

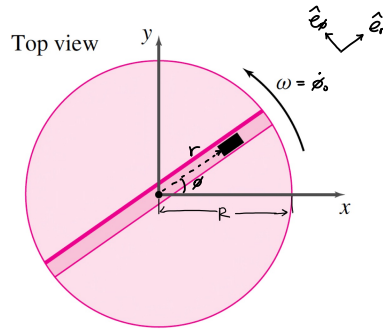
18.3.2 Actual path of bug trying to walk a straight line. A straight line is inscribed on a horizontal turntable. The line goes through the center. Let ϕ be the angle of rotation of the turntable which spins at constant rate $\dot{\phi}_0$. A bug starts on the outside edge of the turntable of radius R and walks towards the center, passes through it, and continues to the opposite edge of the turntable. The bug walks at a constant speed v_A , as measured by how far her feet move per step, on the line inscribed on the table. Ignore gravity.

- a) **Picture.** Make an accurate drawing of the bug's path as seen in the room (which is not rotating with the turntable). In order to make this plot, you will need to assume values of v_A and $\dot{\phi}_0$ and initial values of R and ϕ . You will need to write a parametric equation for the path in terms of variables that you can plot (probably x and y coordinates). You will also

need to pick a range of times. Your plot should include the instant at which the bug walks through the origin. Make sure your x and y -axes are drawn to the same scale. A computer plot would be nice.

- b) Calculate the radius of curvature of the bug's path as it goes through the origin.
- c) Accurately draw (say, on the computer) the osculating circle when the bug is at the origin on the picture you drew for (a) above.
- d) **Force.** What is the force on the bug's feet from the turntable when she starts her trip? Draw this force as an arrow on your picture of the bug's path.
- e) **Force.** What is the force on the bug's feet when she is in the middle of the turntable? Draw this force as an arrow on your picture of the bug's path.

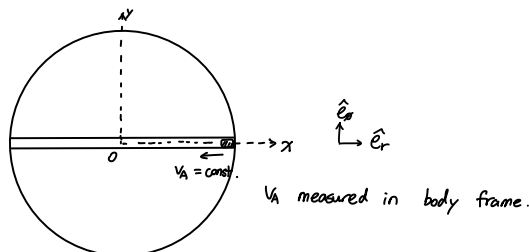
HW # 12 P66 18.3.2



Based on 2019 Old Solution
Modified by Teaya Yang
Corrected May 19, 2021

A bug starts on the outside edge of a turntable. She walks along the inscribed slot with constant speed v_A . No gravity. The table rotates at const rate $\dot{\phi}_0$, where ϕ is the angle of rotation. Assume at time t , the distance between the bug and the center is $r(t)$. $\vec{r} = r\hat{e}_r \Rightarrow \vec{v} = \dot{r}\hat{e}_r + r\dot{\phi}_0\hat{e}_\phi$. The table has turned an angle of $\phi(t)$.

$$\text{Initially, } \begin{cases} r(0) = R \\ \phi(0) = 0 \end{cases}$$



Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

- a) Draw accurate path of the bug relative to the room.
Assume values of V_A and $\dot{\phi}$ and initial values of R and ϕ

The parametric equation for the path in terms of t is:

$$\begin{cases} x(t) = r \cos \phi \\ y(t) = r \sin \phi \end{cases} \quad \text{where} \quad \begin{cases} r(t) = R - V_A t \\ \phi(t) = \dot{\phi}_0 t \end{cases}$$

Write in terms of $x-y$ coordinates

$$\begin{cases} x(t) = (R - V_A t) \cos(\dot{\phi}_0 t) \\ y(t) = (R - V_A t) \sin(\dot{\phi}_0 t) \end{cases}$$

Pick $R = 1 \text{ m}$, $V_A = 0.2 \text{ m/s}$, $\dot{\phi}_0 = 1 \text{ rad/s}$

Code and plots see attached

- b) Find the radius of curvature of the path at origin.

The velocity of the bug is :

$$\begin{aligned} \vec{v} = \dot{\vec{r}} &= \frac{d}{dt} (r(t) \hat{e}_r) = \dot{r}(t) \hat{e}_r + r(t) \dot{\hat{e}}_r \\ &= -V_A \hat{e}_r + r(t) \dot{\phi} \hat{e}_\phi \\ &= -V_A \hat{e}_r + (R - V_A t) \dot{\phi}_0 \hat{e}_\phi \end{aligned}$$

The acceleration is

$$\begin{aligned} \vec{a} = \dot{\vec{v}} &= -V_A \dot{\phi}_0 \hat{e}_\phi + -V_A \dot{\phi}_0 \hat{e}_\phi + (R - V_A t) \dot{\phi}_0^2 \hat{e}_r - (R - V_A t) \dot{\phi}_0^2 \hat{e}_r \\ &= -2V_A \dot{\phi}_0 \hat{e}_\phi - (R - V_A t) \dot{\phi}_0^2 \hat{e}_r \end{aligned}$$

when the bug goes through the center, $r = R - V_A t = 0$

$$\Rightarrow t = \frac{R}{V_A}$$

At this instant,

$$\vec{v} = -V_A \hat{e}_r \quad , \quad \vec{a} = -2V_A \dot{\phi}_0 \hat{e}_\phi$$

compare this to the path coordinates expression

$$\vec{v} = v \hat{e}_t \quad , \quad \vec{a} = \dot{v} \hat{e}_t + \frac{v^2}{\rho} \hat{e}_n$$

where ρ is the radius of curvature

we get when the bug is at the origin

$$\left\{ \begin{array}{l} v = |\vec{v}| = V_A \\ \hat{e}_t = -\hat{e}_r \\ \dot{v} = 0 \\ \frac{v^2}{\rho} = 2V_A \dot{\phi}_0 \\ \hat{e}_n = -\hat{e}_\phi \end{array} \right. \quad (\text{since } \vec{a} \perp \vec{v} \text{ at this instant})$$

