SAMPLE 18.8 A 'T' shaped tube is welded to a massless rigid arm OAB which rotates about O at a constant rate $\omega = 5\,\mathrm{rad/s}$. See also Sample 18.26. At the instant shown a particle P is falling down in the vertical section of the tube with speed $v_{/\mathrm{tube}} = 4\,\mathrm{ft/s}$. Find the absolute velocity of the particle. Take $\ell = 2\,\mathrm{ft}$ in the figure.

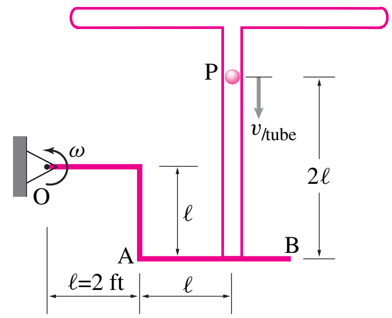


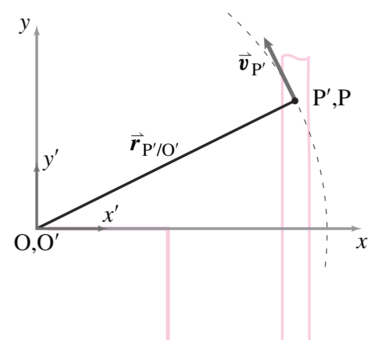
Figure 18.40

Solution Let us attach a frame $\mathcal{B}$ to arm OAB. Thus $\mathcal{B}$ rotates with OAB with angular velocity $\vec{\omega}_{\mathcal{B}} = \omega\hat{\boldsymbol{k}}$ where $\omega = 5\,\mathrm{rad/s}$. To do calculations in $\mathcal{B}$ we attach a coordinate system $x'y'z'$ to $\mathcal{B}$ at point O. At the instant of interest the rotating coordinate system $x'y'z'$ coincides with the fixed coordinate system xyz. (Since the entire motion is in the xy-plane, the z-axis is not shown in the figure). Let P' be a point coincident with P but fixed in $\mathcal{B}$. Now,

$$\vec{\boldsymbol{v}}_P = \vec{\boldsymbol{v}}_{P'} + \vec{\boldsymbol{v}}_{\mathrm{rel}}$$

where

$$\begin{aligned}
\vec{\boldsymbol{v}}_{P'} &= \underbrace{\vec{\boldsymbol{v}}_{O'}}_{\vec{0}} + \vec{\omega}_{\mathcal{B}} \times \vec{\boldsymbol{r}}_{P'/O'} \\
&= \omega\hat{\boldsymbol{k}} \times (2\ell\hat{\boldsymbol{\imath}} + \ell\hat{\boldsymbol{\jmath}}) \\
&= 2\omega\ell\hat{\boldsymbol{\jmath}} - \omega\ell\hat{\boldsymbol{\imath}},
\end{aligned}$$



and

$$\begin{aligned}
\vec{\boldsymbol{v}}_{\mathrm{rel}} &= \text{Velocity relative to the frame } \mathcal{B} \\
&= -v_{/\mathrm{tube}}\hat{\boldsymbol{\jmath}}.
\end{aligned}$$

Thus,

$$\begin{aligned}
\vec{\boldsymbol{v}}_P &= \vec{\boldsymbol{v}}_{P'} + \vec{\boldsymbol{v}}_{rel} \\
&= -\omega\ell\hat{\boldsymbol{\imath}} + (2\omega\ell - v_{/\mathrm{tube}})\hat{\boldsymbol{\jmath}} \\
&= -5\,\mathrm{rad/s}\cdot 2\,\mathrm{ft}\hat{\boldsymbol{\imath}} + (2\cdot 5\,\mathrm{rad/s}\cdot 2\,\mathrm{ft} - 4\,\mathrm{ft/s})\hat{\boldsymbol{\jmath}} \\
&= -10\,\mathrm{ft/s}\hat{\boldsymbol{\imath}} + 16\,\mathrm{ft/s}\hat{\boldsymbol{\jmath}}.
\end{aligned}$$

$$\boxed{\vec{\boldsymbol{v}}_P = -10\,\mathrm{ft/s}\hat{\boldsymbol{\imath}} + 16\,\mathrm{ft/s}\hat{\boldsymbol{\jmath}}}$$

