18.4.1 Slider crank kinematics (No FBD required!). 2-D . Assume $R, \ell, \theta, \dot\theta, \ddot\theta$ are given. The crank mechanism parts move on the xy plane with the x direction being along the piston. Vectors should be expressed in terms of $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ components.

a) What is the angular velocity of the crank OA?

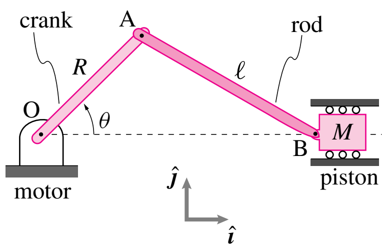
b) What is the angular acceleration of the crank OA?

c) What is the velocity of point A?

d) What is the acceleration of point A?

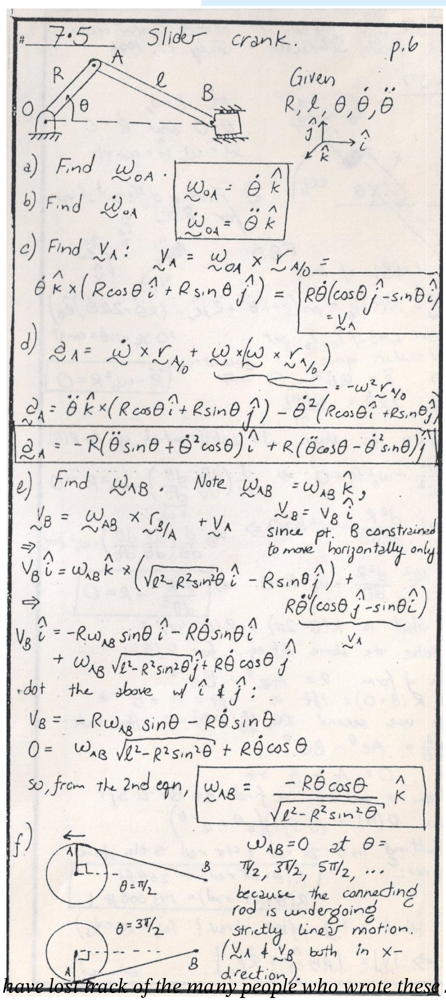
e) What is the angular velocity of the connecting rod AB? [Geometry fact: $\vec{r}_{AB} = \sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}$]

f) For what values of θ is the angular velocity of the connecting rod AB equal to zero (assume $\dot\theta \neq 0$)? (you need not answer part (e) correctly to answer this question correctly.)



Problem 18.1

Answer:
(a) $\vec{\omega}_{OA} = \dot\theta\hat{k}$.
(b) $\vec{\alpha}_{OA} = \ddot\theta\hat{k}$.
(c) $\vec{v}_A = -R\dot\theta\sin\theta\hat{\imath} + R\dot\theta\cos\theta\,\hat{\jmath}$.
(d) $\vec{a}_A = -(R\dot\theta^2\cos\theta + R\ddot\theta\sin\theta)\hat{\imath} + (R\ddot\theta\cos\theta - R\dot\theta^2\sin\theta)\hat{\jmath}$.
(e) $\vec{\omega}_{AB} = \frac{-R\dot\theta\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}$.
(f) $\theta = (2n+1)\frac{\pi}{2}$, $n = 0, \pm1, \pm2, ...$


#7.5 Slider crank. p.6

Given $R, \ell, \theta, \dot\theta, \ddot\theta$

a) Find ω_{OA}.

b) Find $\dot\omega_{OA}$

$\boxed{\omega_{OA} = \dot\theta\hat{k} \quad \dot\omega_{OA} = \ddot\theta\hat{k}}$

c) Find v_A: $v_A = \omega_{OA} \times r_{A/O} =$
$\dot\theta\hat{k} \times (R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}) = \boxed{R\dot\theta(\cos\theta\,\hat{\jmath} - \sin\theta\,\hat{\imath}) = v_A}$

d) $a_A = \dot\omega \times r_{A/O} + \underbrace{\omega \times (\omega \times r_{A/O})}_{= -\omega^2 r_{A/O}}$
$a_A = \ddot\theta\hat{k} \times (R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}) - \dot\theta^2(R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath})$
$\boxed{a_A = -R(\ddot\theta\sin\theta + \dot\theta^2\cos\theta)\hat{\imath} + R(\ddot\theta\cos\theta - \dot\theta^2\sin\theta)\hat{\jmath}}$

e) Find ω_{AB}. Note $\omega_{AB} = \omega_{AB}\hat{k}$,
$v_B = \omega_{AB} \times r_{B/A} + v_A$ $\quad v_B = v_B\hat{\imath}$ since pt. B constrained to move horizontally only.
$\Rightarrow$
$v_B\hat{\imath} = \omega_{AB}\hat{k} \times (\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}) + \underbrace{R\dot\theta(\cos\theta\,\hat{\jmath} - \sin\theta\,\hat{\imath})}_{v_A}$
$\Rightarrow$
$v_B\hat{\imath} = -R\omega_{AB}\sin\theta\,\hat{\imath} - R\dot\theta\sin\theta\,\hat{\imath} + \omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\jmath} + R\dot\theta\cos\theta\,\hat{\jmath}$
dot the above w/ $\hat{\imath}$ & $\hat{\jmath}$:
$v_B = -R\omega_{AB}\sin\theta - R\dot\theta\sin\theta$
$0 = \omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta} + R\dot\theta\cos\theta$
so, from the 2nd eqn, $\boxed{\omega_{AB} = \frac{-R\dot\theta\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}\hat{k}}$

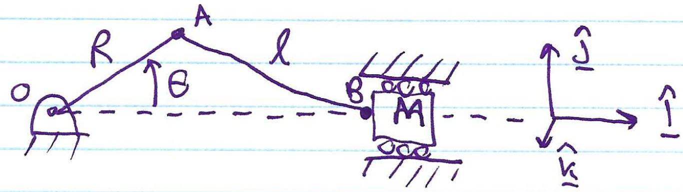
f) $\omega_{AB} = 0$ at $\theta = \pi/2, 3\pi/2, 5\pi/2, \ldots$ because the connecting rod is undergoing strictly linear motion. (v_A & v_B both in x-direction).

A B $\theta = \pi/2$
$\theta = 3\pi/2$ A B

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.4.1 M.R. Buche 5/6/21

Given $R, \ell, \theta, \dot{\theta}, \ddot{\theta}$ for a crank mechanism in 2D.

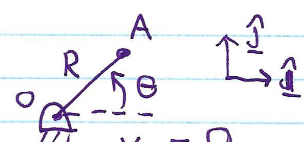


a) Find $\underline{\omega}_{OA}$. $\boxed{\underline{\omega}_{OA} = \dot{\theta}\,\hat{\underline{k}}}$

b) Find $\underline{\alpha}_{OA}$. $\boxed{\underline{\alpha}_{OA} = \ddot{\theta}\,\hat{\underline{k}}}$

c) Find $\underline{v}_A$. $\underline{v}_A = \underline{\omega}_{OA} \times \underline{r}_{A/O}$

$\underline{r}_{A/O} = R\left(\hat{\underline{\imath}}\cos\theta + \hat{\underline{\jmath}}\sin\theta\right)$



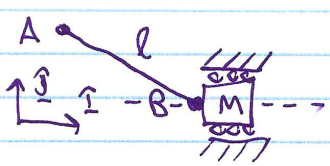
$\underline{v}_O = \underline{0}$

$$\boxed{\underline{v}_A = R\dot{\theta}\left(\hat{\underline{\jmath}}\cos\theta - \hat{\underline{\imath}}\sin\theta\right)}$$

d) Find $\underline{a}_A$. $\underline{a}_A = \underline{\alpha}_{OA} \times \underline{r}_{A/O} - \omega_{OA}^2\,\underline{r}_{A/O}$

$$\underline{a}_A = R\ddot{\theta}\left(\hat{\underline{\jmath}}\cos\theta - \hat{\underline{\imath}}\sin\theta\right) - R\dot{\theta}^2\left(\hat{\underline{\imath}}\cos\theta + \hat{\underline{\jmath}}\sin\theta\right)$$

f)



Since $\underline{v}_B \cdot \hat{\underline{\jmath}} = 0$, we have $\underline{\omega}_{AB} = \underline{0}$ when $\underline{v}_A \cdot \hat{\underline{\jmath}} = 0$

$\underline{v}_A \cdot \hat{\underline{\jmath}} = R\dot{\theta}\cos\theta$

$$\boxed{\text{Assuming } \dot{\theta} \neq 0 \text{ and } R > 0,\quad \underline{\omega}_{AB} = \underline{0} \text{ when } \theta = \pi/2 \text{ and } 3\pi/2.}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.4.1 M. R. Buche 5/6/21

e) Find $\underline{\omega}_{AB}$, use $\underline{r}_{B/A} = \hat{\imath}\sqrt{\ell^2 - R^2\sin^2\theta} - \hat{\jmath} R\sin\theta$.

We know $\underline{v}_A$ (part c) and $\underline{v}_B = |\underline{v}_B|\hat{\imath}$.