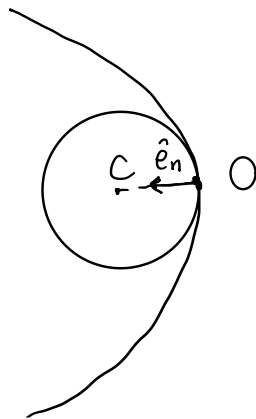
$$\Rightarrow \boxed{\rho = \frac{V^2}{2V_A \dot{\phi}_0} = \frac{V_A^2}{2V_A \dot{\phi}_0} = \frac{V_A}{2 \dot{\phi}_0}}$$

c) Coding to draw the osculating circle when the bug is at the origin on picture drawn in a)

$\Rightarrow$ Find the center & radius of the circle

get in part b)

Now find the center of the osculating circle



C is the center of osculating circle
O is the origin

Recall from part b):

$$t = \frac{R}{V_A} \Rightarrow \theta = \dot{\theta}_0 t = \frac{R\dot{\theta}_0}{V_A}$$

$$\rho = \frac{V_A}{2\dot{\theta}_0}$$

$$\hat{e}_n = -\hat{e}_\theta$$

$$\text{So } \vec{r}_{c/o} = x_c \hat{i} + y_c \hat{j} = -\rho \hat{e}_\theta$$

$$x_c \hat{i} + y_c \hat{j} = -\rho (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

$$\begin{cases} x_c = \frac{V_A}{2\dot{\theta}_0} \sin\left(\frac{R\dot{\theta}_0}{V_A}\right) \\ y_c = -\frac{V_A}{2\dot{\theta}_0} \cos\left(\frac{R\dot{\theta}_0}{V_A}\right) \end{cases}$$

with the position of C, we can draw the circle
see Matlab code attached

d) Find the force on the bug's feet from the turntable at the starting point

Draw the force as an arrow on the path

We know from b) that

$$\vec{a} = -2V_A \dot{\phi}_0 \hat{e}_\phi - r \dot{\phi}_0^2 \hat{e}_r$$

since when $t=0$ $\hat{e}_r = \hat{i}$, $\hat{e}_\phi = \hat{j}$ then

$$\vec{a}(0) = -2V_A \dot{\phi}_0 \hat{j} - R \dot{\phi}_0^2 \hat{i}$$

LMB : $\vec{F} = m \vec{a}$

$$\Rightarrow \boxed{\vec{F} = -m (R \dot{\phi}_0^2 \hat{i} + 2V_A \dot{\phi}_0 \hat{j})}$$

is the force acting on the bug at the beginning.

e) Do the same as d) except at origin

$$t = \frac{R}{V_A} \quad \phi = \phi_0 \frac{R}{V_A}$$

$$a(t = \frac{R}{V_A}) = -2V_A \dot{\phi}_0 \hat{e}_\phi \quad (\text{from b})$$

$$= -2V_A \dot{\phi}_0 (-\sin(\dot{\phi}_0 \frac{R}{V_A}) \hat{i} + \cos(\dot{\phi}_0 \frac{R}{V_A}) \hat{j})$$

$$= 2V_A \dot{\phi}_0 \sin(\dot{\phi}_0 \frac{R}{V_A}) \hat{i} - 2V_A \dot{\phi}_0 \cos(\dot{\phi}_0 \frac{R}{V_A}) \hat{j}$$

LMB: $\vec{F} = m \vec{a}$

$$\boxed{\vec{F} = 2mV_A \dot{\phi}_0 (\sin(\dot{\phi}_0 \frac{R}{V_A}) \hat{i} - \cos(\dot{\phi}_0 \frac{R}{V_A}) \hat{j})}$$

Contents

- [Part a\) Plotting the actual path](#)
- [Part b\) Drawing Osculating circle at origin](#)
- [Part c\)d\) Drawing force vector at t=0 and at the origin](#)

```
% MAE2030 HW12 P66 8.3.2
% 2019 Solution code, modified by Teaya Yang
% Actual path of bug trying to walk a straight line

% Parameters
R=1; % radius of the turntable
vA=0.2; % velocity of the bug on the turntable
phidot=1; % constant angular velocity of the turntable
```

