magnitude of the net force at $t=2s$

LMB $\Sigma\vec{F} = m\vec{a}$

magnitude $|\Sigma\vec{F}| = m|\vec{a}| = 1\,kg\cdot\sqrt{3.5^2 + 5^2}\ m/s^2 = 6.10\,N$

16.21 Given; $\ddot{\theta} = 1$ (constant); $\dot{\theta}(0) = \theta(0) = 0$

$$r = 0.5\theta \Rightarrow r(t) = 0.5\theta(t)$$

Find a_r, a_t, $|\vec{F}|$ at $t = 2s$

Lets solve for $r(t)$ and $\theta(t)$

$$\ddot{\theta} = 1$$

Integrate Twice $\theta(t) = \frac{t^2}{2} + C_1 t + C_2$

But $\dot{\theta}(0) = \theta(0) = 0 \Rightarrow \boxed{\theta(t) = 0.5t^2}$

Also $r(t) = 0.5\theta(t) \Rightarrow \boxed{r(t) = 0.25t^2}$

(a) $\vec{a} = (\ddot{r} - \dot{\theta}^2 r)\hat{e}_r + (2\dot{r}\dot{\theta} + \ddot{\theta}r)\hat{e}_\theta$

$r(2) = 1$ $\quad \dot{r}(t) = 0.5t \Rightarrow \dot{r}(2) = 1$

$\ddot{r}(t) = 0.5 \Rightarrow \ddot{r}(2) = 0.5$

$\dot{\theta}(t) = t \Rightarrow \dot{\theta}(2) = 2$

$\ddot{\theta}(t) = 1 \Rightarrow \ddot{\theta}(2) = 1$

Thus $\vec{a} = (0.5 - 2^2 \cdot 1)\hat{e}_r + (2 \times 1 \times 2 + 1 \times 1)\hat{e}_\theta$

$\boxed{\vec{a} = -3.5\hat{e}_r + 5\hat{e}_\theta}$ (Answer)

(b) $\vec{F} = m\vec{a} = -3.5\hat{e}_r + 5\hat{e}_\theta$ $\{$As $m = 1\,kg\}$

$|\vec{F}| = \sqrt{(-3.5)^2 + 5^2} = 6.1$

$\boxed{|\vec{F}| = 6.1\ N}$

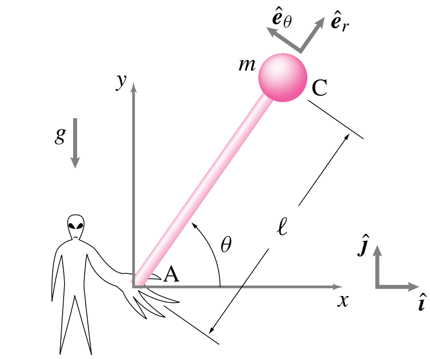
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

19.1.22 A rod is on the palm of your hand at point A. Its length is ℓ. Its mass m is assumed to be concentrated at its end at C. Assume that you know θ and $\dot\theta$ at the instant of interest. Also assume that your hand is accelerating both vertically and horizontally with $\vec{a}_{\text{hand}} = a_{hx}\hat{\imath} + a_{hy}\hat{\jmath}$. Coordinates and directions are as marked in the figure.

a) Draw a Free-Body Diagram of the rod.

b) Assume that the hand is stationary. Solve for $\ddot\theta$ in terms of g, ℓ, m, θ, and $\dot\theta$.

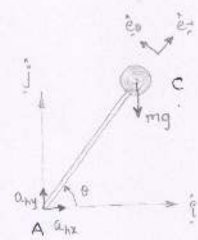
c) If the hand is not stationary but $\ddot\theta$ has been determined somehow, find the vertical force of the hand on the rod in terms of $\ddot\theta$, $\dot\theta$, θ, g, ℓ, m, a_{hx}, and a_{hy}.



Problem 19.22: Rod on a Moving Support(2-D).

