19.1 Mechanics of a constrained particle

19.1.1 A bead slides on a frictionless circular hoop . The mass of the bead $m_{\text{bead}} = 2$ grams, mass of hoop $m_{\text{hoop}} = 1$ kg and radius of hoop $R_{\text{hoop}} = 3$ m. Neglect gravity. The center of the circular hoop is the origin O of a fixed (Newtonian) coordinate system $Oxyz$. The hoop is on the xy plane. The hoop is kept from moving by little angels who let the bead slide by unimpeded. At $t = 0$, the speed of the bead is 4 m/s, it is traveling counter-clockwise (looking down the z-axis), and it is on the $+x$-axis. There are no other external forces applied.

a) At $t = 0$ what is the bead's kinetic energy ?

b) At $t = 0$ what is the bead's linear momentum ?

c) At $t = 0$ what is the bead's angular momentum about the origin?

d) At $t = 0$ what is the bead's acceleration?

e) At $t = 0$ what is the radius of the osculating circle of the bead's path ρ?

f) At $t = 0$ what is the force of the hoop on the bead?

g) At $t = 0$ what is the net force of the angel's hands on the hoop?

h) At $t = 27s$ what is the x-component of the bead's linear momentum?

19.1.2 A warehouse operator wants to move a crate of weight $W = 100$ lb from a 6 ft high platform to ground level by means of a roller conveyer as shown. The rollers on the inclined plane are well lubricated and thus assumed frictionless; the rollers on the horizontal conveyer are frictional, thus providing an effective friction coefficient μ. Assume the rollers are massless.

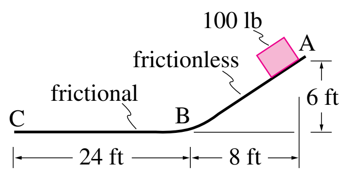
a) What are the kinetic and potential energies(pick a suitable datum) of the crate at the elevated platform, point A?

b) What are the kinetic and potential energies of the crate at the end of the inclined plane just before it moves onto the horizontal conveyor, point B?

c) Using conservation of energy, calculate the maximum speed attained by the crate at point B, assuming it starts moving from rest at point A.

d) If the crate slides a distance x, say, on the horizontal conveyor, what is the energy lost to sliding in terms of μ, W, and x?

e) Calculate the value of the coefficient of friction, μ, such that the crate comes to rest at point C.



Problem 19.1.2

19.1.3 On a wintry evening, a student of mass m starts down the steepest street in town, *Steep Street*, which is of height h and slope θ. At the top of the slope she starts to slide (coefficient of dynamic friction μ).

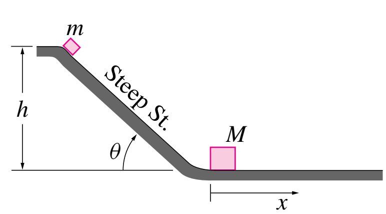
a) What is her initial kinetic and potential energy?

b) What is the energy lost to friction as she slides down the hill?

c) What is her velocity on reaching the bottom of the hill? Ignore air resistance and all cross streets (i.e. assume the hill is of constant slope).

d) If, upon reaching the bottom of the street, she collides with another student of mass M and they embrace, what is their instantaneous mutual velocity just after the embrace?

e) Assuming that friction still acts on the level flats on the bottom, how much time will it take before they come to rest?



Problem 19.1.3

19.1.4 Reconsider the system of blocks in problem 2.3.14, this time with equal mass, $m_1 = m_2 = m$. Also, now, both blocks are frictional and sitting on a frictional surface. Assume that both blocks are sliding to the right with the top block moving faster. The coefficient of friction between the two blocks is μ_1. The coefficient of friction between the lower block and the floor is μ_2. It is known that $\mu_1 > 2\mu_2$.

a) Draw free-body diagrams of the blocks together and separately.

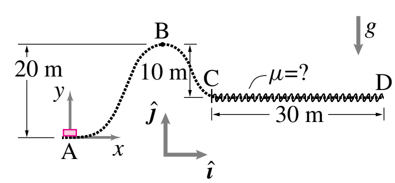
b) Write the equations of linear momentum balance for each block.

c) Find the acceleration of each block for the case of frictionless blocks. For the frictional blocks, find the acceleration of each block. What happens if $\mu_1 >> \mu_2$?

19.1.5 An initially motionless roller-coaster car of mass 20 kg is given a horizontal impulse $\int \vec{F}\,dt = P\hat{\imath}$ at position A, causing it to move along the track, as shown below.