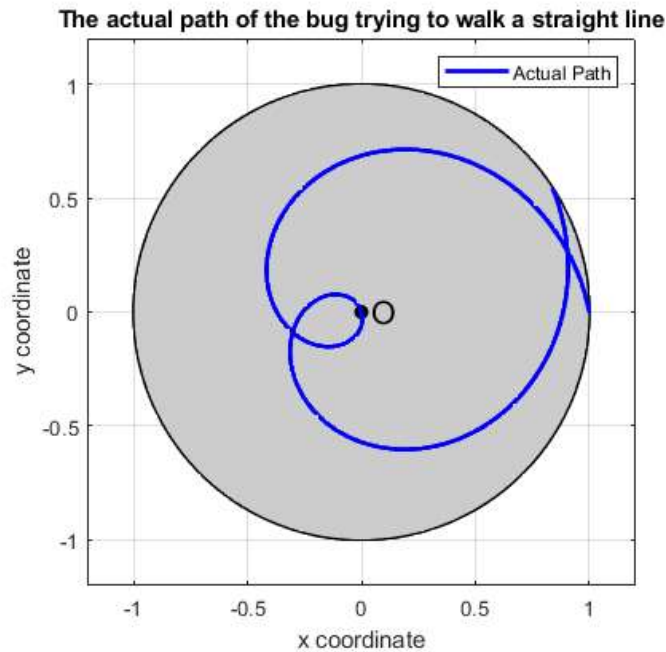
Part a) Plotting the actual path

Draw the turntable

```
figure(1)
box on
hold on
axis equal
grid on
axis([-R R -R R]*1.2)
angle = 0:pi/80:2*pi;
circlex = R*cos(angle);
circley = R*sin(angle);
p(1) = plot(circlex,circley,'k-','linewidth',1.5);
p(2) = fill(circlex,circley,[0.8 0.8 .8]);
p(3) = plot(0,0,'ko','MarkerSize',6,'MarkerFaceColor','k');
text(.04,0,'0','fontsize',15);

% Draw the actual path of the bug
tend = 2*R/vA; % Time when the bug reaches the other end
t = linspace(0,2*R/vA,1000);
x=(R-vA*t).*cos(phidot*t);
y=(R-vA*t).*sin(phidot*t);
p(4) = plot(x,y,'b-','linewidth',2);
title('The actual path of the bug trying to walk a straight line');
xlabel('x coordinate');
ylabel('y coordinate');
legend(p(4),'Actual Path')
```

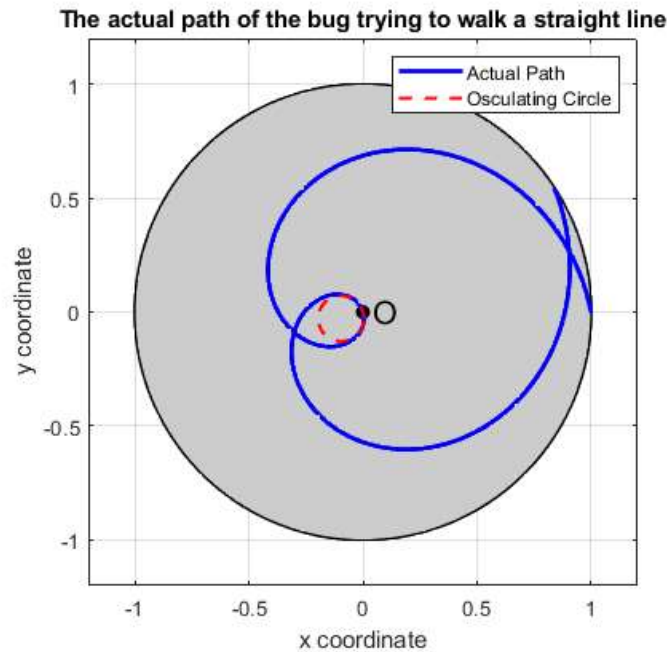
Disclaimer: We have lost track of the many people who wrote these solutions.
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Part b) Drawing Osculating circle at origin

Draw osculating circle when bug goes through the center

```
rho=vA/(2*phidot); % radius of curvature of the path at the origin
xC= vA*sin(phidot*R/vA)/(2*phidot);
yC= -vA*cos(phidot*R/vA)/(2*phidot); % position of the center
theta=0:0.01:2*pi;
x=xC+rho*cos(theta);
y=yC+rho*sin(theta);
hold on;
p(5) = plot(x,y,'r--','linewidth',1.5);
legend([p(4),p(5)], 'Actual Path', 'Osculating Circle')
```