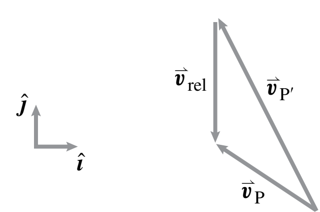


Figure 18.41:

Comments: The kinematics calculation is equivalent to the vector addition shown in Figure 18.41. The velocity of P is the sum of $\vec{\boldsymbol{v}}_{P'}$ and $\vec{\boldsymbol{v}}_{\mathrm{rel}} = \vec{\boldsymbol{v}}_{P/\mathcal{B}}$.

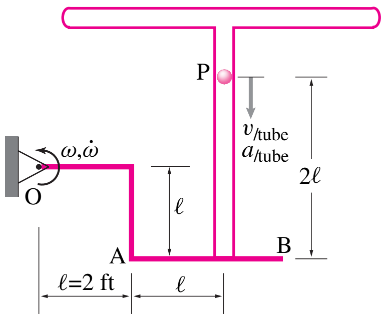
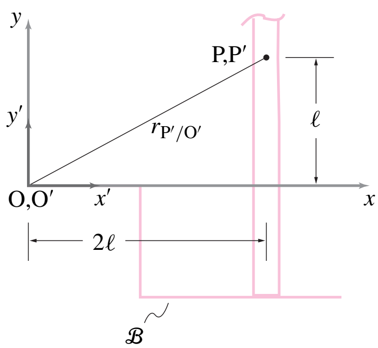


Figure 18.42



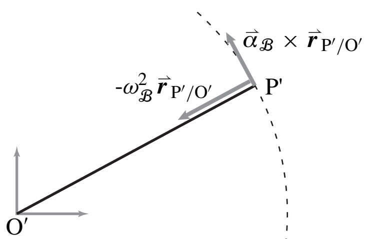


Figure 18.43: Acceleration of point P′. P′ is fixed to the frame $\mathcal{B}$ and at the moment coincides with point P. Therefore, $\vec{a}_{P'} = \vec{\alpha}_{\mathcal{B}} \times \vec{r}_{P'/O'} - \omega^2_{\mathcal{B}}\vec{r}_{P'/O'}$.

SAMPLE 18.9 Acceleration of a point in a rotating frame. Consider the rotating tube of Sample 18.8 again. The arm OAB rotates with counterclockwise angular acceleration $\dot\omega = 3\,\text{rad/s}^2$ and, at the instant shown, its angular speed $\omega = 5\,\text{rad/s}$. Also, at the same instant, the particle P falls down with speed $v_{/\text{tube}} = 4\,\text{ft/s}$ and acceleration $a_{/\text{tube}} = 2\,\text{ft/s}^2$. Find the absolute acceleration of the particle at the given instant. Take $\ell = 2\,\text{ft}$ in the figure.

Solution We consider a frame $\mathcal{B}$, with coordinate axes $x'y'z'$, fixed to the arm OAB and thus rotating with $\vec{\omega}_{\mathcal{B}} = \omega\hat{\boldsymbol{k}} = 5\,\text{rad/s}\hat{\boldsymbol{k}}$ and $\vec{\alpha}_{\mathcal{B}} = \dot\omega\hat{\boldsymbol{k}} = 3\,\text{rad/s}^2\hat{\boldsymbol{k}}$. The acceleration of point P is given by

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{\text{cor}} + \vec{a}_{\text{rel}}$$

where

$$\begin{aligned}
\vec{a}_{P'} &= \text{acceleration of a point P}' \text{ that is fixed in } \mathcal{B} \\
&\quad\ \text{and at the moment coincides with P,} \\
\vec{a}_{\text{cor}} &= \text{Coriolis acceleration, and} \\
\vec{a}_{\text{rel}} &= \text{acceleration of P relative to frame } \mathcal{B}.
\end{aligned}$$