Also: $\underline{v}_B = \underline{v}_A + \underline{\omega}_{AB} \times \underline{r}_{B/A}$, $\underline{\omega}_{AB} = |\underline{\omega}_{AB}|\hat{k}$

Laurie Anderson: "let x = x" ($\underline{v}_B = \underline{v}_B$),

$|\underline{v}_B|\hat{\imath} = \underline{v}_A + \underline{\omega}_{AB} \times \underline{r}_{B/A}$

↳ does this spark joy? (dot with $\hat{\jmath}$)

$\underline{v}_A \cdot \hat{\jmath} = R\dot{\theta}\cos\theta$

$\underline{\omega}_{AB} \times \underline{r}_{B/A} = |\underline{\omega}_{AB}|\left(\hat{\imath} R\sin\theta + \hat{\jmath}\sqrt{\ell^2 - R^2\sin^2\theta}\right)$

$(\underline{\omega}_{AB} \times \underline{r}_{B/A}) \cdot \hat{\jmath} = |\underline{\omega}_{AB}|\sqrt{\ell^2 - R^2\sin^2\theta}$

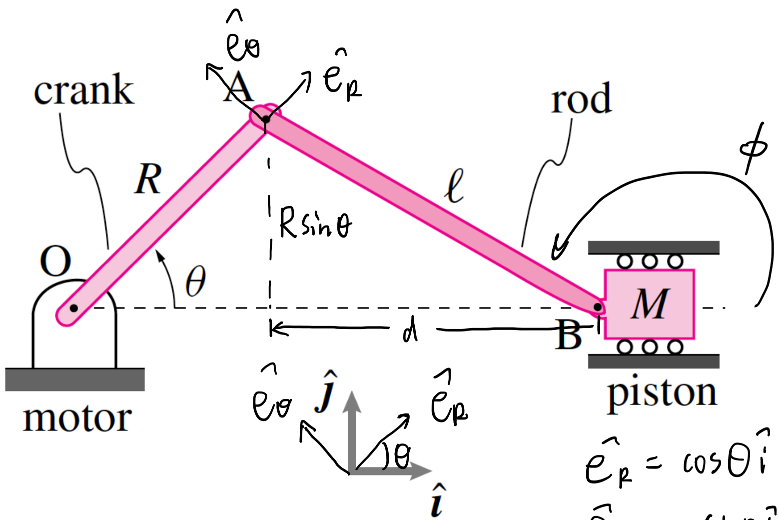
Solving $\underline{v}_A \cdot \hat{\jmath} + (\underline{\omega}_{AB} \times \underline{r}_{B/A}) \cdot \hat{\jmath} = 0$,

we see $$\boxed{\underline{\omega}_{AB} = \frac{-\dot{\theta}\cos\theta\,\hat{k}}{\sqrt{(\ell/R)^2 - \sin^2\theta}}}$$

Observe that for $\ell = R$, we receive singular configurations (also called dead positions) at $\theta = 0$ and π. More generally, we receive them at $\pi - \operatorname{asin}(\ell/R)$ for $\ell \le R$.

17.4.1

Saturday, May 4, 2019 5:29 PM



$\hat{e}_R = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

Given: $R, l, \theta, \dot{\theta}, \ddot{\theta}$

Find :

a) angular velocity of OA

$\vec{\omega}_{OA} = \dot{\theta}\hat{k}$

b) angular acceleration of OA

$\dot{\vec{\omega}}_{OA} = \ddot{\theta}\hat{k}$

c) velocity of A

$\vec{V}_A = \dot{\theta}R\hat{e}_\theta = \dot{\theta}R(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

d) Acceleration of A

$\vec{a}_A = \dot{\vec{V}}_A = \ddot{\theta}R\hat{e}_\theta - \dot{\theta}^2 R\hat{e}_R$

$= \ddot{\theta}R(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) - \dot{\theta}^2 R(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$= (-\ddot{\theta}R\sin\theta - \dot{\theta}^2 R\cos\theta)\,\hat{i} + (\ddot{\theta}R\cos\theta - \dot{\theta}^2 R\sin\theta)\,\hat{j}$

e) angular velocity of AB ? $\vec{\omega}_{AB} = \omega_{AB}\hat{k}$

$\vec{V}_A = \vec{V}_A$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\{\dot\theta R\hat e_\theta = V_B\hat i + \dot\phi\hat k \times \vec r_{BA}\}$$

$$\{\}\cdot\hat j \Rightarrow \underbrace{\dot\theta R\hat e_\theta\cdot\hat j}_{\cos\theta} = 0 + (\dot\phi\hat k\times\vec r_{BA})\cdot\hat j$$

$$\vec r_{BA} = -\underbrace{d}_{\sqrt{l^2-R^2\sin^2\theta}}\hat i + R\sin\theta\,\hat j$$

$$\Rightarrow \dot\phi = \frac{-\dot\theta R\cos\theta}{\sqrt{l^2-R^2\sin^2\theta}}$$

$$\vec\omega_{AB} = \dot\phi\hat k$$

$$\{\dot\theta R(-\sin\theta\,\hat i + \cos\theta\,\hat j) = \vec V_B - \omega_{AB}\sqrt{l^2-R^2\sin^2\theta}\,\hat j - \omega_{AB}R\sin\theta\,\hat i\}$$

$$\{\}\cdot\hat j \Rightarrow \dot\theta R\cos\theta = -\omega_{AB}\sqrt{l^2-R^2\sin^2\theta}$$

$$\Rightarrow \omega_{AB} = -\frac{\dot\theta R\cos\theta}{\sqrt{l^2-R^2\sin^2\theta}}$$

$$\therefore \vec\omega_{AB} = -\frac{\dot\theta R\cos\theta}{\sqrt{l^2-R^2\sin^2\theta}}\hat k$$

f) $\theta = ?$ makes the angular velocity of AB equal to zero ($\dot\theta \neq 0$)

$\theta = \pm 90^\circ$.

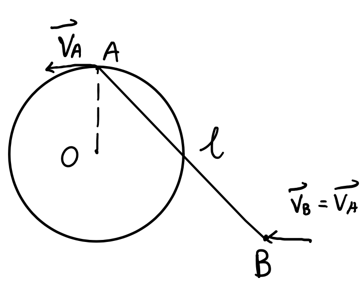
physically, when $\theta = \pm 90^\circ$, $\vec V_A = \vec\omega_{OA}\times\vec r_{A/O} \parallel \hat i$

thus also parallel to the velocity of $\vec V_B$.

Thus at this instant, the rod AB is not rotating, just translating. (No need to get (e) correctly).

Mathematically from part e)

Set $\omega_{AB} = 0 \Rightarrow \cos\theta = 0 \quad \theta = \pm 90^\circ$.

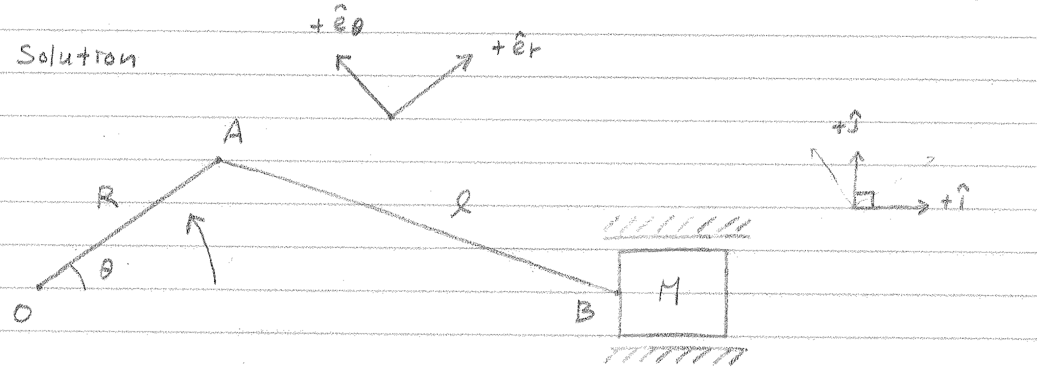


Sec 205

[redacted]

You Won Park
TAM 2030
10:10 AM Sec 205
TA: Pranav Bhounsule
HW22, Apr. 14, 2009

15.29 Solution



$R, \ell, \theta, \dot\theta, \ddot\theta$ given

a) Angular velocity of OA?

$$\boxed{\vec\omega_{OA} = \dot\theta \hat k}$$

b) Angular acceleration of OA?

$$\boxed{\dot{\vec\omega}_{OA} = \ddot\theta \hat k}$$

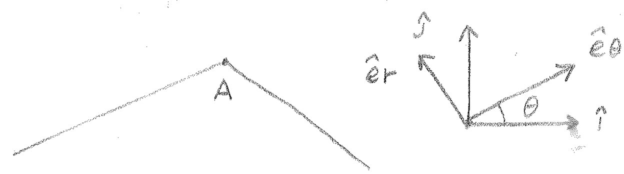
c) Velocity of A?

$$\vec v_A = \dot\theta R \hat e_\theta = \boxed{\dot\theta R(-\sin\theta \hat\imath + \cos\theta \hat\jmath)}$$

d) Acceleration of A?

$$\vec a_A = \dot{\vec v}_A = \boxed{(-\ddot\theta R\sin\theta - \dot\theta^2 R\cos\theta)\hat\imath + (\ddot\theta R\cos\theta - \dot\theta^2 R\sin\theta)\hat\jmath}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*



e) Angular velocity of AB?

$$\vec{r}_{AB} = \sqrt{\ell^2 - R^2\sin^2\theta}\ \hat{\imath} - R\sin\theta\ \hat{\jmath}$$

$$\vec{v}_A = \vec{r}_{AB} \times \vec{\omega}_{AB}$$

$$= \left(\sqrt{\ell^2 - R^2\sin^2\theta}\ \hat{\imath} - R\sin\theta\ \hat{\jmath}\right) \times \omega_{AB}\hat{k}$$

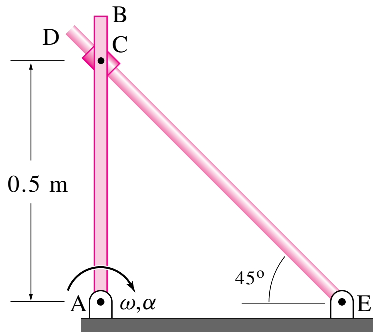
$$\left\{ -R\dot{\theta}\sin\theta\ \hat{\imath} + R\dot{\theta}\cos\theta\ \hat{\jmath} = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}\ \hat{\jmath} - \omega_{AB}R\sin\theta\ \hat{\imath} \right\}$$

$$\{\}\cdot\hat{\jmath} \Rightarrow R\dot{\theta}\cos\theta = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}$$

$$\therefore \boxed{\vec{\omega}_{AB} = -\frac{R\dot{\theta}\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}\hat{k}}$$

f) for $\boxed{\theta = \pm 90^\circ}$

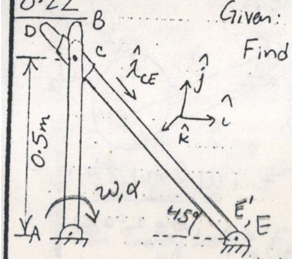
18.4.4 The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3\,\text{rad/s}$ and angular acceleration $\alpha = 2\,\text{rad/s}^2$. Find the angular velocity of rod DE.