Part c)d) Drawing force vector at $t=0$ and at the origin

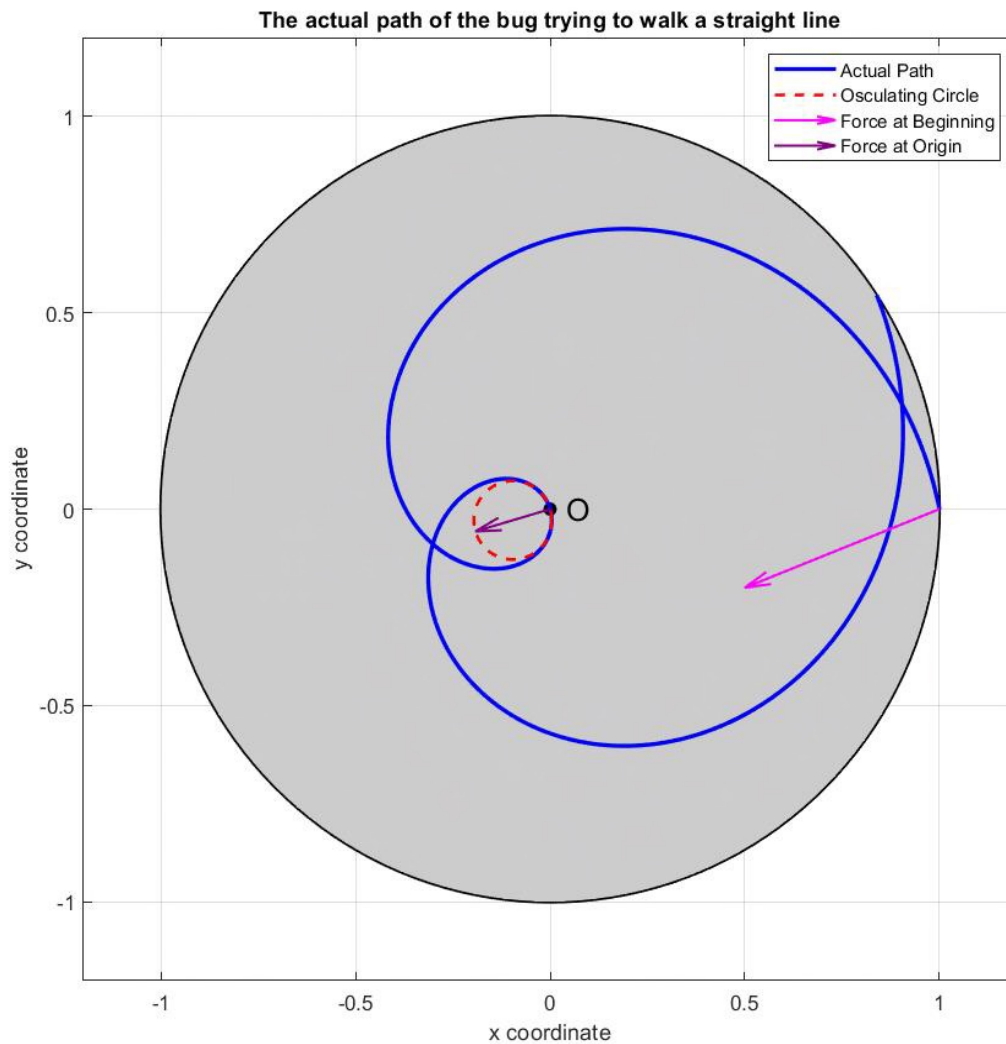
draw force vector

```

m=1; %mass of the bug
scale=0.5; % scale for graphics
f1x=-m*R*phidot; → f1x = -m*R*phidot^2;
f1y=-m*vA*phidot;
% draw force at the beginning
p(6) = quiver(R,0,f1x,f1y,scale,'m','linewidth',1.2,'MaxHeadSize',.4);
f2x=2*m*vA*phidot*sin(phidot*R/vA);
f2y=-2*m*vA*phidot*cos(phidot*R/vA);
%draw force at the origin
p(7) = quiver(0,0,f2x,f2y,scale,'color',[.5 0 .5],'linewidth',1.2,'MaxHeadSize',1.2);
legend([p(4),p(5),p(6),p(7)], 'Actual Path', 'Osculating Circle', 'Force at Beginning', 'Force at Origin')

```

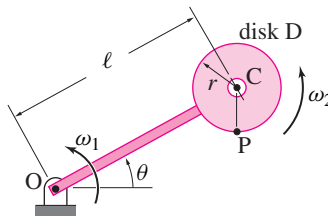
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18.3.4 Arm OC rotates with constant rate ω_1 . Disc D of radius r rotates about point C at constant rate ω_2 measured with respect to the arm OC. What are the absolute velocity and acceleration of point P on the disc, $\vec{v}_P$ and $\vec{a}_P$? (To do this problem will require defining a moving frame of reference. More than one choice is possible.)

redo the problem. Make sure the answers are the same.



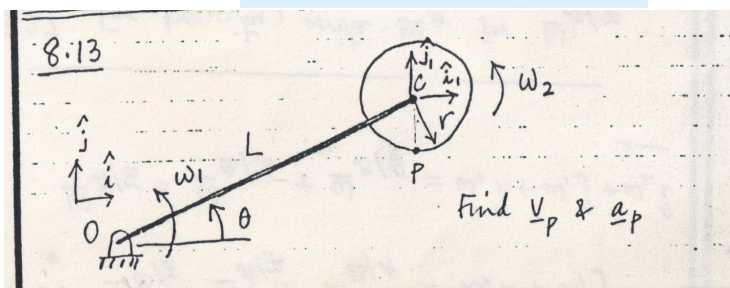
Problem 18.4

- Pick a suitable moving frame and do the problem.
- Pick another suitable moving frame and

Answer:

$$\vec{v}_P = \omega_1 L (\cos \theta \hat{j} - \sin \theta \hat{i}) + (\omega_1 + \omega_2) r \hat{i},$$

$$\vec{a}_P = -\omega_1^2 L (\cos \theta \hat{i} + \sin \theta \hat{j}) + (\omega_1 + \omega_2)^2 \hat{j}.$$