Now we calculate each of these terms separately. For calculating $\vec{a}_{P'}$, imagine a rigid rod from point O to point P, rotating with the frame $\mathcal{B}$. Mark the far end of the rod as P′ (same as point P). The acceleration of this end of the rod is $\vec{a}_{P'}$. To find the relative terms $\vec{v}_{\text{rel}}$ and $\vec{a}_{\text{rel}}$, freeze the motion of the frame $\mathcal{B}$ at the given moment and watch the motion of point P. The non-intuitive term $\vec{a}_{\text{cor}}$ has no such simple physical interpretation but has a simple formula. Thus,

$$\begin{aligned}
\vec{a}_{P'} &= \overbrace{\vec{a}_{O'}}^{\vec{0}} + \vec{\alpha}_{\mathcal{B}} \times \vec{r}_{P'/O'} + \overbrace{\vec{\omega}_{\mathcal{B}} \times (\vec{\omega}_{\mathcal{B}} \times \vec{r}_{P'/O'})}^{-\omega^2_{\mathcal{B}}\vec{r}_{P'/O'}} \\
&= \dot\omega\hat{\boldsymbol{k}} \times (2\ell\hat{\boldsymbol{\imath}} + \ell\hat{\boldsymbol{\jmath}}) - \omega^2(2\ell\hat{\boldsymbol{\imath}} + \ell\hat{\boldsymbol{\jmath}}) \\
&= 2\dot\omega\ell\hat{\boldsymbol{\jmath}} - \dot\omega\ell\hat{\boldsymbol{\imath}} - 2\omega^2\ell\hat{\boldsymbol{\imath}} - \omega^2\ell\hat{\boldsymbol{\jmath}} \\
&= -\ell(\dot\omega + 2\omega^2)\hat{\boldsymbol{\imath}} + \ell(2\dot\omega - \omega^2)\hat{\boldsymbol{\jmath}} \\
&= -(106\hat{\boldsymbol{\imath}} + 38\hat{\boldsymbol{\jmath}})\,\text{ft/s}^2,
\end{aligned}$$

$$\begin{aligned}
\vec{a}_{\text{cor}} &= 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{\text{rel}} \\
&= 2\omega\hat{\boldsymbol{k}} \times v_{/\text{tube}}(-\hat{\boldsymbol{\jmath}}) \\
&= 2\omega v_{/\text{tube}}\hat{\boldsymbol{\imath}} = 2 \cdot 5\,\text{rad/s} \cdot 4\,\text{ft/s} \\
&= 40\,\text{ft/s}^2\hat{\boldsymbol{\imath}},
\end{aligned}$$

$$\vec{a}_{\text{rel}} = a_{/\text{tube}}(-\hat{\boldsymbol{\jmath}}) = -2\,\text{ft/s}^2\hat{\boldsymbol{\jmath}}.$$

Adding the three terms together, we get

$$\begin{aligned}
\vec{a}_P &= -106\,\text{ft/s}^2\hat{\boldsymbol{\imath}} - 38\,\text{ft/s}^2\hat{\boldsymbol{\jmath}} + 40\,\text{ft/s}^2\hat{\boldsymbol{\imath}} - 2\,\text{ft/s}^2\hat{\boldsymbol{\jmath}} \\
&= -66\,\text{ft/s}^2\hat{\boldsymbol{\imath}} - 40\,\text{ft/s}^2\hat{\boldsymbol{\jmath}}.
\end{aligned}$$

$$\boxed{\vec{a}_P = -(66\hat{\boldsymbol{\imath}} + 40\hat{\boldsymbol{\jmath}})\,\text{ft/s}^2}$$

Note that the single term $\vec{a}_{P'}$ encompasses three terms of the five term acceleration formula.