Problem 18.4

Answer:
$\vec{\omega}_{DE} = -\frac{\omega}{2}\hat{\boldsymbol{k}} = -1.5\,\text{rad/s}\hat{\boldsymbol{k}}$

8·22



Given: $\omega_{AB} = 3\,\text{rad/s}$,
Find ω_{DE}.

First, find $\underset{\sim}{v}_C$ from motion of rod AB:

$$\underset{\sim}{v}_C = \omega_{AB}\, r_{C/A}\, \hat{i} \quad ①$$

Now, attach a frame $\mathcal{B}$ to rod DE. This frame rotates with rod DE.

Choose a coordinate system $x'y'z'$ for this rod with its origin at pt. E. The $x'y'z'$ coordinate system is aligned with the xyz coordinate system at this instant in time.

Then: $\underset{\sim}{v}_C = \underset{\sim}{v}_{C'} + \underset{\sim}{v}_{rel}$

where $\underset{\sim}{v}_{rel} = v_{C/E'}\hat{\lambda}_{CE} = v_{rel}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ } we know the direction of v_{rel} since the collar slides along the rod.

$\underset{\sim}{v}_{C'} = \underset{\sim}{v}_{E'/E} + \underset{\sim}{\omega}_{\mathcal{B}} \times \underset{\sim}{r}_{C'/E'}$

0: pt. E' does not move relative to the fixed point E

$\Rightarrow \underset{\sim}{v}_{C'} = \omega_{\mathcal{B}}\hat{k} \times (-0.5\text{m}\,\hat{i} + 0.5\text{m}\,\hat{j})$

thus $\underset{\sim}{v}_C = -(0.5\text{m})\omega_{\mathcal{B}}(\hat{j}+\hat{i}) + \frac{v_{rel}}{\sqrt{2}}(\hat{i}-\hat{j}) \quad ②$

now; $\underset{\sim}{v}_C = \underset{\sim}{v}_C$ (equate ① & ②)

continued

8·22 continued p. 10

$$\omega_{AB}\, r_{C/A}\,\hat{i} = -(0.5\text{m})\,\omega_{\mathcal{B}}(\hat{j}+\hat{i}) + \frac{v_{rel}}{\sqrt{2}}(\hat{i}-\hat{j}) \quad ③$$

$\hat{i}\cdot③ \Rightarrow \quad 1.5\,\text{m/s} = -0.5\text{m}(\omega_{\mathcal{B}}) + \frac{v_{rel}}{\sqrt{2}} \quad ④$

$\hat{j}\cdot③ \Rightarrow \quad 0 = -0.5\text{m}(\omega_{\mathcal{B}}) - \frac{v_{rel}}{\sqrt{2}} \quad ⑤$

$④ + ⑤ \rightarrow \quad 1.5\,\text{m/s} = -1\text{m}(\omega_{\mathcal{B}}) \Rightarrow \omega_{\mathcal{B}} = -1.5\,\text{rad/s}$

thus, since $\omega_{\mathcal{B}} = \omega_{DE}$, $\boxed{\underset{\sim}{\omega}_{DE} = -1.5\,\text{rad/s}\,\hat{k}}$

(Note: $④ - ⑤ \Rightarrow 1.5\,\text{m/s} = \frac{2v_{rel}}{\sqrt{2}} \Rightarrow v_{rel} = 0.75\sqrt{2}\,\text{m/s}$

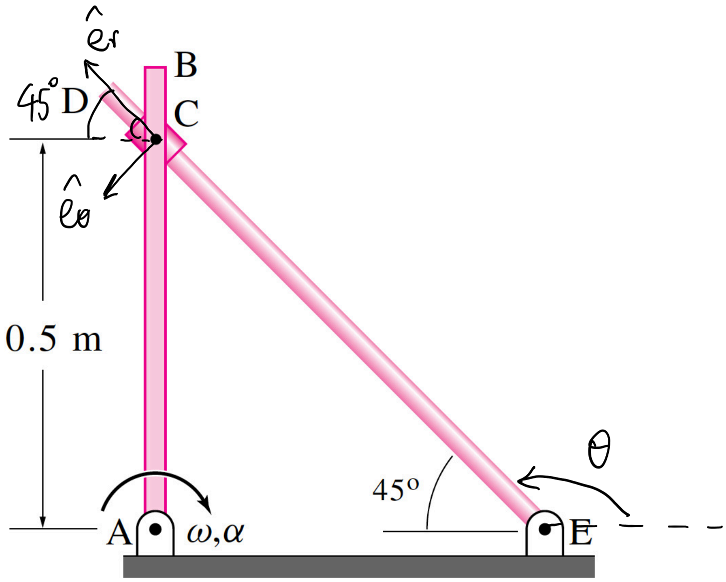
$\Rightarrow \underset{\sim}{v}_{rel} = v_{rel}\hat{\lambda}_{CE} = 0.75\sqrt{2}\,\text{m/s}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right) =$

$0.75\,\text{m/s}(\hat{i}-\hat{j}) = \underset{\sim}{v}_{rel}$ * (used in prob 8·30)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.4.4

Saturday, May 4, 2019 6:11 PM



C is pinned to AB but free on DE.

At this instant, given:

angular velocity of AB : $\vec{\omega}_{AB} = \omega \hat{k}$ $\quad \omega = -3$ rad/s

angular acceleration of AB : $\vec{\alpha}_{AB} = \alpha \hat{k}$ $\quad \alpha = -2$ rad/s^2

Find : angular velocity of DE. $\vec{\omega}_{DE} = \dot{\theta} \hat{k}$

$$\vec{v}_C = \vec{v}_C$$

$$\{ \vec{\omega}_{AB} \times \vec{r}_{C/A} = \vec{\omega}_{DE} \times \vec{r}_{C/E} + V_{cr} \hat{e}_r \}$$

$\{\}\cdot \hat{e}_\theta \Rightarrow$ $\quad \omega \hat{k} \times 0.5 \hat{j} \cdot \hat{e}_\theta = (\dot{\theta} \hat{k} \times \frac{0.5}{\sin 45^\circ} \hat{e}_r + V_{cr} \hat{e}_r) \cdot \hat{e}_\theta$

$$0.5\,\omega \cdot \sin 45^\circ = \dot{\theta} \frac{0.5}{\sin 45^\circ}$$

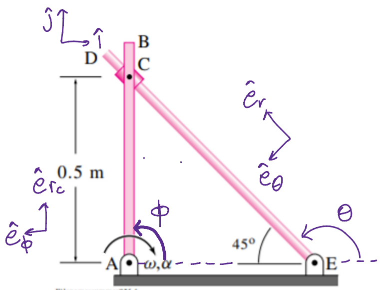
$$\Rightarrow \quad \dot{\theta} = \frac{1}{2}\omega = -1.5 \,[\text{rad/s}]$$

$$\therefore \quad \vec{\omega}_{DE} = \dot{\theta} \hat{k} = -1.5 \hat{k} \quad [\text{rad/s}]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

#69 Solution Michelle deSouza

18.4.4 The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3\,\mathrm{rad/s}$ and angular acceleration $\alpha = 2\,\mathrm{rad/s^2}$. Find the angular velocity of rod DE. *



Problem 18.4.4

Given: $r_{C/A} = 0.5\,m$

$\vec{\omega}_{AB} = -3\hat{k}\ \mathrm{rad/s} = -\omega_{AB}\hat{k}$

$\vec{\alpha}_{AB} = -2\hat{k}\ \mathrm{rad/s^2} = -\alpha_{AB}\hat{k}$

$\theta = 135°$

$\phi = 90°$

Find: ω_{DE}

Hint: $\vec{V}_c = \vec{V}_c$

find from AB — find from DE

Instantaneous unit vectors:

$\hat{e}_{r_c} = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} = \hat{\jmath}$

$\hat{e}_\phi = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} = -\hat{\imath}$

$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} = -\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$

$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath} = -\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$

Note: $\dot{\phi} = -\omega$, $\ddot{\phi} = -\alpha$

(ω_{AB}, α_{AB} defined as CW +. We will use $\vec{\dot{\Phi}}$ defined CCW +, so $\vec{\omega}_{AB} = -\omega_{AB}\hat{k} = \dot{\phi}\hat{k}$)

From AB: C is pinned to AB

Rigid object kinematics:

$\vec{V}_c = \vec{V}_A + \vec{V}_{C/A}$

$\vec{V}_c = \cancel{\vec{V}_A}^{\,0} + \dot{\phi}\hat{k} \times \vec{r}_{C/A}$

$\vec{V}_c = -\omega_{AB}\hat{k} \times r_{C/A}\hat{e}_{r_c}$

① $\vec{V}_c = -\omega_{AB}\, r_{C/A}\,\hat{e}_\phi$

From DE: C is free to slide, $r_{C/D} \neq$ const.

