18.4 Kinematics of 2-D mechanisms

18.4.1 Slider crank kinematics (No FBD required!). 2-D . Assume $R, \ell, \theta, \dot{\theta}, \ddot{\theta}$ are given. The crank mechanism parts move on the xy plane with the x direction being along the piston. Vectors should be expressed in terms of $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ components.

a) What is the angular velocity of the crank OA? *

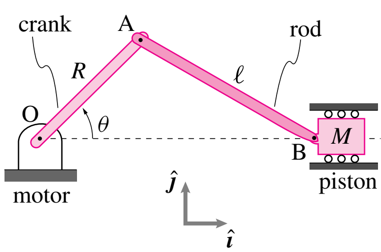
b) What is the angular acceleration of the crank OA? *

c) What is the velocity of point A? *

d) What is the acceleration of point A? *

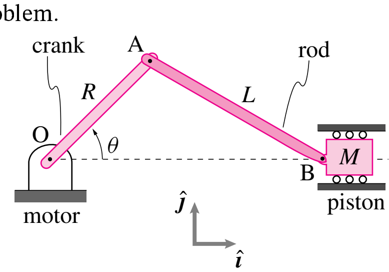
e) What is the angular velocity of the connecting rod AB? [Geometry fact: $\vec{r}_{AB} = \sqrt{\ell^2 - R^2 \sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}$] *

f) For what values of θ is the angular velocity of the connecting rod AB equal to zero (assume $\dot{\theta} \neq 0$)? (you need not answer part (e) correctly to answer this question correctly.) *



Problem 18.4.1

18.4.2 Slider-Crank. Consider a slider-crank mechanism. Given $\theta, \dot{\theta}, \ddot{\theta}, L$, and R, can you find the velocity and acceleration of B? There are many ways to do this problem.



Problem 18.4.2

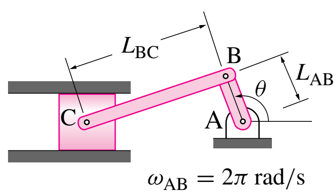
18.4.3 The crank AB with length $L_{AB} =$ 1 inch in the crank mechanism shown rotates at a constant rate $\dot{\theta}_{AB} = \omega_{AB} = 2\pi$ rad/s counter-clockwise. The initial angle of rotation is $\theta = 0$ at $t = 0$. The connecting rod BC has a length of $L_{BC} = 2 \in$.

a) What is the velocity of point B at the end of the crank when $\theta = \pi/2$ rad?

b) What is the velocity of point C at the end of the connecting rod when $\theta = \pi/2$ rad?

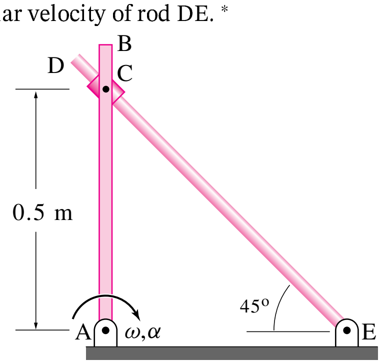
c) What is the angular velocity of the connecting rod BC when $\theta = \pi/2$ rad?

d) What is the angular velocity of the connecting rod BC as a function of time?



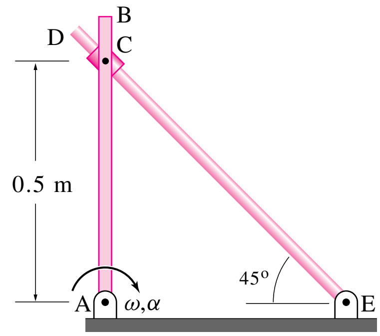
Problem 18.4.3

18.4.4 The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3$ rad/s and angular acceleration $\alpha = 2$ rad/s^2. Find the angular velocity of rod DE. *



Problem 18.4.4

18.4.5 Reconsider problem 18.4.4. The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3$ rad/s and angular acceleration $\alpha = 2$ rad/s^2. Find the angular acceleration of rod DE. *

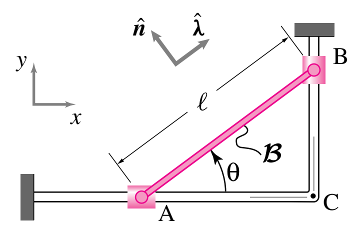


Problem 18.4.5

18.4.6 Collar A is constrained to slide on a horizontal rod to the right at constant speed v_A. It is connected by a pin joint to one end of a rigid bar AB with length ℓ which makes an angle θ with the horizontal at the instant of interest. A second collar B connected by a pin joint to the other end of the rigid bar AB slides on a vertical rod.

a) Find the velocity of point B. Answer in terms of $v_A, \theta, \ell, \hat{\imath}$ and $\hat{\jmath}$.

b) Find the angular speed $\dot{\theta}$ of the rod? Answer in terms of $v_A, \theta, \ell, \hat{\imath}$ and $\hat{\jmath}$.