Attach a moving frame at point C, with unit vectors $\hat{i}_1, \hat{j}_1$, called $\mathcal{B}$

a) First, assume that the moving frame does not rotate (it just translates; unit vectors don't change direction).

$$\omega_{\text{disc}/\mathcal{B}} = (\omega_1 + \omega_2) \hat{k}$$

$$\therefore \vec{v}_{P/\mathcal{B}} = (\omega_1 + \omega_2) \hat{k} \times \vec{r}_{CP}$$

$$= (\omega_1 + \omega_2) r \hat{k} \times (-\hat{j}_1)$$

$$= (\omega_1 + \omega_2) r \hat{i}_1 \leftarrow$$

$$\vec{v}_C = \omega_1 \hat{k} \times L (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$= \omega_1 L (\cos \theta \hat{j} - \sin \theta \hat{i})$$

$$\vec{v}_P = \vec{v}_C + \vec{v}_{P/\mathcal{B}} + \cancel{\omega_{\mathcal{B}} \times \vec{r}_{CP}}$$

$$\vec{v}_P = \omega_1 L (\cos \theta \hat{j} - \sin \theta \hat{i}) + (\omega_1 + \omega_2) r \hat{i}$$

$$\vec{a}_{P/\mathcal{B}} = (\omega_1 + \omega_2) \hat{k} \times [(\omega_1 + \omega_2) \hat{k} \times \vec{r}_{CP}]$$

$$= (\omega_1 + \omega_2)^2 r \hat{j}_1$$

$$\vec{a}_C = \omega_1 \hat{k} \times [\omega_1 \hat{k} \times L (\cos \theta \hat{i} + \sin \theta \hat{j})]$$

$$\vec{a}_P = \vec{a}_C + \vec{a}_{P/\mathcal{B}} + \text{bunch of zeros}$$

$$\vec{a}_P = -\omega_1^2 L (\cos \theta \hat{i} + \sin \theta \hat{j}) + (\omega_1 + \omega_2)^2 r \hat{j}$$

b) Next, consider the situation where moving frame rotates with the rod, i.e., $\omega_{\mathcal{B}} = \omega_1 \hat{k}$ and $\omega_{\text{disc}/\mathcal{B}} = \omega_2 \hat{k}$

$$\therefore \vec{v}_{P/\mathcal{B}} = \omega_2 \hat{k} \times \vec{r}_{CP} = \omega_2 r \hat{i}$$

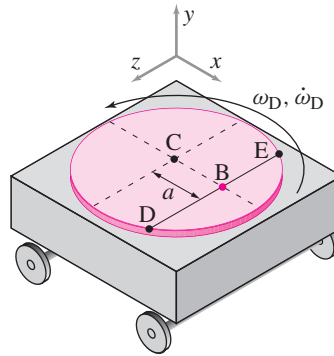
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18.3.5 For the configuration in problem 18.3.4 what is the absolute angular velocity of the disk, D ?

Answer:

$$\bar{\omega}_D = (\omega_1 + \omega_2)\hat{k}.$$

18.3.7 A turntable oscillates with displacement $\vec{x}_C(t) = A \sin(st)\hat{i}$. The disc of the turntable rotates with angular speed and acceleration ω_D and $\dot{\omega}_D$. A small bug walks along line DE with velocity v_b relative to the turntable. At the instant shown, the turntable is at its maximum amplitude $x = A$, the line DE is currently aligned with the z -axis, and the bug is passing through point B on line DE . Point B is a distance a from the center of the turntable, point C . Find the absolute acceleration of the bug, $\vec{a}_b$.



Problem 18.7

Answer:

$$\vec{a}_b = -(s^2 A + a\omega_D^2 + 2\omega_D v_b)\hat{i} - a\dot{\omega}_D\hat{k}.$$

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Find the acceleration of a bug, at B at the instant shown, but moving with velocity v_b along the line DE .

Solution:
Let B' be a point fixed on the turntable but coincident with B at the instant shown.

Then

$$\vec{v}_B = \vec{v}_{B'} + \vec{v}_{rel}$$

$$\vec{a}_B = \vec{a}_{B'} + \vec{a}_{rel} + \vec{a}_{cor}$$

$$\vec{a}_{B'} = \vec{a}_C + \dot{\omega}_{D/T} \times \vec{r}_{B'/C} + \omega_{D/T} \times (\omega_{D/T} \times \vec{r}_{B'/C})$$

$$\omega_{D/T} = \omega_D \hat{j} \quad \dot{\omega}_{D/T} = \dot{\omega}_D \hat{j}$$

And $\vec{r}_{B'/C} = a \hat{i}$

$$\vec{a}_C = \ddot{x}_C \hat{i} = -s^2 A \sin(st) \hat{i}$$

at its extreme position.