$\vec{r}_c = \vec{r}_E + r_{C/E}\hat{e}_r$

② $\vec{V}_c = \cancel{\vec{V}_E}^{\,0} + \dot{r}_{C/E}\,\hat{e}_r + r_{C/E}\,\omega_{DE}\,\hat{e}_\theta$

① $\vec{V}_c = \vec{V}_c$ ②

$-\omega_{AB}\, r_{C/A}\,\hat{e}_\phi = \dot{r}_{C/E}\,\hat{e}_r + r_{C/E}\,\omega_{DE}\,\hat{e}_\theta$ — dot with $\hat{e}_\theta$ to isolate ω_{DE}

$\{\}\cdot\hat{e}_\theta$ $\quad -\omega_{AB}\, r_{C/A}(\hat{e}_\phi\cdot\hat{e}_\theta) = r_{C/E}\,\omega_{DE}$

$\omega_{DE} = -\omega_{AB}\frac{r_{C/A}}{r_{C/E}}(\hat{e}_\phi\cdot\hat{e}_\theta)$

$r_{C/E} = \frac{r_{C/A}}{\sin\theta}$

$\hat{e}_\phi\cdot\hat{e}_\theta = -\hat{\imath}\cdot\hat{e}_\theta = \sin\theta$

$\omega_{DE} = -\omega_{AB}\frac{\cancel{r_{C/A}}}{\cancel{r_{C/A}}/\sin\theta}\sin\theta = -\omega_{AB}\sin^2\theta = -3\left(\frac{1}{2}\right) = \boxed{-1.5\ \mathrm{rad/s}}$

$\omega_{DE} = \omega_{AB}\frac{r_{C/A}}{r_{C/E}}(\hat{e}_\phi\cdot\hat{e}_\theta)$ $\quad |\omega_{DE}|$ trends:

- ↑ for more // rods
- ↑ for higher ω_{AB}
- ↑ for longer $r_{C/A}$
- ↓ for longer $r_{C/D}$

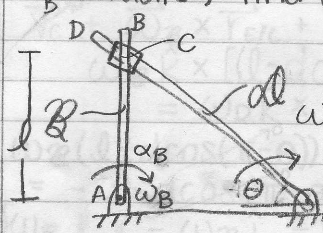
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

15.32, 15.38, 16.1, 16.12, 16.21

Kim Dunphy
HW #1-5 due 12/2/08
TAM 2030
Section 2 at 1:25
TA: Rong Long

1) 15.32 → SOLUTION

Two rods AB & DE connected at collar C (pinned to AB, can slide on DE frictionlessly); AB rotating CCW, $\omega_B = 3$ rad/s, $\alpha_B = 2$ rad/s^2, find angular velocity of DE



$\ell = 0.5$m
$\theta = 45°$

$$\vec{V}_C = \vec{V}_C$$

$$\vec{V}_A^{\,0} + \vec{\omega}_B \times \vec{r}_{C/A} + \vec{V}_{C/B}^{\,0} = \vec{V}_E^{\,0} + \vec{\omega}_D \times \vec{r}_{C/E} + \vec{V}_{C/\alpha\ell}$$

$$(-\omega_B \hat{k}) \times (\ell \hat{j}) = \omega_D \hat{k} \times (-\ell \hat{\imath} + \ell \hat{j}) + V_{C/\alpha\ell}(\cos\theta \hat{\imath} - \sin\theta \hat{j})$$

$$-\omega_B \ell \hat{\imath} = -\omega_D \ell (\hat{\imath} + \hat{j}) + V_{C/\alpha\ell}(\cos\theta \hat{\imath} - \sin\theta \hat{j})$$

$$-(3)(0.5)\hat{\imath} = -\omega_D(0.5\hat{\imath} + 0.5\hat{j}) + V_{C/\alpha\ell}(\cos 45° \hat{\imath} - \sin 45° \hat{j})$$

$$\{-1.5\hat{\imath} = -\omega_D(0.5\hat{\imath} + 0.5\hat{j}) + V_{C/\alpha\ell}(\tfrac{\sqrt{2}}{2}\hat{\imath} - \tfrac{\sqrt{2}}{2}\hat{j})\}$$

$$\{\} \cdot \hat{\imath} \Rightarrow -1.5 = -0.5\omega_D + \tfrac{\sqrt{2}}{2} V_{C/\alpha\ell}$$

$$\omega_D = 3 + \sqrt{2}\, V_{C/\alpha\ell}$$

$$\{\} \cdot \hat{j} \Rightarrow 0 = -0.5\omega_D - \tfrac{\sqrt{2}}{2} V_{C/\alpha\ell}$$

$$\omega_D = -\sqrt{2}\, V_{C/\alpha\ell}$$

$$V_{C/\alpha\ell} = -\tfrac{1}{\sqrt{2}}\omega_D$$

$$\omega_D = 3 + \sqrt{2}\left(\tfrac{-1}{\sqrt{2}}\omega_D\right)$$

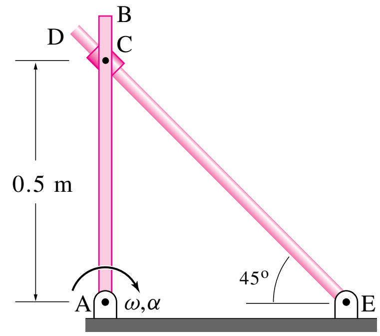
$$\omega_D = 3 - \omega_D$$

$$2\omega_D = 3$$

$\omega_D = 1.5$ rad/s in $-\hat{k}$ direction

$$\boxed{\vec{\omega}_D = -1.5 \text{ rad/s } \hat{k}}$$

18.4.5 Reconsider problem 18.4.4. The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3\,\mathrm{rad/s}$ and angular acceleration $\alpha = 2\,\mathrm{rad/s^2}$. Find the angular acceleration of rod DE.



Problem 18.5

Answer:
$\vec{\alpha}_{DE} = 3.5\,\mathrm{rad/s^2}\,\hat{\boldsymbol{k}}$

8.30 Same setup as prob 8.22; use same frame $\mathcal{B}$ attached to rod DE

from the kinematics of rod AB:

$$\underset{\sim}{a}_c = \underset{\sim}{\omega}_{AB} \times \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} + \dot{\underset{\sim}{\omega}}_{AB} \times \underset{\sim}{r}_{C/A} = -r_{C/A}\,\omega_{AB}^2\,\hat{j} + r_{C/A}\,\alpha_{AB}\,\hat{i}$$

$$\Rightarrow \underset{\sim}{a}_c = -4.5\,\mathrm{m/s^2}\,\hat{j} + 1\,\mathrm{m/s^2}\,\hat{i}$$

considering frame $\mathcal{B}$ (attached to rod DE):

$$\underset{\sim}{a}_c = \underset{\sim}{a}_{E'/E} + \underset{\sim}{\omega}_{\mathcal{B}} \times (\underset{\sim}{\omega}_{\mathcal{B}} \times \underset{\sim}{r}_{C/E'}) + \underset{\sim}{\alpha}_{\mathcal{B}} \times \underset{\sim}{r}_{C/E'} + \underset{\sim}{a}_{C/\mathcal{B}} + 2\underset{\sim}{\omega}_{\mathcal{B}} \times \underset{\sim}{v}_{C/\mathcal{B}}$$

$\underset{\sim}{0}$: E' immobile

#8.30 continued p.12

$$\Rightarrow \underset{\sim}{a}_c = \underset{\sim}{0} + (-1.5\,\mathrm{rad/s}\,\hat{k} \times (-1.5\,\mathrm{rad/s}\,\hat{k} \times (-0.5\mathrm{m}\,\hat{i} + 0.5\mathrm{m}\,\hat{j})))$$
$$+ \alpha_{\mathcal{B}}\,\hat{k} \times (-0.5\mathrm{m}\,\hat{i} + 0.5\mathrm{m}\,\hat{j}) + a_{C/\mathcal{B}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$
$$+ 2(-1.5\,\mathrm{rad/s})\,\hat{k} \times 0.75\,\mathrm{m/s}\,(\hat{i}-\hat{j})$$

Where I have used:

$\underset{\sim}{\omega}_{\mathcal{B}} = -1.5\,\mathrm{rad/s}\,\hat{k}$ (from prob 8.22)

$\underset{\sim}{v}_{rel} = \underset{\sim}{v}_{C/\mathcal{B}} = 0.75\,\mathrm{m/s}\,(\hat{i}-\hat{j})$ (from prob 8.22)

$\underset{\sim}{a}_{C/\mathcal{B}} = a_{C/\mathcal{B}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ ⇐ collar must accelerate along rod DE.

⇒ using $\underset{\sim}{a}_c$ (calc. from rod AB) = $\underset{\sim}{a}_c$ (from rod DE) and doing some cross-products yields

$$(-4.5\,\hat{j} + 1\,\hat{i})\,\mathrm{m/s^2} = 1.125\,\mathrm{m/s^2}(\hat{i}-\hat{j}) - 0.5\mathrm{m}\,\alpha_{\mathcal{B}}(\hat{i}+\hat{j})$$
$$+ \frac{a_{C/\mathcal{B}}}{\sqrt{2}}(\hat{i}-\hat{j}) - 2.25\,\mathrm{m/s^2}(\hat{i}+\hat{j})$$

dotting the above w/ $\hat{i}$ & $\hat{j}$:

$$\hat{i}\cdot(\,) \Rightarrow 1\,\mathrm{m/s^2} = 1.125\,\mathrm{m/s^2} - 0.5\mathrm{m}\,\alpha_{\mathcal{B}} + \frac{a_{C/\mathcal{B}}}{\sqrt{2}} - 2.25\,\mathrm{m/s^2}$$

$$\hat{j}\cdot(\,) \Rightarrow -4.5\,\mathrm{m/s^2} = -1.125\,\mathrm{m/s^2} - 0.5\mathrm{m}\,\alpha_{\mathcal{B}} - \frac{a_{C/\mathcal{B}}}{\sqrt{2}} - 2.25\,\mathrm{m/s^2}$$

adding these two eqns together yields:

$$-3.5\,\mathrm{m/s^2} = -1\mathrm{m}\,\alpha_{\mathcal{B}} - 4.5\,\mathrm{m/s^2}$$

$$\Rightarrow \alpha_{\mathcal{B}} = -1\,\mathrm{rad/s^2} \rightarrow \boxed{\underset{\sim}{\alpha}_{\mathcal{B}} = -1\,\mathrm{rad/s^2}\,\hat{k}}$$

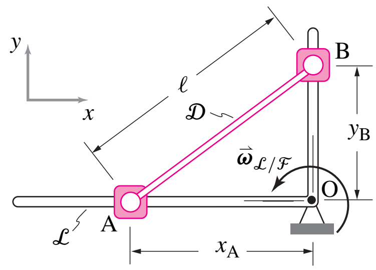
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.4.7 A bar of length $\ell = 5\,\text{ft}$, body $\mathcal{D}$, connects sliders A and B on an L-shaped frame, body $\mathcal{L}$, which itself is rotating at constant speed about an axle perpendicular to the plane of the figure through the point O and relative to a fixed frame $\mathcal{F}$, $\vec{\omega}_{\mathcal{L}/\mathcal{F}} = 0.5\,\text{rad/s}\,\boldsymbol{k}$. At the instant shown, body $\mathcal{L}$ is aligned with the $x-y$ axes, slider A is $x_A = 4\,\text{ft}$ from point O, slider B is $y_B = 3\,\text{ft}$ from O. The speed and acceleration of slider B relative to the frame $\mathcal{L}$ are $\vec{v}_{B/\mathcal{L}} = -2\,\text{ft/s}\,\hat{\boldsymbol{j}}$ and $\vec{a}_{B/\mathcal{L}} = -2\,\text{ft/s}^2\,\hat{\boldsymbol{j}}$, respectively.