SAMPLE 18.10 A small collar P is pinned to a rigid rod AB at length $\ell = 1\,\text{m}$ along the rod. The collar is free to slide in a straight track on a disk of radius $r = 400\,\text{mm}$. The disk rotates about its center O at a constant $\omega = 2\,\text{rad/s}$. At the instant shown, when $\theta = 45^\circ$ and the collar is at a distance $\frac{3}{4}r$ in the track from the center O, find

1. the angular velocity of the rod AB and
2. the velocity of point P relative to the disk.

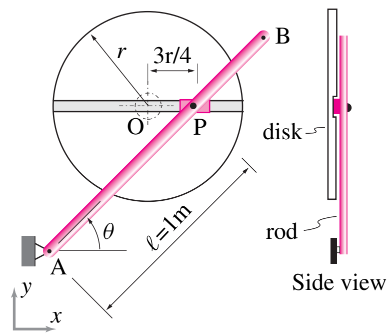
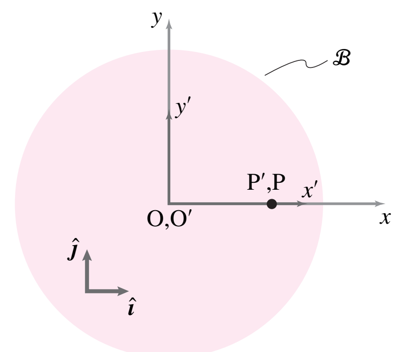


Figure 18.44:



xy is the fixed frame. $x'y'$ rotates with the disk with ω_B. P′ is fixed in the rotating frame.

Solution We will think of P in two ways: one as attached to the rod and the other as sliding in the slot. First, let us attach a frame $\mathcal{B}$ to the disk. Thus $\mathcal{B}$ rotates with the disk with angular velocity $\vec{\omega}_{\mathcal{B}} = \omega\hat{\boldsymbol{k}} = 2\,\text{rad/s}\hat{\boldsymbol{k}}$. We attach a coordinate system $x'y'z'$ to $\mathcal{B}$ at point O. At the instant of interest, the rotating coordinate system $x'y'z'$ coincides with the fixed coordinate system xyz. Now let us consider point P' which is fixed on the disk (and hence in $\mathcal{B}$) and coincides with point P at the instant of interest. We can write the velocity of P as:

$$\vec{\boldsymbol{v}}_P = \vec{\boldsymbol{v}}_{P'} + \vec{\boldsymbol{v}}_{\text{rel}}$$

where

$$\begin{aligned}
\vec{\boldsymbol{v}}_{P'} &= \overbrace{\vec{\boldsymbol{v}}_{O'}}^{\vec{\mathbf{0}}} + \vec{\omega}_{\mathcal{B}} \times \vec{\boldsymbol{r}}_{P'/O'} = \omega\hat{\boldsymbol{k}} \times \left(\frac{3}{4}r\hat{\boldsymbol{\imath}}'\right) = \frac{3}{4}\omega r\,\hat{\boldsymbol{\jmath}}' = \frac{3}{4}\omega r\,\hat{\boldsymbol{\jmath}},\\
\vec{\boldsymbol{v}}_{\text{rel}} &\equiv \vec{\boldsymbol{v}}_{P/\mathcal{B}} = v_{\text{rel}}\hat{\boldsymbol{\imath}}' = v_{\text{rel}}\hat{\boldsymbol{\imath}}.
\end{aligned}$$

In the last expression, $\vec{\boldsymbol{v}}_{\text{rel}} = v_{\text{rel}}\hat{\boldsymbol{\imath}}$, we do not know the magnitude of $\vec{\boldsymbol{v}}_{\text{rel}}$ and hence have left it as an unknown v_{rel}, but its direction is known because $\vec{\boldsymbol{v}}_{\text{rel}}$ has to be along the track and the track at the given instant is along the x-axis. Thus,

$$\vec{\boldsymbol{v}}_P = \frac{3}{4}\omega r\,\hat{\boldsymbol{\jmath}} + v_{\text{rel}}\hat{\boldsymbol{\imath}}. \tag{18.33}$$