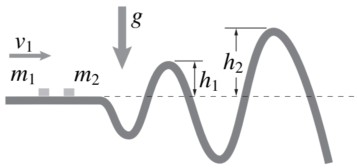
a) Assuming that the track is perfectly frictionless from point A to C and that the car never leaves the track, determine the magnitude of the impulse I so that the car "just makes it" over the hill at B.

b) In the ensuing motion, assume that the horizontal track C-D is frictional and determine the coefficient of friction required to bring the car to a stop at D.



Problem 19.1.5

19.1.6 Masses $m_1 = 1$ kg and m_2 kg move on the frictionless varied terrain shown. Initially, m_1 has speed $v_1 = 12$ m/s and m_2 is at rest. The two masses collide on the flat section. The coefficient of restitution in the collision is $e = 0.5$. Find the speed of m_2 at the top of the first hill of elevation $h_1 = 1$ m. Does m_2 make it over the second hill of elevation $h_2 = 4$ m?



Problem 19.1.6

19.1.7 A **Scotch yoke** is a device for converting rotary motion into linear motion. In this case it is used to make a horizontal platform go up and down. The motion of the platform is $y = A\sin(\omega t)$ with constant ω. A dead bug with mass m is standing on the platform. Her feet have no glue on them. There is gravity.

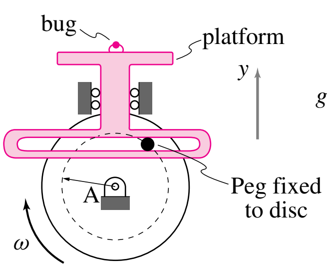
a) Draw a free-body diagram of the bug.

b) What is the acceleration of the platform and, hence, the bug?

c) Write the equation of linear momentum balance for the bug.

d) What condition must be met so that the bug does not bounce the platform?

e) What is the maximum spin rate ω that can be used if the bug is not to bounce off the platform?



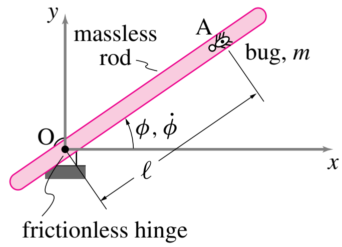
Problem 19.1.7

19.1.8 A bug of mass m walks straight-forwards with speed v_A and rate of change of speed $\dot{v}_A$ on a straight light (assumed to be massless) stick. The stick is hinged at the origin so that it is always horizontal but is free to rotate about the z axis. Assume that the distance the bug is from the origin, ℓ, the angle the stick makes with the x axis, ϕ, and its rate of change, $\dot{\phi}$, are known at the instant of interest. Ignore gravity. Answer the following questions in terms of ϕ, $\dot{\phi}$, ℓ, m, v_A, and $\dot{v}_A$.

a) What is $\ddot{\phi}$?

b) What is the force exerted by the rod on the support?

c) What is the acceleration of the bug?



Problem 19.1.8

19.1.9 A new kind of gun. Assume $\omega = \omega_0$ is a constant for the rod in the figure. Assume the mass is free to slide. At $t = 0$, the rod is aligned with the x-axis and the bead is one foot from the origin and has no radial velocity ($dR/dt = 0$).

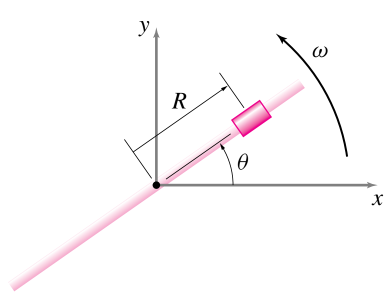
a) Find a differential equation for $R(t)$. *

b) Turn this equation into a differential equation for $R(\theta)$. *

c) How far will the bead have moved after one revolution of the rod? How far after two? *

d) What is the speed of the bead after one revolution of the rod (use $\omega_0 = 2\pi$ rad/s = 1 rev/sec)? *

e) How much kinetic energy does the bead have after one revolution and where did it come from? *