$$\therefore \vec{a}_C = -s^2 A \hat{i}$$

$$\dot{\omega}_{D/T} \times \vec{r}_{B'/C} = (a\dot{\omega}_D)(-\hat{k})$$

$$\omega_{D/T} \times (\omega_{D/T} \times \vec{r}_{B'/C}) = -(a\omega_D^2) \hat{i}$$

$$\vec{a}_{rel} = 0 \quad \vec{a}_{cor} = 2\omega_{D/T} \times v_b (-\hat{k})$$

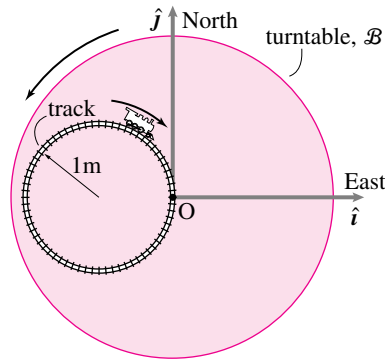
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$$\therefore \vec{a}_B = -(s^2 A + a\omega_D^2 + 2\omega_D v_b)\hat{i} - a\dot{\omega}_D \hat{k}$$

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18.3.8 A small 0.1 kg toy train engine is going *clockwise* at a constant rate (relative to the track) of 2 m/s on a circular track of radius 1 m. The track itself is on a turntable B that is rotating *counter-clockwise* at a constant rate of 1 rad/s. The dimensions are as shown. At the instant of interest the train is pointing due south ($-\hat{j}$) and is at the center O of the turntable.

- e) What is the total force acting on the train?
- f) Sketch the path of the train for one revolution of the turntable (surprise)?



- a) What is the velocity of the train relative to the turntable $\vec{v}_{T/B}$?
- b) What is the absolute velocity of the train $\vec{v}$?
- c) What is the acceleration of the train relative to the turntable $\vec{a}_{T/B}$?
- d) What is the absolute acceleration of the train $\vec{a}$?

Problem 18.8

8.4B Train. Note that the directions of rotation shown in the book's sketch are not consistent with those shown in the text. I'll go with the ones shown in the sketch.

Attach frame B to the turntable. Its basis vectors $\hat{i}, \hat{j}$ are aligned with $\hat{e}_1, \hat{e}_2$ at this instant. This frame rotates with $\vec{\omega}_B = -1 \text{ rad/s } \hat{k}$.

a) $\vec{v}_{T/B} = -2 \text{ m/s } \hat{j}$

b) $\vec{v}_T = \vec{v}_{T/B} + \vec{v}_{T/B} + \vec{\omega}_B \times \vec{r}_{T/O}$

$\Rightarrow \vec{v}_T = \vec{v}_{T/B} = -2 \text{ m/s } \hat{j}$

continued

8.4B continued

c) $\vec{a}_{T/B} = \vec{\omega}_{T/B} \times \vec{r}_{T/B} + \vec{\omega}_{T/B} \times \vec{r}_{T/B}$

$\vec{\omega}_{T/B} = \vec{\omega}_{T/B} \hat{k} = \frac{v_{T/B}}{r} \hat{k} = \frac{2 \text{ m/s}}{1 \text{ m}} \hat{k} = 2 \text{ rad/s } \hat{k}$

$\Rightarrow \vec{a}_{T/B} = 2 \text{ rad/s } \hat{k} \times 2 \text{ rad/s } \hat{k} \times -1 \text{ m } \hat{i} =$

$\vec{a}_{T/B} = 4 \text{ m/s}^2 \hat{i}$

d) $\vec{a}_T = \vec{a}_{T/B} + \vec{\omega}_B \times \vec{v}_{T/B} + \vec{\omega}_B \times \vec{v}_{T/B}$

$\Rightarrow \vec{a}_T = 4 \text{ m/s}^2 \hat{i} + 2(-1 \text{ rad/s } \hat{k} \times -2 \text{ m/s } \hat{j})$

$\vec{a}_T = (4 \text{ m/s}^2 + 4 \text{ m/s}^2) \hat{i} = \vec{0} = \vec{a}_T$

e) $\sum \vec{F}_T = m_T \vec{a}_T = \vec{0}$

f) Numbers denote train posn after:

①: 0 rev (start)

②: 1/4 rev of turntable

③: 1/2 rev " " "

④: 3/4 rev " " "

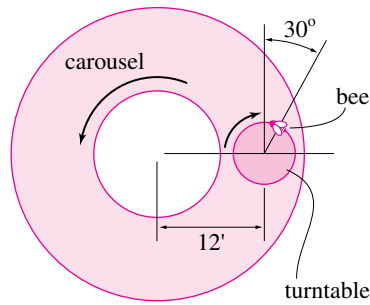
⑤: 1 rev " " "

(This occurs only when $\vec{\omega}_{T/B} = -2 \vec{\omega}_B$)

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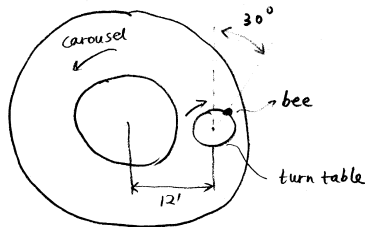
18.3.11 A nostalgic honeybee alights on a phonograph turntable that is being carried by a carnival goer who is riding on a carousel. The situation is sketched below. The carousel has angular velocity of 5 rpm, which is increasing (accelerating) at 10 rev/min^2 ; the phonograph rotates at a constant $33 \frac{1}{3} \text{ rpm}$. The honeybee is at the outer edge of the phonograph record in the position shown in the figure; the radius of the record is 7 inches. Calculate the magnitude of the accelera-

tion of the honeybee.



