CHAPTER 19

Mechanics of constrained particles and rigid objects

The dynamics of particles and rigid objects is studied using the relative-motion kinematics ideas from chapter chapter 18. This is the capstone chapter for a two-dimensional dynamics course. After this chapter a good student should be able to navigate through and use most of the skills in the concept map inside the back cover.

We have studied the mechanics of particles and rigid bodies with constraints that require progressively more involved kinematics. We now proceed to study the mechanics of more complex systems: particles with constrained paths, particles moving relative to moving frames, and mechanisms with several parts.

The basic strategy throughout is to use, in combination, the following skills:

1. Basic modeling. Describe a system in an appropriate way using the language of particle and rigid body mechanics. As described in Chapter 2, the force modeling and kinematic modeling are coupled. Where relative motion is freely allowed there is no force. And where motion is caused or prevented there is a force. Here is where you decide the constitutive (force) laws you are using for springs, contact, gravity, etc.
2. Draw free-body diagrams of the system of interest and of its parts. These diagrams should show what you do and do not know about the constraint forces (*e.g.,* at a pin connection cut free in a free-body diagram the FBD should show an arbitrary force and no moment). These are exactly the same free-body diagrams that one would draw for statics.
3. Kinematics calculations. Pick appropriate configuration variables, as many as there are degrees of freedom. Then write the velocities, accelerations, angular velocities and angular accelerations of interest in terms of the configuration variables and their first and second time derivatives, possibly using methods from Chapter 10. Often this is the hardest part of the analysis.
4. Use appropriate balance equations: linear momentum, angular momentum, or power balance equations.
5. Solve the balance equations for unknown forces or accelerations of interest. Sometimes this can be done by hand by writing out components and solving simultaneous equations or by using appropriate dot products. And sometimes it is best done by setting up a matrix equation and solving on the computer.
6. Solve the differential equations to find how the basic configuration variables change with time. For some special problems this can be done by hand, but most often involves computer solution.
7. Plug the ODE solution from (6) above into the equations from kinematics (3 above) and the balance laws (4 above). This is not a different skill from (3) or (4), it is just applied at a different time in the work.
8. Make plots of how forces, positions and velocities change with time, or of trajectories. Animations are also often nice.

These skills are used to solve dynamics problems which often fall into one of these 4 categories.

a. Kinematics. These are problems where *only* geometry is used, where the kinematics constraints determine what you are interested in, independent of the forces or time history. A classic example is determining the path of a point on a given four-bar linkage. More basic examples include finding position or acceleration from a given velocity history.

b. Instantaneous dynamics. These are problems where the positions and velocities of all points are given and you need to find forces or accelerations. Often these are "first-motion" problems: what are accelerations and forces immediately after something is released from rest?

c. "Inverse dynamics." These problems are called "inverse" because they are backwards of the original hard dynamics problems ((d) below). In these problems the motion is given as a function of time, and you have to calculate the forces. These problems are easier than non "inverse" problems because the differential equations from the balance laws don't need to be solved. A classic example is a slider-crank where the motion of the crank is known *a priori* to be at constant rate and you need to find the torque required to keep that motion. Usually in science "inverse" problems are harder. In dynamics this kind of "inverse" problem is easier than the non "inverse" problems.

d. Dynamics analysis. You are given some information about forces and constraints and you have to find the motion and more about the forces. These are the capstone problems that require use of all the skills.

A flow chart showing how these problem types are solved using the basic skill components ideas is shown in the chart in the inside back cover. As you solve a problem, at any instant in time you should be able to place your work on this chart.

In the sense of putting all the basic ideas together, this chapter completes the book. But in this chapter we only consider two-dimensional models and motions. Three-dimensional models and motions involve the same assembly of basic ideas, but more difficult kinematics, so are postponed.