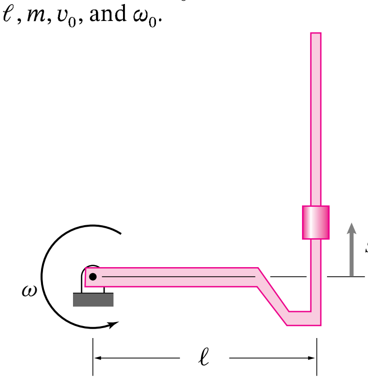
Problem 19.1.9

19.1.10 The new gun gets old and rusty. Reconsider the bead on a rod in problem 19.1.9. This time, friction cannot be neglected. The friction coefficient is μ. At the instant of interest the bead with mass m has radius R_0 with rate of change $\dot{R}_0$. The angle θ is zero and ω is a constant. Neglect gravity.

a) What is $\ddot{R}$ at this instant? Give your answer in terms of any or all of R_0, $\dot{R}_0$, ω, m, μ, $\hat{\boldsymbol{e}}_\theta$, $\hat{\boldsymbol{e}}_R$, $\hat{\boldsymbol{\imath}}$, and $\hat{\boldsymbol{\jmath}}$. *

b) After a very long time it is observed that the angle ϕ between the path of the bead and the rod/trough is nearly constant. What, in terms of μ, is this ϕ? *

19.1.11 A newer kind of gun. As an attempt to make an improvement on the 'new gun' demonstrated in problem 19.1.9, a person adds a length ℓ to the shaft on which the bead slides. Assume there is no friction between the bead (mass m) and the wire. Assume the bead starts at $s = 0$ with $\dot{s} = v_0$. The rigid rod, on which the bead slides, rotates at a constant rate $\omega = \omega_0$. Find $s(t)$ in terms of ℓ, m, v_0, and ω_0.



Problem 19.1.11

19.1.12 Slippery bead on straight rotating stick. A long stick with mass m_s rotates at a constant rate and a bead, modeled as a point mass, slides on the stick. The stick rotates at constant rate $\vec{\omega} = \omega\hat{\boldsymbol{k}}$. Neglect gravity. The bead has mass m_b. Initially the mass is at a distance R_0 from the hinge point on the stick and has no radial velocity ($\dot{R} = 0$). The initial angle of the stick is $\theta = 0$, measured counterclockwise from the positive x axis.

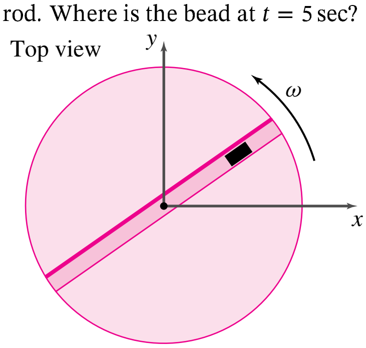
a) What is the torque, as a function of the net angle the stick has rotated, θ, required in order to keep the stick rotating at a constant rate ?

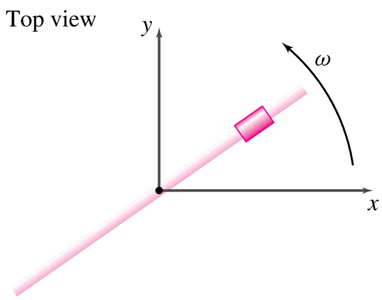
b) What is the path of the bead in the xy-plane? (Draw an accurate picture showing about one half of one revolution.)

c) How long should the stick be if the bead is to fly in the negative x-direction when it gets to the end of the stick?

d) **Add friction.** How does the speed of the bead over one revolution depend on μ, the coefficient of friction between the bead and the wire? Make a plot of $|\vec{v}_{\text{one revolution}}|$ versus μ. [Note, you have been working with $\mu = 0$ in the problems above.]

19.1.13 Mass on a lightly greased slotted turntable or spinning uniform rod. Assume that the rod/turntable in the figure is massless and also free to rotate. Assume that at $t = 0$, the angular velocity of the rod/turntable is 1 rad/s, that the radius of the bead is one meter, and that the radial velocity of the bead, dR/dt, is zero. The bead is free to slide on the rod. Where is the bead at $t = 5$ sec?

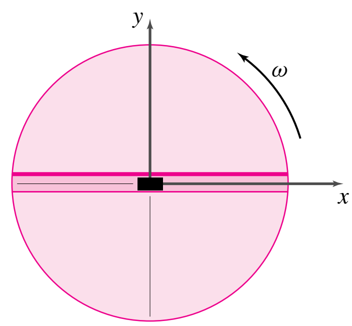




Problem 19.1.13

19.1.14 A bead slides in a frictionless slot in a turntable. The turntable spins a constant rate ω. The slot is straight and goes through the center of the turntable. If the bead is at radius R_o with $\dot{R} = 0$ at $t = 0$, what are the components of the acceleration vector in the directions normal and tangential to the path of the bead after one revolution? Neglect gravity.