Problem 18.11

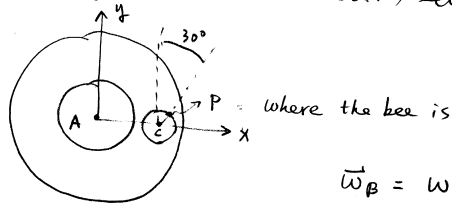
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At this instant, the carousel has angular velocity of $\omega_c = 5 \text{ rpm} = 10\pi \text{ rad/min}$
angular acceleration $\alpha_c = 10 \text{ r/min}^2 = 20\pi \text{ rad/min}^2$

The turntable rotates at $\omega_t = 33 \frac{1}{3} \text{ rpm} = 66 \frac{2}{3} \pi \text{ rad/min}$, $\alpha_t = 0$

Use the moving frame glued to the carousel. Let's call it $\mathcal{B}$.



$$\vec{\omega}_{\mathcal{B}} = \omega_c \hat{k} = 10\pi \text{ rad/min}$$

$$\vec{\alpha}_{\mathcal{B}} = \alpha_c \hat{k} = 20\pi \text{ rad/min}^2$$

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/\mathcal{B}} - \omega_{\mathcal{B}}^2 \vec{r}_{P/A} + \dot{\omega}_{\mathcal{B}} \times \vec{r}_{P/A} + 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{P/\mathcal{B}}$$

$$\text{i) } \vec{a}_A = 0 \quad \text{since A is fixed}$$

$$\text{ii) } \vec{a}_{P/\mathcal{B}} = -\omega_t^2 \vec{r}_{P/C} = -(66 \frac{2}{3} \pi)^2 7 \left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j} \right) \text{ in/min}^2$$

Since P rotates with the turntable at a constant angular velocity.

$$\therefore \vec{a}_{P/\mathcal{B}} = (-1.5353 \times 10^5 \hat{i} - 2.6952 \times 10^5 \hat{j}) \text{ in/min}^2$$

$$\text{iii) } -\omega_{\mathcal{B}}^2 \vec{r}_{P/A} = -\omega_{\mathcal{B}}^2 (\vec{r}_{P/C} + \vec{r}_{C/A})$$

$$= - (10\pi)^2 \left(\frac{7}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} + 12 \times 12 \hat{i} \right)$$

$$= -1.4558 \times 10^5 \hat{i} - 5.9831 \times 10^3 \hat{j} \quad \text{in/min}^2$$

$$\text{iv) } \dot{\vec{\omega}}_B \times \vec{r}_{P/A}$$

$$= 20\pi \hat{k} \times \left[\left(\frac{7}{2} + 144 \right) \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} \right]$$

$$= -380.898 \hat{i} + 9.2677 \times 10^3 \hat{j} \quad \text{in/min}^2$$

$$\text{v) } 2 \vec{\omega}_B \times \vec{v}_{P/B}$$

$$\vec{v}_{P/B} = (-\omega_t \hat{k}) \times \vec{r}_{P/C}$$

$$\therefore 2 \vec{\omega}_B \times \vec{v}_{P/B} = 2 \omega_c \omega_t \vec{r}_{P/C}$$

$$= 2(10\pi) \times (66\frac{2}{3}\pi) \times \left(\frac{7}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} \right)$$

$$= 4.6058 \times 10^4 \hat{i} + 7.9775 \times 10^4 \hat{j} \quad \text{in/min}^2$$

Sum all the five terms up.

$$\vec{a}_P = (-2.5343 \times 10^5 \hat{i} - 1.8646 \times 10^5 \hat{j}) \quad \text{in/min}^2$$

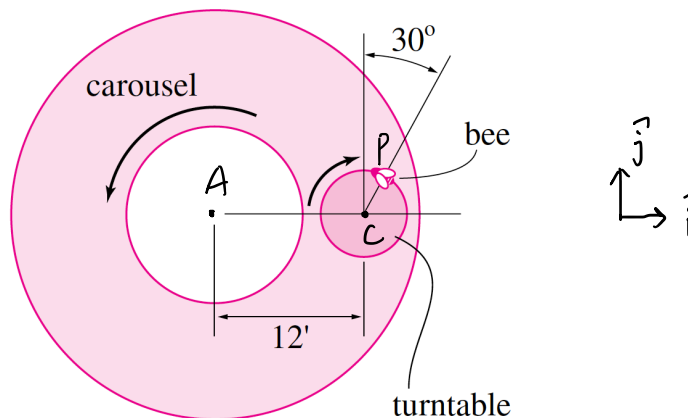
$\therefore$ the magnitude of acceleration

$$\begin{aligned} |\vec{a}_P| &= 3.1464 \times 10^5 \text{ in/min}^2 \\ &= 87.4 \text{ in/sec}^2 \end{aligned}$$

17.3.11

Saturday, April 27, 2019

17:16



Carousel : angular velocity $\omega_c = 5 \text{ rpm} = 10\pi \text{ rad/min}$
 angular acceleration $\alpha_c = 10 \text{ r/min}^2 = 20 \text{ rad/min}^2$

Turntable : const angular velocity $\omega_t = 33\frac{1}{3} \text{ rpm} = 66\frac{2}{3}\pi \text{ rad/min}$
 radius $r_t = 7'$