Now let us consider the motion of rod AB. Let $\vec{\boldsymbol{\Omega}} = \Omega\hat{\boldsymbol{k}}$ be the angular velocity of AB at the instant of interest where Ω is unknown. Since P is pinned to the rod, it executes circular motion about A with radius $AP = \ell$. Therefore,

$$\vec{\boldsymbol{v}}_P = \vec{\boldsymbol{\Omega}} \times \vec{\boldsymbol{r}}_{P/A} = \Omega\hat{\boldsymbol{k}} \times \ell(\cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}) = \Omega\ell(\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\hat{\boldsymbol{\imath}}). \tag{18.34}$$

But, and this trivial formula is the key, $\vec{\boldsymbol{v}}_P = \vec{\boldsymbol{v}}_P$. Therefore, from Eqn. (18.33) and (18.34),

$$\frac{3}{4}\omega r\,\hat{\boldsymbol{\jmath}} + v_{\text{rel}}\hat{\boldsymbol{\imath}} = \Omega\ell(\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\hat{\boldsymbol{\imath}}). \tag{18.35}$$

Taking dot product of both sides of the above equation with $\hat{\boldsymbol{\jmath}}$ we get

$$\begin{aligned}
\frac{3}{4}\omega r &= \Omega\ell\cos\theta\\
\Rightarrow \quad \Omega &= \frac{3\omega r}{4\ell\cos\theta} = \frac{3\cdot 2\,\text{rad/s}\cdot 0.4\,\text{m}}{4\cdot 1\,\text{m}\cdot\frac{1}{\sqrt{2}}} = 0.85\,\text{rad/s}.
\end{aligned}$$

Again taking the dot product of both sides of Eqn. (18.35) with $\hat{\boldsymbol{\imath}}$ we get

$$v_{\text{rel}} = -\Omega\ell\sin\theta = -0.85\,\text{rad/s}\cdot 1\,\text{m}\cdot\frac{1}{\sqrt{2}} = -0.6\,\text{m/s}.$$

$$(i)\ \vec{\boldsymbol{\Omega}} = 0.85\,\text{rad/s}\hat{\boldsymbol{k}}, \quad (ii)\ \vec{\boldsymbol{v}}_{\text{rel}} = -0.6\,\text{m/s}\hat{\boldsymbol{\imath}}$$

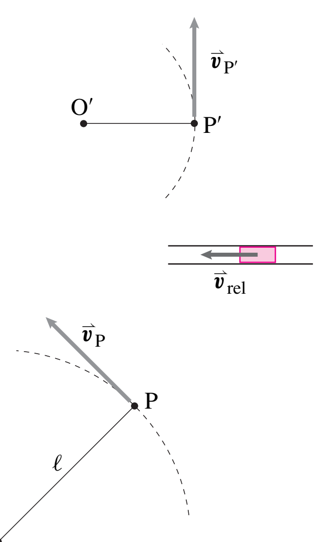


Figure 18.45:

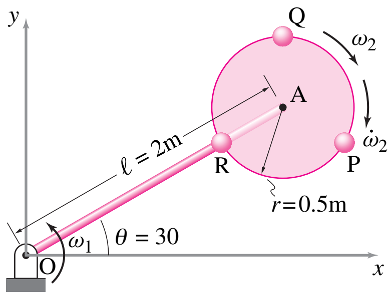


Figure 18.46

SAMPLE 18.11 **Spinning wheel on a rotating rod in 2-D .** A rigid body OA is attached to a wheel that is massless except for three point masses P, Q, and R, placed symmetrically on the wheel. Each of the three masses is $m = 0.5$ kg. The rod OA rotates about point O in the counterclockwise direction at a constant rate $\omega_1 = 3$ rad/s. The wheel rotates with respect to the arm about point A with angular acceleration $\dot\omega_2 = 1\,\text{rad/s}^2$ and at the instant shown it has angular speed $\omega_2 = 5$ rad/s. Note that both ω_2 and $\dot\omega_2$ are given with respect to the arm.