Determine:

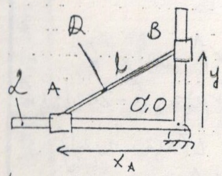
a) The absolute velocity of the slider A, $\vec{v}_{A/\mathcal{F}}$, and

b) The absolute angular velocity of bar AB, body $\mathcal{D}$, $\vec{\omega}_{\mathcal{D}/\mathcal{F}}$.



Problem 18.7

8.21 Consider the moving frame $\mathcal{L}$, which is glued to the bent rod AOB; its coordinate axes $x'y'z'$ are coincident with the fixed axes x, y, z at this instant.



Given: $\underset{\sim}{\omega}_{\mathcal{L}/\mathcal{F}}$, $\underset{\sim}{V}_{B/\mathcal{L}}$, $\underset{\sim}{a}_{B/\mathcal{L}}$

Find $\underset{\sim}{V}_{A/\mathcal{F}}$, $\underset{\sim}{\omega}_{D/\mathcal{F}}$

'3-term velocity formula': $\underset{\sim}{V}_A = \underset{\sim}{V}_{A'} + \underset{\sim}{V}_{rel}$

Find $\underset{\sim}{V}_{rel} = \underset{\sim}{V}_{A/\mathcal{L}}$ by noting that length of bar $AB = \text{const}$; thus $x_A^2 + y_B^2 = \text{const}$.

differentiate: $x_A\dot{x}_A + y_B\dot{y}_B = 0$;

we're given: $x_A = 4\text{ft}$, $y_B = 3\text{ft}$, $\dot{y}_B = V_{B/\mathcal{L}} = -2\,\text{ft/s}$

$\Rightarrow$ thus $\dot{x}_A = V_{A/\mathcal{L}} = 1.5\,\text{ft/s} \Rightarrow \underset{\sim}{V}_{A/O'} = -1.5\,\text{ft/s}\,\hat{\imath}$

so $\underset{\sim}{V}_A = \underset{\sim}{V}_{O'/O} + \underset{\sim}{\omega}_{\mathcal{L}} \times \underset{\sim}{r}_{A/O'} + \underset{\sim}{V}_{A/O'}$

$\underset{\sim}{V}_A = \underset{\sim}{0} + (0.5\,\text{rad/s}\,\hat{k}) \times (-4\text{ft}\,\hat{\imath}) - 1.5\,\text{ft/s}\,\hat{\imath}$

(pt. O' does not move) $\Rightarrow \boxed{\underset{\sim}{V}_A = -2\,\text{ft/s}\,\hat{\jmath} - 1.5\,\text{ft/s}\,\hat{\imath}}$

\# 8.21 continued p. 9

b) find $\underset{\sim}{\omega}_{D/\mathcal{F}}$ $\Rightarrow$ note that

$(\underset{\sim}{V}_A = \underset{\sim}{V}_B + \underset{\sim}{V}_{A/B})$ relative to frame $\mathcal{L}$

$\Updownarrow$

$(\underset{\sim}{V}_A = \underset{\sim}{V}_B + \underset{\sim}{\omega}_D \times \underset{\sim}{r}_{A/B})$ relative to $\mathcal{L}$

$\Updownarrow$

$\underset{\sim}{V}_{A/\mathcal{L}} = \underset{\sim}{V}_{B/\mathcal{L}} + \underset{\sim}{\omega}_{D/\mathcal{L}} \times \underset{\sim}{r}_{A/B}$ $\leftarrow$ $\underset{\sim}{\omega}_{D/\mathcal{L}}$ is the only unknown

putting in #s:

$(-1.5\,\text{ft/s})\hat{\imath} = (-2\,\text{ft/s})\hat{\jmath} + \omega_{D/\mathcal{L}}\hat{k} \times (-3\text{ft}\,\hat{\jmath} - 4\text{ft}\,\hat{\imath})$

$\Rightarrow -1.5\,\text{ft/s}\,\hat{\imath} = -2\,\text{ft/s}\,\hat{\jmath} + \omega_{D/\mathcal{L}}(3\text{ft}\,\hat{\imath} - 4\text{ft}\,\hat{\jmath})$

Dotting the above with either $\hat{\imath}$ or $\hat{\jmath}$ yields $\Rightarrow \underset{\sim}{\omega}_{D/\mathcal{L}} = -0.5\,\text{rad/s}\,\hat{k}$

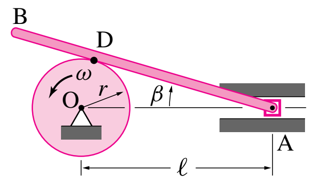
Now $\underset{\sim}{\omega}_{D/\mathcal{F}} = \underset{\sim}{\omega}_{D/\mathcal{L}} + \underset{\sim}{\omega}_{\mathcal{L}/\mathcal{F}} = (-0.5 + 0.5)\,\text{rad/s}\,\hat{k}$

$\Rightarrow \boxed{\underset{\sim}{\omega}_{D/\mathcal{F}} = \underset{\sim}{0}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

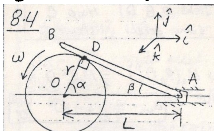
18.4.8 The link AB is supported by a wheel at D and its end A is constrained so that it only has horizontal velocity. No slipping occurs between the wheel and the link. The wheel has an angular velocity ω and radius r. The distance $OA = \ell$.

Given: ω, r, and ℓ. ($\beta = \sin^{-1}\frac{r}{\ell}$). Determine the angular velocity of the link AB and velocity of the point A.



Problem 18.8

8.4



find ω_{AB}.

Note:

$\sin\beta = \cos\alpha = r/L$

$\cos\beta = \sin\alpha = \frac{\sqrt{L^2-r^2}}{L}$

looking at the disk:

$\underset{\sim}{v}_D = \underset{\sim}{\omega} \times \underset{\sim}{r} = \omega\hat{k} \times r\left(\frac{r}{L}\hat{i} + \frac{\sqrt{L^2-r^2}}{L}\hat{j}\right)$

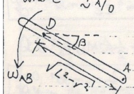
$\underset{\sim}{v}_D = \frac{\omega r}{L}\left(r\hat{j} - \sqrt{L^2-r^2}\,\hat{i}\right)$

Link AB:

$\underset{\sim}{V}_A = \underset{\sim}{V}_D + \underset{\sim}{V}_{A/D} \Rightarrow$ we know that $\underset{\sim}{V}_A = V_A\hat{i}$

$V_A\hat{i} = \frac{\omega r}{L}\left(r\hat{j} - \sqrt{L^2-r^2}\,\hat{i}\right) + \omega_{AB}\hat{k} \times \underset{\sim}{r}_{A/D}$

where $\underset{\sim}{r}_{A/D} = \sqrt{L^2-r^2}\left(\cos\beta\,\hat{i} - \sin\beta\,\hat{j}\right)$



Thus:

(1) $V_A\hat{i} = \frac{\omega r}{L}\left(r\hat{j} - \sqrt{L^2-r^2}\,\hat{i}\right) + \omega_{AB}\sqrt{L^2-r^2}\left(\cos\beta\,\hat{j} - \sin\beta\,\hat{i}\right)$

$\hat{j}\cdot$(1) $\Rightarrow 0 = \frac{\omega r^2}{L} + \omega_{AB}\sqrt{L^2-r^2}\cos\beta$

$\Rightarrow \omega_{AB} = \frac{-\omega r^2}{L\sqrt{L^2-r^2}\cos\beta} = \boxed{\frac{-\omega r^2}{L^2-r^2}\hat{k} = \underset{\sim}{\omega}_{AB}}$

$\hat{i}\cdot$(1) $\Rightarrow V_A = -\frac{\omega r\sqrt{L^2-r^2}}{L} + \omega_{AB}\sqrt{L^2-r^2}\sin\beta$

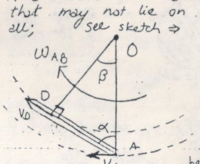
$V_A = \frac{-\omega r}{L}\sqrt{L^2-r^2} - \frac{\omega r^2}{\sqrt{L^2-r^2}}(r/L) =$ continued

8.4 continued p. 6

$V_A = \frac{-\omega r(L^2-r^2) - \omega r^3}{L\sqrt{L^2-r^2}} = \frac{-\omega r L}{\sqrt{L^2-r^2}} \Rightarrow$

$\boxed{\underset{\sim}{V}_A = \frac{-\omega r L}{\sqrt{L^2-r^2}}\hat{i}}$

b. Instantaneous center of rotation: There is some point (call it O) (which may change with time) about which all the points on rod ADB rotate. This is a fictitous point that may not lie on rod ADB at all; see sketch $\Rightarrow$



At this instant, point D behaves like it's going in circles of radius OD about pt. O, while pt. A behaves like it's going in circles of radius OA about O.