Find : the magnitude of the acceleration a_b of the bee

Using the rotating frame β which is fixed to the carousel

$$\text{Thus } \vec{\omega}_\beta = \omega_\beta \hat{k} = \omega_c \hat{k} = 10\pi \text{ rad/min}$$

$$\vec{\omega}_\beta = \alpha_\beta \hat{k} = \alpha_c \hat{k} = 20\pi \text{ rad/min}^2$$

$$\vec{a}_p = \vec{a}_A + \vec{a}_{p/\beta} - \omega_\beta^2 \vec{r}_{p/A} + \dot{\vec{\omega}}_\beta \times \vec{r}_{p/A} + 2\vec{\omega}_\beta \times \vec{v}_{p/\beta}$$

$$\text{i) } \vec{a}_A = 0 \quad \text{since } A \text{ is fixed}$$

$$\text{ii) } \vec{a}_{p/\beta} = -\omega_t^2 \vec{r}_{p/C} = -\left(66\frac{2}{3}\pi\right)^2 7 \left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j}\right) [\text{in/min}^2]$$

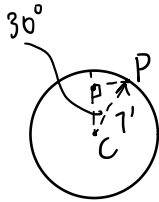
Since P rotates with the turntable at a const angular velocity

$$\therefore \vec{a}_{P/\beta} = (-1.5353 \times 10^5 \hat{i} - 2.6952 \times 10^5 \hat{j}) [\text{in}/\text{min}^2]$$

$$\text{iii) } -\omega_\beta^2 \vec{r}_{P/A} = -\omega_\beta^2 (\vec{r}_{P/C} + \vec{r}_{C/A})$$

$$= -(10\pi)^2 \left(\frac{7}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} + 12 \hat{i} \right) [\text{in}/\text{min}^2]$$

$$= -1.5298 \times 10^4 \hat{i} - 5.9831 \times 10^3 \hat{j} [\text{in}/\text{min}^2]$$



$$|\vec{r}_{P/C}| = 7, \quad \vec{r}_{P/C} = \frac{7}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} [\text{in}]$$

$$A \xrightarrow{12'} C \quad \vec{r}_{C/A} = 12 \hat{i} [\text{in}]$$

$$\text{iv) } \vec{\omega}_\beta \times \vec{r}_{P/A}$$

$$= 20\pi \hat{k} \times \left(\left(\frac{7}{2} + 12 \right) \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} \right) [\text{in}/\text{min}^2]$$

$$= -380.898 \hat{i} + 973.8937 \hat{j} [\text{in}/\text{min}^2]$$

$$\text{v) } 2 \vec{\omega}_\beta \times \vec{v}_{P/\beta} \quad (-\omega_\beta \hat{k}) \times \vec{r}_{P/C}$$

$$= 2\omega_\beta \hat{k} \times (-\omega_\beta \hat{k}) \times \vec{r}_{P/C}$$

$$= 2\omega_\beta \omega_\beta \vec{r}_{P/C}$$

$$= 2 \times 10\pi \times 66 \frac{2}{3} \pi \times \left(\frac{7}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j} \right) [\text{in}/\text{min}^2]$$

$$= 4.6058 \times 10^4 \hat{i} + 7.9775 \times 10^4 \hat{j} [\text{in}/\text{min}^2]$$

Sum all the five terms

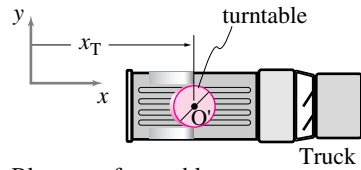
$$\vec{a}_P = -1.2315 \times 10^5 \hat{i} - 1.9475 \times 10^5 \hat{j} [\text{in}/\text{min}^2]$$

$$\therefore |\vec{a}_P| = 2.3042 \times 10^5 [\text{in}/\text{min}^2] = 64.0056 [\text{in}/\text{sec}^2]$$

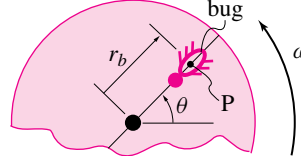
18.3.12 Consider a turntable on the back of a pick-up truck. A bug walks on a line on the turntable. The line may or may not be drawn through the center of the turntable. The truck may or may not be going at constant rate. The turntable may or may not be spinning, and, if it spins, it may or may not go at a constant rate. The bug may be anywhere on the line and may or may not be walking at a constant rate.

- Draw a picture of the situation.** Clearly define all variables you are going to use. What is the moving frame, and what is the reference point on that frame?
- One term at a time.** For each term in the five term acceleration formula, find a situation where all but one term is zero. Use the turntable as the moving frame, the bug as the particle of interest.
- Two terms at a time.** (harder) How many situations can you find where a pair of terms is not zero but all other terms are zero?

(Don't try to do all 10 cases unless you really think this infamous formula is fun. Try at least one or two.)



Blow-up of turntable:



Problem 18.12: Picking apart the five term acceleration formula.

Answer:

(b) One case: $\vec{a}_P = \vec{a}_{O'}$, where P refers to the bug; the bug stands still on the turntable, the truck accelerates, and the turntable does not rotate.

(c) One case: $\vec{a}_P = \vec{a}_{O'} + \vec{a}_{P/D}$, where D is the turntable; the bug accelerates along a straight line on the turntable, the truck accelerates, and the turntable does not rotate.