19.1.15 A bead of mass $m = 1$ gm is constrained to slide in a straight frictionless slot in a disk which is spinning counterclockwise at constant rate $\omega = 3$ rad/s. At time $t = 0$, the slot is parallel to the x-axis and the bead is in the center of the disk moving out (in the plus x direction) at a rate $\dot{R}_o = .5$ m/s. After a net rotation θ of one and one eighth (1.125) revolutions, what is the force $\vec{F}$ of the disk on the bead? Express this answer in terms of $\hat{\imath}$ and $\hat{\jmath}$. Make the unreasonable assumption that the slot is long enough to contain the bead for this motion.

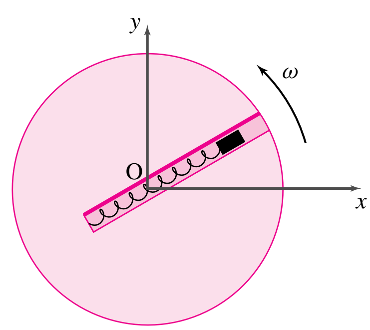


Problem 19.1.15

19.1.16 Bead on springy leash in a slot on a turntable. The bead in the figure is held by a spring that is relaxed when the bead is at the origin. The constant of the spring is k. The turntable speed is controlled by a strong stiff motor.

a) Assume $\omega = 0$ for all time. What are possible motions of the bead?

b) Assume $\omega = \omega_0$ is a constant. What are possible motions of the bead? Notice there are two cases depending on the value of ω_0. What is going on here?



Problem 19.1.16: Mass in a slot.

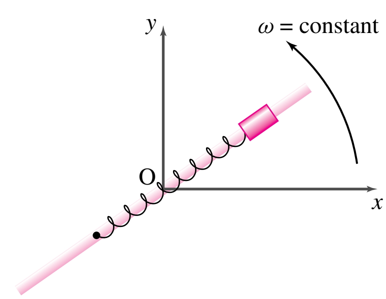
19.1.17 A small bead with mass m slides without friction on a rigid rod which rotates about the z axis with constant ω (maintained by a stiff motor not shown in the figure). The bead is also attached to a spring with constant K, the other end of which is attached to the rod. The spring is relaxed when the bead is at the center position. Assume the bead is pulled to a distance d from the center of the rod and then released with an initial $\dot{R} = 0$. If needed, you may assume that $K > m\omega^2$.

a) Derive the equation of motion for the position of the bead $R(t)$.

b) How is the motion affected by large versus small values of K?

c) What is the magnitude of the force of the rod on the bead as the bead passes through the center position? Neglect gravity. Answer in terms of m, ω, d, K.

d) Write an expression for the Coriolis acceleration. Give an example of a situation in which this acceleration is important and explain why it arises.



Problem 19.1.17

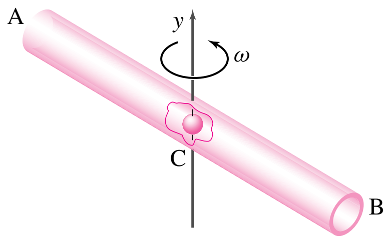
19.1.18 A small ball of mass m_b = 500 grams may slide in a slender tube of length $l = 1.2$ m. and of mass $m_t = 1.5$ kg. The tube rotates freely about a vertical axis passing through its center C. (Hint: Treat the tube as a slender rod.)

a) If the angular velocity of the tube is $\omega = 8$ rad/s as the ball passes through C with speed relative to the tube, v_0, calculate the angular velocity of the tube just before the ball leaves the tube. *

b) Calculate the angular velocity of the tube just after the ball leaves the tube. *

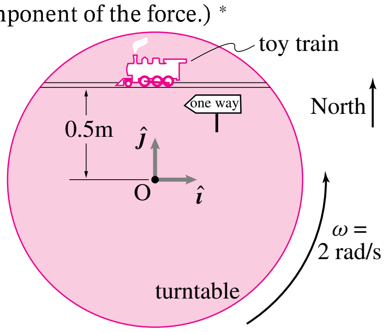
c) If the speed of the ball as it passes C is $v = 1.8$ m/s, determine the transverse and radial components of velocity of the ball as it leaves the tube. *

d) After the ball leaves the tube, what constant torque must be applied to the tube (about its axis of rotation) to bring it to rest in 10 s? *