Using a rotating frame $\mathcal{B}$ attached to the rod and a coordinate system attached to the frame with origin at O, find

1. the velocity of the mass P and
2. the acceleration of the mass P.

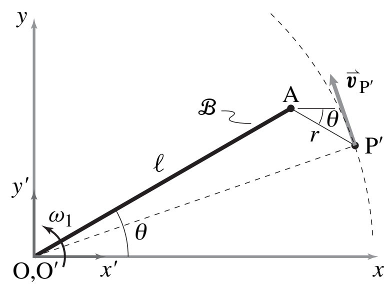


Figure 18.47: P′ goes in circles around point O with radius OP′. Therefore, $\vec{v}_{P'}$ is tangential to the circular path at P′.

Solution Frame $\mathcal{B}$ is attached to the rod. We choose a coordinate system $x'y'z'$ in frame $\mathcal{B}$ with its origin at O and, at the instant, aligned with the fixed coordinate system xyz. We consider a point P′ momentarily coincident with point P but fixed in frame $\mathcal{B}$. Since P′ is fixed in $\mathcal{B}$, it rotates with $\mathcal{B}$ with $\vec{\omega}_{\mathcal{B}} = \omega_1\hat{\boldsymbol{k}}$. To visualize the motion of P′ imagine a rigid rod from O to P′ (see Fig. 18.47). Now we can calculate the velocity and acceleration of point P′ as follows.

1. **Velocity of point P:**

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{\text{rel}}.$$

Now we calculate the two terms separately:

$$\begin{aligned}
\vec{v}_{P'} &= \underbrace{\vec{v}_{O'}}_{\vec{0}} + \vec{\omega}_{\mathcal{B}} \times \vec{r}_{P'/O'} \\
&= \omega_1\hat{\boldsymbol{k}} \times (\vec{r}_{A/O'} + \vec{r}_{P'/A}) \\
&= \omega_1\hat{\boldsymbol{k}} \times [\underbrace{\ell(\cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\hat{\boldsymbol{\jmath}})}_{\vec{r}_{A/O'}} + \underbrace{r(\cos\theta\hat{\boldsymbol{\imath}} - \sin\theta\hat{\boldsymbol{\jmath}})}_{\vec{r}_{P'/A}}] \\
&= \omega_1(\ell + r)\cos\theta\hat{\boldsymbol{\jmath}} - \omega_1(\ell - r)\sin\theta\hat{\boldsymbol{\imath}} \\
&= 3\,\text{rad/s}\cdot 2.5\,\text{m}\cdot\cos 30^\circ\,\hat{\boldsymbol{\jmath}} - 3\,\text{rad/s}\cdot 1.5\,\text{m}\cdot\sin 30^\circ\,\hat{\boldsymbol{\imath}} \\
&= (6.50\hat{\boldsymbol{\jmath}} - 2.25\hat{\boldsymbol{\imath}})\,\text{m/s}.
\end{aligned}$$

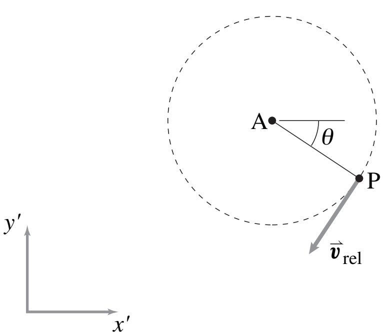


Figure 18.48: Velocity of point P with respect to the frame $\mathcal{B}$.