18.1.22

a) Free body diagram:



b) Assume $\underline{a}_{hand} = \underline{0}$, $\ddot\theta = ?$

Applying AMB about A: $\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$

$\Rightarrow l\hat{e}_r \times mg(-\hat{j}) = l\hat{e}_r \times m\underline{a}$

$\underline{a} = \underline{a}_{hand} + \ddot\theta\hat{k} \times l\hat{e}_r + \dot\theta\hat{k} \times (\dot\theta\hat{k} \times l\hat{e}_r)$

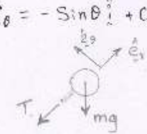
$\Rightarrow \underline{a} = l\ddot\theta\hat{e}_\theta - l\omega^2\hat{e}_r$

$\therefore \{-mgl\cos\theta\,\hat{k} = ml^2\ddot\theta\,\hat{k}\}$

$\{\}\cdot\hat{k} \Rightarrow \ddot\theta = -\frac{g}{l}\cos\theta$ //

$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$



c) Applying LMB on mass: $\sum \underline{F} = m\underline{a}$

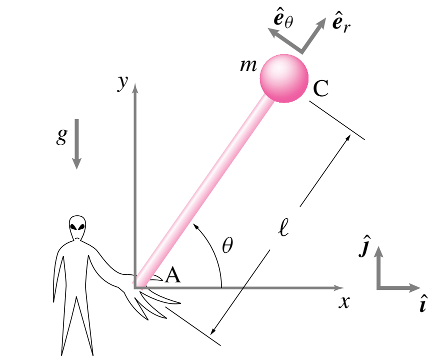
$\Rightarrow \{-T\hat{e}_r - mg\hat{j} = m(a_{hx}\hat{i} + a_{hy}\hat{j} + \ddot\theta l\hat{e}_\theta - l\dot\theta^2\hat{e}_r)\}\cdot\hat{e}_r$

$\{\}\cdot\hat{e}_r \Rightarrow -T - mg\sin\theta = m(a_{hx}\cos\theta + a_{hy}\sin\theta - l\dot\theta^2)$

$\Rightarrow T = m[l\dot\theta^2 - a_{hx}\cos\theta - (a_{hy}+g)\sin\theta]$

$\therefore$ Force on rod, $\underline{F} = -T\hat{e}_r = m[a_{hx}\cos\theta + (a_{hy}+g)\sin\theta - l\dot\theta^2]\,\hat{e}_r$

19.1.23 Balancing a broom. Assume the hand is accelerating to the right with acceleration $\boldsymbol{a} = a\hat{\boldsymbol{\imath}}$. What is the force of the hand on the broom in terms of m, ℓ, θ, $\dot{\theta}$, a, $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and g? *(You may not have any $\hat{\boldsymbol{e}}_R$ or $\hat{\boldsymbol{e}}_\theta$ in your answer.)*



Problem 19.23

18.1.23

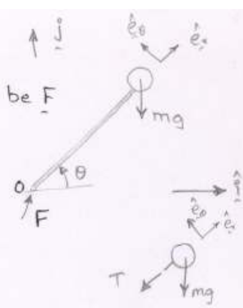
Let the force of the hand on the broom be $\underline{F}$

Applying L.M.B : $\sum \underline{F} = m\underline{a}$

$$mg(-\hat{\jmath}) - T\hat{e}_r = m\left[\underline{a}_0 + \ddot{\theta}\,l\,\hat{e}_\theta - l\dot{\theta}^2\hat{e}_r\right]$$

$\{\}\cdot\hat{e}_r \Rightarrow -T - mg\sin\theta = m\{a\cos\theta - l\dot{\theta}^2\}$

$\Rightarrow T = m\left[l\dot{\theta}^2 - g\sin\theta - a\cos\theta\right](\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$



$\underline{a}_0 = a\hat{\imath}$

$\Rightarrow$ Force on broom, $F = m\cos\theta\left[g\sin\theta + a\cos\theta - l\dot{\theta}^2\right]\hat{\imath} + m\sin\theta\left[g\sin\theta + a\cos\theta - l\dot{\theta}^2\right]\hat{\jmath}$