We will focus our efforts on triangle ODA.

From the kinematics of the disk, $V_D = r\omega$.

thus, $\omega_{AB} = \frac{v_D}{R_{D/O}}$. Find $R_{D/O}$ using Law of Sines:

$\frac{OD}{\sin\alpha} = \frac{AD}{\sin\beta} \Rightarrow \frac{(OD)(L)}{\sqrt{L^2-r^2}} = \frac{L\sqrt{L^2-r^2}}{r}$

$\Rightarrow OD = \frac{L^2-r^2}{r}$ thus $\omega_{AB} = \frac{V_D}{OD} =$

$\frac{r\omega(r)}{L^2-r^2} = \omega_{AB} = \frac{r^2\omega}{L^2-r^2}$ Noting that $\underset{\sim}{\omega}_{AB} = -\omega_{AB}\hat{k}$,

$\boxed{\underset{\sim}{\omega}_{AB} = \frac{-r^2\omega}{L^2-r^2}\hat{k}}$ (same as before)

Now, note that $V_A = R_{A/O}\,\omega_{AB}$. Find $R_{A/O} = OA$:

$\frac{OA}{\sin 90^\circ} = \frac{AD}{\sin\beta} \Rightarrow OA = \frac{L\sqrt{L^2-r^2}}{r}$

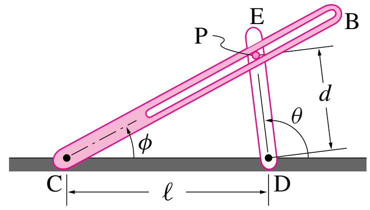
thus, $V_A = (OA)(\omega_{AB}) = \frac{L\sqrt{L^2-r^2}}{r}\left(\frac{r^2\omega}{L^2-r^2}\right)$

so $V_A = \frac{rL\omega}{\sqrt{L^2-r^2}}$

note that $\underset{\sim}{V}_A = -V_A\hat{i}$: $\boxed{\underset{\sim}{V}_A = \frac{-rL\omega}{\sqrt{L^2-r^2}}\hat{i}}$

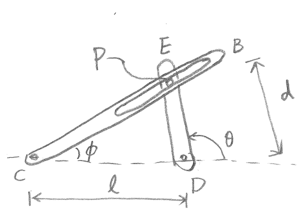
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.4.10 The slotted link CB is driven in an oscillatory motion by the link ED which rotates about D with constant angular velocity $\dot\theta = \omega_D$. The pin P is attached to ED at fixed radius d and engages the slot on CB as shown. Find the angular velocity and acceleration $\dot\phi$ and $\ddot\phi$ of CB when $\theta = \pi/2$.



Problem 18.10

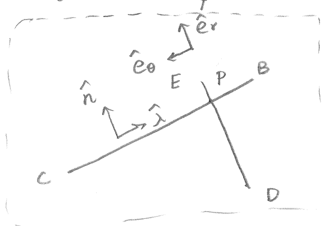
15.38



P is pinned on DE and can slide along CB , $\dot\theta = \omega_D$

Find $\dot\phi$, $\ddot\phi$ when $\theta = \frac{\pi}{2}$

i) Find $\dot\phi$



Build up two local coordinate system $\hat e_\theta , \hat e_r$ and $\hat\lambda , \hat n$.

Call the moving frame attaching to CB , $\mathcal{B}$

the moving frame attaching to DE , $\mathcal{D}$

(a)

$$\vec V_P = \vec V_P$$

$$\cancel{\vec V_C}^{\,0} + \vec V_{P/\mathcal{B}} + \vec\omega_{\mathcal{B}} \times \vec r_{P/C} = \cancel{\vec V_D}^{\,0} + \cancel{\vec V_{P/\mathcal{D}}}^{\,0} + \vec\omega_{\mathcal{D}} \times \vec r_{P/D}$$

$$\Rightarrow \quad V_{P/\mathcal{B}}\,\hat\lambda + \dot\phi\,\hat k \times r_{P/C}\,\hat\lambda = \omega_D\,\hat k \times d\,\hat e_r$$

when $\theta = \frac{\pi}{2}$, $r_{P/C} = \dfrac{\ell}{\cos\phi}$

$$\Rightarrow \left\{ V_{P/\mathcal{B}}\,\hat\lambda + \dot\phi\,\frac{\ell}{\cos\phi}\,\hat n = \omega_D\, d\,\hat e_\theta \right\}$$

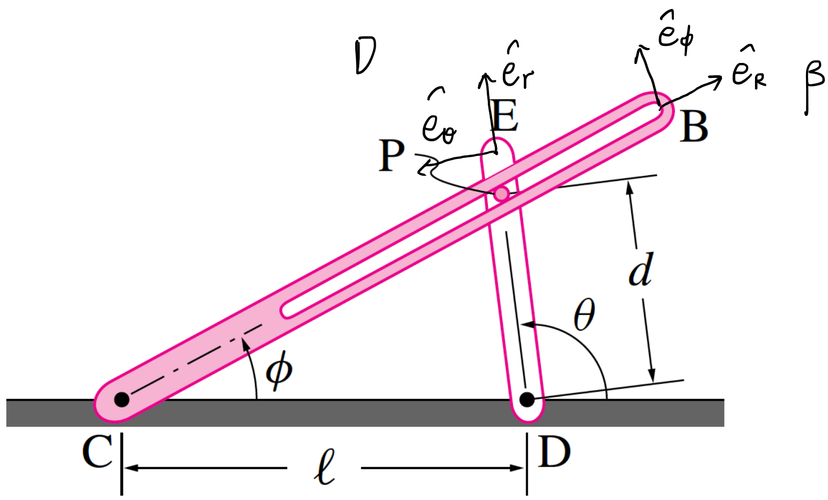
$$\{\ \}\cdot\hat n \quad \Rightarrow \quad \dot\phi\,\frac{\ell}{\cos\phi} = \omega_D\, d\,(\hat e_\theta\cdot\hat n)$$

$$\{\ \}\cdot\hat\lambda \quad \Rightarrow \quad V_{P/\mathcal{B}} = \omega_D\, d\,(\hat e_\theta\cdot\hat\lambda)$$

when $\theta = \frac{\pi}{2}$,
$$\hat e_\theta\cdot\hat n = \sin\phi$$
$$\hat e_\theta\cdot\hat\lambda = -\cos\phi$$

17.4.10

Saturday, May 4, 2019 6:38 PM



P is attached to DE

P can slide along CB

DE rotates with const angular velocity $\dot{\theta} = \omega_0$

Given: d, l, $\dot{\theta} = \omega_0$ $\theta = \frac{\pi}{2}$

Find a) angular velocity of CB: $\dot{\phi}$ when $\theta = \frac{\pi}{2}$.

b) angular acceleration of CB: $\ddot{\phi}$

Build up two local body fixed frame β and $\mathcal{D}$ as shown in the figure

$$\vec{v}_P = \vec{v}_P$$

$$\cancelto{0}{\vec{v}_C} + \vec{v}_{P/\beta} + \vec{\omega}_\beta \times \vec{r}_{P/C} = \cancelto{0}{\vec{v}_D} + \cancelto{0}{\vec{v}_{P/D}} + \vec{\omega}_D \times \vec{r}_{P/D}$$

$$\Rightarrow \quad v_{P/\beta}\,\hat{e}_R + \dot{\phi}\,\hat{k} \times r_{P/C}\,\hat{e}_R = \dot{\theta}\,\hat{k} \times d\,\hat{e}_r$$

when $\theta = \frac{\pi}{2}$ $\quad r_{P/C} = \frac{l}{\cos\phi}$ $\quad \dot{\theta} = \omega_0$ plug in

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\{ v_{P/B}\,\hat{e}_R + \dot{\phi}\frac{l}{\cos\phi}\,\hat{e}_\phi = \omega_D d\,\hat{e}_\theta \}$$

$$\{\}\cdot\hat{e}_\phi \Rightarrow \quad \dot{\phi}\frac{l}{\cos\phi} = \omega_D d\,(\hat{e}_\theta\cdot\hat{e}_\phi)$$

when $\theta=\frac{\pi}{2}$ $\quad \hat{e}_\theta\cdot\hat{e}_\phi = \sin\phi$

$$\therefore\ \dot{\phi} = \omega_D\frac{d}{l}\sin\phi\cos\phi$$

Since $\cos\phi = \frac{d}{\sqrt{l^2+d^2}}$ $\quad \sin\phi = \frac{l}{\sqrt{l^2+d^2}}$

$$\therefore\ \boxed{\dot{\phi} = \omega_D\frac{d^2}{d^2+l^2} \quad \text{when } \theta=\frac{\pi}{2}}$$

b) $\quad \vec{a}_P = \vec{a}_P$

$$\cancel{\vec{a}_C}^{\,0} + \vec{a}_{P/B} + \dot{\vec{\omega}}_B\times\vec{r}_{P/C} - \omega_B^2\,\vec{r}_{P/C} + 2\,\vec{\omega}_B\times\vec{v}_{P/B}$$

$$= \cancel{\vec{a}_D}^{\,0} + \cancel{\vec{a}_{P/D}}^{\,0} + \cancel{\dot{\vec{\omega}}_D}^{\,0}\times\vec{r}_{P/D} - \vec{\omega}_D^{\,2}\,\vec{r}_{P/D} + 2\,\omega_D\times\cancel{\vec{v}_{P/D}}^{\,0}$$