Since the wheel rotates with angular speed ω_2 with respect to the rod, an observer sitting in frame $\mathcal{B}$ would see a circular motion of point P about point A. Therefore,

$$\begin{aligned}
\vec{v}_{\text{rel}} &= \vec{\omega}_{\text{wheel}/\mathcal{B}} \times \vec{r}_{P/A} \\
&= -\omega_2\hat{\boldsymbol{k}}' \times r(\cos\theta\hat{\boldsymbol{\imath}}' - \sin\theta\hat{\boldsymbol{\jmath}}') \\
&= -\omega_2 r(\cos\theta\hat{\boldsymbol{\jmath}}' + \sin\theta\hat{\boldsymbol{\imath}}') \\
&= -(2.16\hat{\boldsymbol{\jmath}}' + 1.25\hat{\boldsymbol{\imath}}')\,\text{m/s}.
\end{aligned}$$

But at the instant of interest, $\hat{\boldsymbol{\imath}}' = \hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}' = \hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}' = \hat{\boldsymbol{k}}$. So,

$$\vec{v}_{\text{rel}} = -(2.16\hat{\boldsymbol{\jmath}} + 1.25\hat{\boldsymbol{\imath}})\ \text{m/s}$$

Therefore,

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{\text{rel}} = 4.33\,\text{m/s}\,\hat{\boldsymbol{\jmath}} - 3.50\,\text{m/s}\,\hat{\boldsymbol{\imath}}.$$

$$\boxed{\vec{v}_P = (-3.50\hat{\boldsymbol{\imath}} + 4.33\hat{\boldsymbol{\jmath}})\,\text{m/s}}$$

2. **Acceleration of point P:** We can similarly find the acceleration of point P:

$$\vec{\boldsymbol{a}}_P = \vec{\boldsymbol{a}}_{P'} + \vec{\boldsymbol{a}}_{\text{cor}} + \vec{\boldsymbol{a}}_{\text{rel}}$$

where

$$\begin{aligned}
\vec{\boldsymbol{a}}_{P'} &= \text{acceleration of point P}' \\
&= \underbrace{\vec{\boldsymbol{a}}_{O'}}_{\vec{\mathbf{0}}} + \underbrace{\vec{\boldsymbol{\alpha}}_{\mathcal{B}}}_{\vec{\mathbf{0}}} \times \vec{\boldsymbol{r}}_{P'/O'} + \vec{\boldsymbol{\omega}}_{\mathcal{B}} \times (\vec{\boldsymbol{\omega}}_{\mathcal{B}} \times \vec{\boldsymbol{r}}_{P'/O'}) \\
&= -\omega_{\mathcal{B}}^2 \vec{\boldsymbol{r}}_{P'/O'} \\
&= -\omega_1^2[(\ell + r)\cos\theta \hat{\boldsymbol{\imath}} + (\ell - r)\sin\theta \hat{\boldsymbol{\jmath}}] \\
&= -9(\,\text{rad/s})^2[2.5\,\text{m} \cdot \cos 30^\circ \hat{\boldsymbol{\imath}} + 1.5\,\text{m} \cdot \sin 30^\circ \hat{\boldsymbol{\jmath}}] \\
&= -(19.48\hat{\boldsymbol{\imath}} + 6.75\hat{\boldsymbol{\jmath}})\,\text{m/s}^2,
\end{aligned}$$

$$\begin{aligned}
\vec{\boldsymbol{a}}_{\text{cor}} &= \text{Coriolis acceleration} \\
&= 2\vec{\boldsymbol{\omega}}_{\mathcal{B}} \times \vec{\boldsymbol{v}}_{\text{rel}} \\
&= 2\omega_1 \hat{\boldsymbol{k}} \times \vec{\boldsymbol{v}}_{\text{rel}} \qquad \text{(see part (a) above for } \vec{\boldsymbol{v}}_{\text{rel}}). \\
&= (6\,\text{rad/s})\hat{\boldsymbol{k}} \times (-2.16\hat{\boldsymbol{\jmath}} - 1.25\hat{\boldsymbol{\imath}})\,\text{m/s} \\
&= (12.99\hat{\boldsymbol{\imath}} - 7.50\hat{\boldsymbol{\jmath}})\,\text{m/s}^2,
\end{aligned}$$