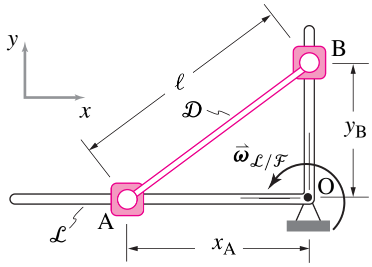
Problem 18.4.6

18.4.7 A bar of length $\ell = 5$ ft, body $\mathcal{D}$, connects sliders A and B on an L-shaped frame, body $\mathcal{L}$, which itself is rotating at constant speed about an axle perpendicular to the plane of the figure through the point O and relative to a fixed frame $\mathcal{F}$, $\vec{\omega}_{\mathcal{L}/\mathcal{F}} = 0.5$ rad/s$\mathbf{k}$. At the instant shown, body $\mathcal{L}$ is aligned with the $x-y$ axes, slider A is $x_A = 4$ ft from point O, slider B is $y_B = 3$ ft from O. The speed and acceleration of slider B relative to the frame $\mathcal{L}$ are $\vec{v}_{B/\mathcal{L}} = -2$ ft/s$\hat{\jmath}$ and $\vec{a}_{B/\mathcal{L}} = -2$ ft/s$^2\,\hat{\jmath}$, respectively.

Determine:

a) The absolute velocity of the slider A, $\vec{\boldsymbol{v}}_{A/\mathcal{F}}$, and

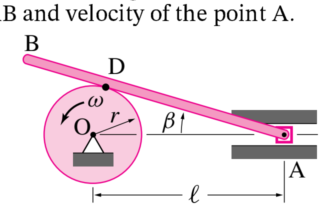
b) The absolute angular velocity of bar AB, body $\mathcal{D}$, $\vec{\boldsymbol{\omega}}_{\mathcal{D}/\mathcal{F}}$.



Problem 18.4.7

18.4.8 The link AB is supported by a wheel at D and its end A is constrained so that it only has horizontal velocity. No slipping occurs between the wheel and the link. The wheel has an angular velocity ω and radius r. The distance $OA = \ell$.

Given: ω, r, and ℓ. ($\beta = \sin^{-1}\frac{r}{\ell}$). Determine the angular velocity of the link AB and velocity of the point A.

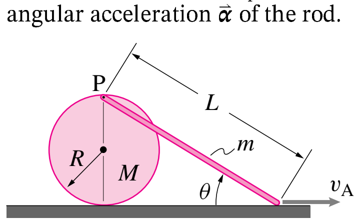


Problem 18.4.8

18.4.9 A solid cylinder of radius R and mass M rolls without slip along the ground. A thin rod of mass m and length L is attached by a frictionless pin P to the cylinder's rim and its right end is dragged at a constant speed $\vec{\boldsymbol{v}}_A$ along the (frictionless) horizontal ground.

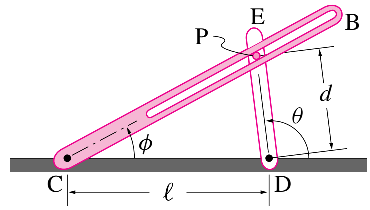
a) For the position shown (where P is directly above the contact point), find P's velocity $\vec{\boldsymbol{v}}_P$ and the rod's angular velocity $\vec{\boldsymbol{\omega}}$.

b) Find P's acceleration $\vec{\boldsymbol{a}}_P$ and the angular acceleration $\vec{\boldsymbol{\alpha}}$ of the rod.



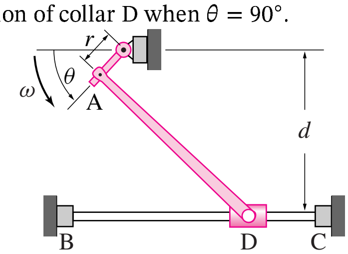
Problem 18.4.9

18.4.10 The slotted link CB is driven in an oscillatory motion by the link ED which rotates about D with constant angular velocity $\dot{\theta} = \omega_D$. The pin P is attached to ED at fixed radius d and engages the slot on CB as shown. Find the angular velocity and acceleration $\dot{\phi}$ and $\ddot{\phi}$ of CB when $\theta = \pi/2$.



Problem 18.4.10

18.4.11 The rod of radius $r = 50\,\text{mm}$ shown has a constant angular velocity of $\omega = 30\,\text{rad/s}$ counterclockwise. Knowing that rod AD is 250 mm long and distance $d = 150\,\text{mm}$, determine the acceleration of collar D when $\theta = 90°$.



Problem 18.4.11