$\dot{\vec{\omega}}_D = \vec{0}$ since $\dot{\theta}=\omega_D = \text{const}$

$$\vec{a}_{P/B} + \dot{\vec{\omega}}_B\times\vec{r}_{P/C} - \omega_B^2\,\vec{r}_{P/C} + 2\vec{\omega}_B\times\vec{v}_{P/B} = -\omega_D^2\,\vec{r}_{P/D}$$

$$\Rightarrow \{ a_{P/B}\,\hat{e}_R + \ddot{\phi}\,\hat{k}\times\frac{l}{\cos\phi}\,\hat{e}_R - \dot{\phi}^2\frac{l}{\cos\phi}\,\hat{e}_R + 2\dot{\phi}\,\hat{k}\times(-\omega_D d\cos\phi)\,\hat{e}_R = -\omega_D^2 d\,\hat{e}_r \}$$

$$\{\}\cdot\hat{e}_\phi \Rightarrow \quad \ddot{\phi}\frac{l}{\cos\phi} - 2\,\omega_D\,\dot{\phi}\,d\cos\phi = -\omega_D^2 d\,(\hat{e}_r\cdot\hat{e}_\phi)$$

when $\theta=\frac{\pi}{2}$ $\quad \hat{e}_r\cdot\hat{e}_\phi = \cos\phi$

$$\therefore\ \ddot{\phi} = \frac{2\omega_D\dot{\phi}\,d\cos^2\phi}{l} - \omega_D^2\frac{d}{l}\cos^2\phi$$

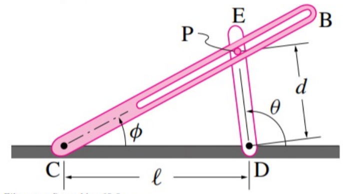
$$= \omega_D^2\,\frac{dl\,(d^2-l^2)}{(l^2+d^2)^2}$$

Thus angular acceleration of CB is $\boxed{\ddot{\phi} = \omega_D^2\,\frac{dl\,(d^2-l^2)}{(l^2+d^2)^2}}$ when $\theta=\frac{\pi}{2}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

HW 13 P70 18.4.10 Teaya Yong (YY449)

18.4.10 The slotted link CB is driven in an oscillatory motion by the link ED which rotates about D with constant angular velocity $\dot{\theta} = \omega_D$. The pin P is attached to ED at fixed radius d and engages the slot on CB as shown. Find the angular velocity and acceleration $\dot{\phi}$ and $\ddot{\phi}$ of CB when $\theta = \pi/2$.

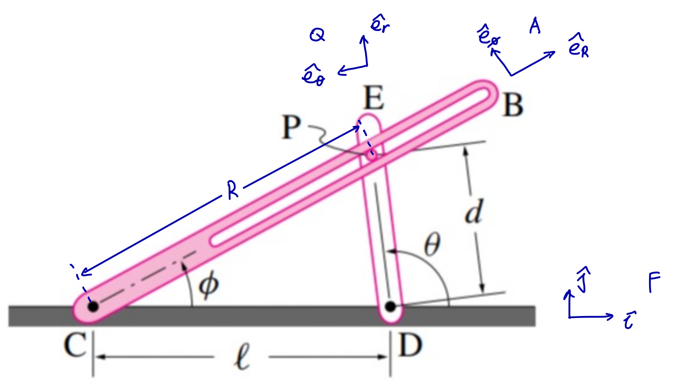


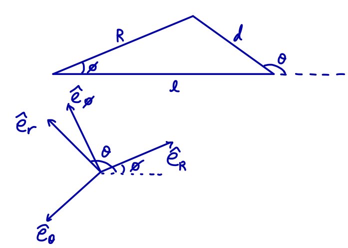
Problem 18.4.10

Given: link ED rotates at $\dot{\theta} = \omega_D$
P is attached to ED with radius d
P slides along BC

Find: a) $\dot{\phi}(\theta = \frac{\pi}{2})$ of CB
b) $\ddot{\phi}(\theta = \frac{\pi}{2})$ of CB

Define one reference frame for each object:
① F is the fixed frame
② A is the body frame for link CB (unit vectors $\hat{e}_R$, $\hat{e}_\phi$)
③ Q is the body frame for link DE (unit vectors $\hat{e}_r$, $\hat{e}_\theta$)





a) Finding $\dot{\phi}\ (\theta = \frac{\pi}{2})$

$$\vec{V}_{P/F} = \vec{V}_{P/F} \qquad \text{* using velocity of pin P as constraint}$$

$$\frac{d}{dt}^F(\vec{r}_{P/C}) = \frac{d}{dt}^F(\vec{r}_{P/D} + \vec{r}_{D/C})$$

This $\dot{R}$ is the key for part b)

$$\underbrace{\vec{V}_{P/A}}_{\dot{R}\hat{e}_R} + \underbrace{\vec{\omega}_{A/F}}_{\dot{\phi}\hat{k}} \times \underbrace{\vec{r}_{P/C}}_{R\hat{e}_R} = \cancel{\vec{V}_{P/Q}}^{0} + \underbrace{\vec{\omega}_{Q/F}}_{\dot{\theta}\hat{k}} \times \underbrace{\vec{r}_{P/D}}_{d\hat{e}_r} + \cancel{\vec{V}_{D/F}}^{0}$$

$$\{\dot{R}\hat{e}_R + \dot{\phi} R \hat{e}_\phi = \dot{\theta} d \hat{e}_\theta\}$$

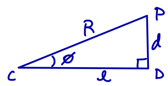
$$\{\ \} \cdot \hat{e}_\phi : \quad \dot{\phi} R = \dot{\theta} d (\hat{e}_\theta \cdot \hat{e}_\phi)$$

To solve $\dot{\phi}$, need to first find R and $\hat{e}_\theta \cdot \hat{e}_\phi$:

↳ BUT the general case for these are not so pretty so let's only consider what R and $\hat{e}_\theta \cdot \hat{e}_\phi$ would be for the orientation at $\theta = \frac{\pi}{2}$:

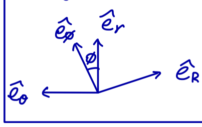
(next page)

Finding $R(\theta = \frac{\pi}{2})$:



$$R = \sqrt{d^2+l^2}$$

Finding $\hat{e}_\theta \cdot \hat{e}_\phi$ at $\theta = \frac{\pi}{2}$:



$$\hat{e}_\theta \cdot \hat{e}_\phi = \sin\phi = \frac{d}{\sqrt{d^2+l^2}}$$

plugging in: $\dot{\phi}\sqrt{d^2+l^2} = \omega_D\, d \dfrac{d}{\sqrt{d^2+l^2}}$

$$\boxed{\dot{\phi}(\theta=\tfrac{\pi}{2}) = \frac{\omega_D d^2}{d^2+l^2}}$$

b) Finding $\ddot{\phi}(\theta = \frac{\pi}{2})$

$$\vec{a}_{P/F} = \vec{a}_{P/F}$$

* using acceleration of pin P as constraint.

Using 5-term acceleration formula:

$$\cancel{\vec{a}_{C/O}}^{\,0} + \vec{a}_{P/A} + \dot{\vec{\omega}}_{A/F} \times \vec{r}_{P/C} - \omega_{A/F}^2\, \vec{r}_{P/C} + 2\vec{\omega}_{A/F} \times \vec{v}_{P/A}$$

$$= \cancel{\vec{a}_{D/O}}^{\,0} + \cancel{\vec{a}_{P/Q}}^{\,0} + \underbrace{\dot{\vec{\omega}}_{A/Q}}_{\cancel{\ddot{\phi}}^{\,0}\hat{k}} \times \vec{r}_{P/D} - \omega_{Q/F}^2\, \vec{r}_{P/D} + 2\vec{\omega}_{Q/F} \times \cancel{\vec{v}_{P/Q}}^{\,0}$$

since $\dot{\theta} = \omega_D$ = const.

$$\vec{a}_{P/A} + \dot{\vec{\omega}}_{A/F} \times \vec{r}_{P/C} - \omega_{A/F}^2\, \vec{r}_{P/C} + 2\vec{\omega}_{A/F} \times \vec{v}_{P/A} = -\omega_{Q/F}^2\, \vec{r}_{P/D}$$

$$(\ddot{R}\,\hat{e}_R) + (\ddot{\phi}\hat{k}) \times (R\hat{e}_R) - \dot{\phi}^2(R\hat{e}_R) + 2(\dot{\phi}\hat{k}) \times (\dot{R}\,\hat{e}_R) = -\omega_D^2(d\,\hat{e}_r)$$

$$\{\ddot{R}\,\hat{e}_R + \ddot{\phi} R\,\hat{e}_\phi - \dot{\phi}^2 R\,\hat{e}_R + 2\dot{\phi}\dot{R}\,\hat{e}_\phi = -\omega_D^2 d\,\hat{e}_r\}$$

$$\{\,\}\cdot\hat{e}_\phi: \quad \ddot{\phi}R + 2\dot{\phi}\dot{R} = -\omega_D^2\, d(\hat{e}_r\cdot\hat{e}_\phi)$$

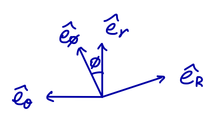
Now to solve for $\ddot{\phi}$, we need R, $\dot{R}$, $\dot{\phi}$ (answer to a)), and $\hat{e}_r\cdot\hat{e}_\phi$

Recall at $\theta = \frac{\pi}{2}$:

$\sin\phi = \dfrac{d}{\sqrt{l^2+d^2}}$ $\quad\sin\theta = 1$

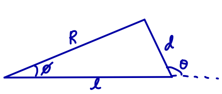
$\cos\phi = \dfrac{l}{\sqrt{l^2+d^2}}$ $\quad\cos\theta = 0$

$$\boxed{\hat{e}_r\cdot\hat{e}_\phi = \cos\phi = \frac{l}{\sqrt{l^2+d^2}}}$$



$$\boxed{R = \sqrt{l^2+d^2}} \qquad \boxed{\dot{\phi}(\theta=\tfrac{\pi}{2}) = \frac{\omega_D d^2}{d^2+l^2}}$$

We are left with $\dot{R}$, which needs a bit of work to get:



Law of Sines: $\dfrac{d}{\sin\phi} = \dfrac{R}{\sin(\pi-\theta)} = \dfrac{R}{\sin\theta}$

$R = d\dfrac{\sin\theta}{\sin\phi}$ (general expression for R at any θ, ϕ)