$$\begin{aligned}
\vec{\boldsymbol{a}}_{\text{rel}} &= \text{acceleration of P relative to frame } \mathcal{B} \\
&= \vec{\boldsymbol{a}}_{P/\mathcal{B}} = \dot{\vec{\boldsymbol{\omega}}}_2 \times \vec{\boldsymbol{r}}_{P/A} - \omega_2^2 \vec{\boldsymbol{r}}_{P/A} \\
&= -\dot{\omega}_2 \hat{\boldsymbol{k}}' \times r(\cos\theta \hat{\boldsymbol{\imath}}' - \sin\theta \hat{\boldsymbol{\jmath}}') - \omega_2^2 r(\cos\theta \hat{\boldsymbol{\imath}}' - \sin\theta \hat{\boldsymbol{\jmath}}') \\
&= -r[(\dot{\omega}_2 \sin\theta + \omega_2^2 \cos\theta)\hat{\boldsymbol{\imath}}' + (\dot{\omega}_2 \cos\theta - \omega_2^2 \sin\theta)\hat{\boldsymbol{\jmath}}'] \\
&= -0.5\,\text{m}[(1\,\text{rad/s}^2 \cdot \sin 30^\circ + 25(\,\text{rad/s})^2 \cdot \cos 30^\circ)\hat{\boldsymbol{\imath}}' \\
&\qquad +(1\,\text{rad/s}^2 \cdot \cos 30^\circ - 25(\,\text{rad/s})^2 \cdot \sin 30^\circ)\hat{\boldsymbol{\jmath}}'] \\
&= (-11.08\hat{\boldsymbol{\imath}}' + 5.82\hat{\boldsymbol{\jmath}}')\,\text{m/s}^2 \\
&= (-11.08\hat{\boldsymbol{\imath}} + 5.82\hat{\boldsymbol{\jmath}})\,\text{m/s}^2.
\end{aligned}$$

The term $\vec{\boldsymbol{a}}_{P'}$ encompasses three terms of the five term acceleration formula. The last line in the calculation of $\vec{\boldsymbol{a}}_{\text{rel}}$ follows from the fact that at the instant of interest $\hat{\boldsymbol{\imath}}' = \hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}' = \hat{\boldsymbol{\jmath}}$.
Now adding the three parts of $\vec{\boldsymbol{a}}_P$ we get

$$\begin{aligned}
\vec{\boldsymbol{a}}_P &= \vec{\boldsymbol{a}}_{P'} + \vec{\boldsymbol{a}}_{\text{cor}} + \vec{\boldsymbol{a}}_{\text{rel}} \\
&= -17.57\,\text{m/s}^2 \hat{\boldsymbol{\imath}} - 8.43\,\text{m/s}^2 \hat{\boldsymbol{\jmath}}.
\end{aligned}$$

$$\vec{\boldsymbol{a}}_P = -(17.57\hat{\boldsymbol{\imath}} + 8.43\hat{\boldsymbol{\jmath}})\,\text{m/s}^2$$

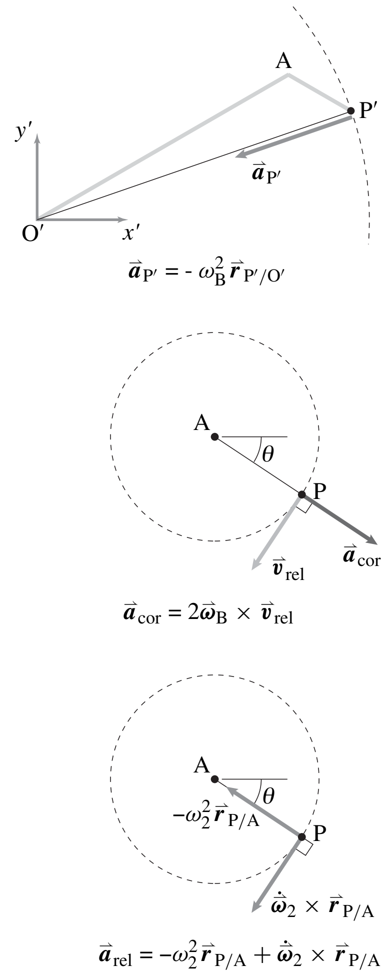


Figure 18.49