Problem 19.1.18

19.1.19 Toy train car on a turn-around. The 0.1 kg toy train's speedometer reads a constant $1 m/s$ when, heading west, it passes due north of point O. The train is on a level straight track which is mounted on a spinning turntable whose center is 'pinned' to the ground. The turntable spins at the constant rate of 2 rad/s. What is the force of the turntable on the train? (Don't worry about the z-component of the force.) *



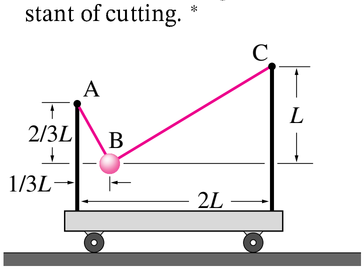
Problem 19.1.19

19.1.20 Due to forces not shown, the cart moves to the right with constant acceleration a_x. The ball B has mass m_B. At time $t = 0$, the string AB is cut. Find

a) the tension in string BC before cutting, *

b) the absolute acceleration of the mass at the instant of cutting, *

c) the tension in string BC at the instant of cutting. *

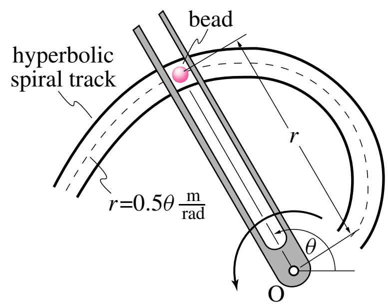


Problem 19.1.20

19.1.21 The forked arm mechanism pushes the bead of mass 1 kg along a frictionless hyperbolic spiral track given by $r = 0.5\theta(\text{ m/ rad})$. The arm rotates about its pivot point at O with constant angular acceleration $\ddot{\theta} = 1\,\text{rad/s}^2$ driven by a motor (not shown). The arm starts from rest at $\theta = 0^\circ$.

a) Determine the radial and transverse components of the acceleration of the bead after 2 s have elapsed from the start of its motion.

b) Determine the magnitude of the net force on the mass at the same instant in time.



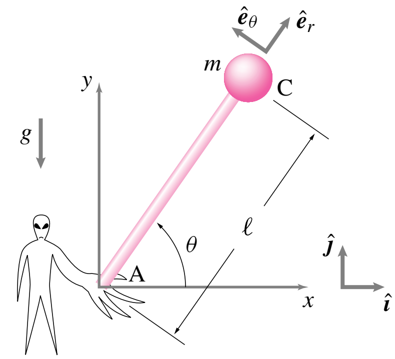
Problem 19.1.21

19.1.22 A rod is on the palm of your hand at point A. Its length is ℓ. Its mass m is assumed to be concentrated at its end at C. Assume that you know θ and $\dot{\theta}$ at the instant of interest. Also assume that your hand is accelerating both vertically and horizontally with $\vec{a}_{\text{hand}} = a_{hx}\hat{\imath} + a_{hy}\hat{\jmath}$. Coordinates and directions are as marked in the figure.

a) Draw a Free-Body Diagram of the rod.

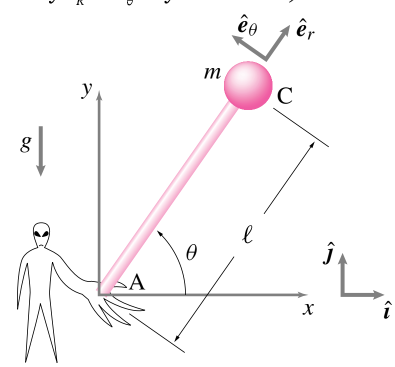
b) Assume that the hand is stationary. Solve for $\ddot{\theta}$ in terms of g, ℓ, m, θ, and $\dot{\theta}$.

c) If the hand is not stationary but $\ddot{\theta}$ has been determined somehow, find the vertical force of the hand on the rod in terms of $\ddot{\theta}$, $\dot{\theta}$, θ, g, ℓ, m, a_{hx}, and a_{hy}.



Problem 19.1.22: Rod on a Moving Support(2-D).

19.1.23 Balancing a broom. Assume the hand is accelerating to the right with acceleration $\boldsymbol{a} = a\hat{\boldsymbol{\imath}}$. What is the force of the hand on the broom in terms of m, ℓ, θ, $\dot{\theta}$, a, $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and g? *(You may not have any $\hat{\boldsymbol{e}}_R$ or $\hat{\boldsymbol{e}}_\theta$ in your answer.)*



Problem 19.1.23