Deferentiate to find $\dot{R}$:

$$\dot{R} = d\,\frac{\dot{\theta}\cancel{\cos\theta}^{\,0} - \dot{\phi}\cos\phi}{\sin^2\phi} = -\frac{d\dot{\phi}\cos\phi}{\sin^2\phi} = -d\dot{\phi}\left(\frac{l}{d^2}\sqrt{l^2+d^2}\right) \Rightarrow \boxed{\dot{R} = -\frac{\omega_D d\, l}{\sqrt{d^2+l^2}}}$$

$$\ddot{\phi}R + 2\dot{\phi}\dot{R} = -\omega_D^2\, d(\hat{e}_r\cdot\hat{e}_\phi)$$

$$\ddot{\phi}R + 2\left(\omega_D\frac{d^2}{R^2}\right)\left(-\frac{\omega_D d\,l}{R}\right) = -\omega_D^2\frac{l d}{R}$$

$$\ddot{\phi} = 2\omega_D^2\frac{d^3 l}{R^4} - \omega_D^2\frac{l d}{R^2}$$

$$= \omega_D^2\left(\frac{2d^3l - l d R^2}{R^4}\right) = \omega_D^2\left(\frac{2d^3l - ld(d^2+l^2)}{R^4}\right) = \omega_D^2\left(\frac{d^3l - l^3d}{R^4}\right)$$

$$\boxed{\ddot{\phi}(\theta=\tfrac{\pi}{2}) = \omega_D^2\frac{dl(d^2-l^2)}{(l^2+d^2)^2}}$$

Recall that we simplified many terms in this problem by only considering the case where $\theta = \frac{\pi}{2}$. The general case solution would be much more complicated.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$\therefore \quad \dot{\phi} = \omega_D \frac{d}{\ell} \cos\phi \sin\phi \quad , \quad \vec{V}_{P/B} = -\omega_D\, d \cos\phi\, \hat{\lambda}$

$\because \quad \cos\phi = \frac{\ell}{\sqrt{d^2+\ell^2}} \, , \quad \sin\phi = \frac{d}{\sqrt{d^2+\ell^2}}$ when $\theta = \frac{\pi}{2}$, $\Rightarrow \boxed{\dot{\phi} = \omega_D \frac{d^2}{d^2+\ell^2}}$

(b) $\vec{a}_P = \vec{a}_P$ (Note: $\dot{\vec{\omega}}_D = 0$ because $\dot{\theta} = \omega_D = \text{constant}$)

$$\cancelto{0}{\vec{a}_C} + \vec{a}_{P/B} + \dot{\vec{\omega}}_B \times \vec{r}_{P/C} - \omega_B^2\, \vec{r}_{P/C} + 2\vec{\omega}_B \times \vec{V}_{P/B} = \cancelto{0}{\vec{a}_D} + \cancelto{0}{\vec{a}_{P/D}} + \cancelto{0}{\dot{\vec{\omega}}_D \times \vec{r}_{P/D}} - \omega_D^2\, \vec{r}_{P/D} + \cancelto{0}{2\vec{\omega}_D \times \vec{V}_{P/D}}$$

$\Rightarrow$

$$a_{P/B}\,\hat{\lambda} + \ddot{\phi}\,\hat{k} \times \frac{\ell}{\cos\phi}\hat{\lambda} - \dot{\phi}^2 \frac{\ell}{\cos\phi}\hat{\lambda} + 2\dot{\phi}\,\hat{k} \times (-\omega_D\, d\cos\phi)\hat{\lambda} = -\omega_D^2\, d\, \hat{e}_r$$

$$\Rightarrow \left\{ a_{P/B}\,\hat{\lambda} + \ddot{\phi}\frac{\ell}{\cos\phi}\hat{n} - \dot{\phi}^2\frac{\ell}{\cos\phi}\hat{\lambda} - 2\omega_D\,\dot{\phi}\, d\cos\phi\, \hat{n} = -\omega_D^2\, d\, \hat{e}_r \right\}$$

$$\{\ \} \cdot \hat{n} \quad \Rightarrow \quad \ddot{\phi}\frac{\ell}{\cos\phi} - 2\omega_D\,\dot{\phi}\, d\cos\phi = -\omega_D^2\, d\,(\hat{e}_r \cdot \hat{n})$$

$\because \hat{e}_r \cdot \hat{n} = \cos\phi$

$$\therefore \quad \ddot{\phi} = \frac{2\omega_D\,\dot{\phi}\, d\cos^2\phi}{\ell} - \omega_D^2 \frac{d}{\ell}\cos^2\phi$$

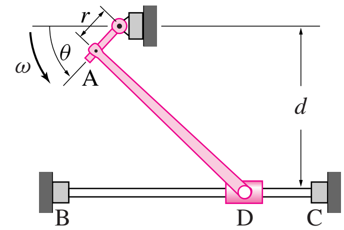
$$= \frac{2\omega_D d}{\ell}\, \frac{\ell^2}{\ell^2+d^2}\, \omega_D \frac{d^2}{d^2+\ell^2} - \omega_D^2 \frac{d}{\ell}\, \frac{\ell^2}{\ell^2+d^2}$$

$$= \omega_D^2 \frac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$$

$\therefore$ Angular acceleration of CB when $\theta = \frac{\pi}{2}$, is $\ddot{\phi} = \omega_D^2 \frac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$

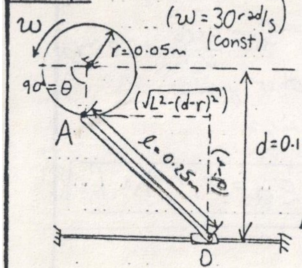
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.4.11 The rod of radius $r = 50\,\mathrm{mm}$ shown has a constant angular velocity of $\omega = 30\,\mathrm{rad/s}$ counterclockwise. Knowing that rod AD is 250 mm long and distance $d = 150\,\mathrm{mm}$, determine the acceleration of collar D when $\theta = 90°$.



Problem 18.11

8.14



Find $\underset{\sim}{a}_D$ when $\theta = 90°$

First find $\underset{\sim}{V}_A$ & $\underset{\sim}{a}_A$ from disk:

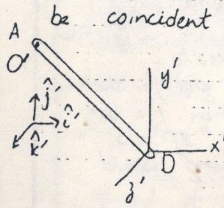
when $\theta = 90°$,

$\underset{\sim}{V}_A = r\omega\,\hat{i}$,

$\underset{\sim}{a}_A = r\omega^2\,\hat{j}$

Attach frame $\mathcal{B}$ to rod AD.

The frame translates and rotates with the rod. Let its coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ be coincident with $(\hat{i}, \hat{j}, \hat{k})$ at this instant.



Then $\underset{\sim}{V}_D = \underset{\sim}{V}_{O'} + \underset{\sim}{V}_{rel}$

thus $\quad$ O: D fixed on frame $\mathcal{B}$

$\underset{\sim}{V}_D = \underset{\sim}{V}_{O'} + \underset{\sim}{\omega}_{\mathcal{B}} \times \underset{\sim}{r}_{D/O'}$

we know $\underset{\sim}{V}_A$, so let's take pt. A to be pt O':

$\underset{\sim}{V}_{O'} = R\omega\,\hat{i}$

$\Rightarrow V_D\,\hat{i} = r\omega\,\hat{i} + \omega_{\mathcal{B}}\,\hat{k} \times \left(\sqrt{L^2-(d-r)^2}\,\hat{i} - (d-r)\hat{j}\right)$

$\Rightarrow V_D\,\hat{i} = r\omega\,\hat{i} + \omega_{\mathcal{B}}\left(\sqrt{L^2-(d-r)^2}\,\hat{j} + (d-r)\hat{i}\right) \;*$

cont'd

8.14 continued

p. 8

$\hat{j}\cdot(*) \Rightarrow 0 = \omega_{\mathcal{B}}\left(\sqrt{L^2-(d-r)^2}\right) \Rightarrow \omega_{\mathcal{B}} = 0$

rod undergoes straight line motion at this instant in time.

Acceleration:

$\underset{\sim}{a}_D = \underset{\sim}{a}_{O'} + \cancel{\underset{\sim}{a}_{rel}} + \cancel{\underset{\sim}{a}_{cor}}$

O: D fixed on $\mathcal{B}$ $\quad = 2\omega_{\mathcal{B}}^{\,0} + \underset{\sim}{v}_{D/\mathcal{B}}^{\,0}$ definitely zero!

$\Rightarrow a_D\,\hat{i} = \underset{\sim}{a}_{O'} + \underset{\sim}{\omega}_{\mathcal{B}} \times \cancel{(\underset{\sim}{\omega}_{\mathcal{B}} \times \underset{\sim}{r}_{D/O'})} + \underbrace{\dot{\underset{\sim}{\omega}}_{\mathcal{B}}}_{\text{unknown}} \times \underset{\sim}{r}_{D/O'}$

again let pt. O' = point A; $\quad$ O: $\underset{\sim}{\omega}_{\mathcal{B}} = \underset{\sim}{0}$

$a_D\,\hat{i} = r\omega^2\,\hat{j} + \dot{\omega}_{\mathcal{B}}\,\hat{k} \times \left(\sqrt{L^2-(d-r)^2}\,\hat{i} - (d-r)\hat{j}\right)$

$a_D\,\hat{i} = r\omega^2\,\hat{j} + \dot{\omega}_{\mathcal{B}}\left(\sqrt{L^2-(d-r)^2}\,\hat{j} + (d-r)\hat{i}\right) \;**$

$\hat{j}\cdot(**) \Rightarrow \dot{\omega}_{\mathcal{B}} = \dfrac{-r\omega^2}{\sqrt{L^2-(d-r)^2}}$

$\hat{i}\cdot(**) \Rightarrow a_D = \dot{\omega}_{\mathcal{B}}(d-r) = \dfrac{-r\omega^2(d-r)}{\sqrt{L^2-(d-r)^2}}$

plugging in #s: $\boxed{\underset{\sim}{a}_D = -19.6\,\mathrm{m/s^2}\,\hat{i}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*