18.3 General expressions for velocity and acceleration

18.3.1 A bug walks on a straight line engraved on a rotating turntable (the bug's path in the room is not a straight line). The line passes through the center of the turntable. The bug's speed on this line is 1 in/s (the bug's absolute speed is *not* 1 in/s). The turntable rotates at a constant rate of 2 revolutions every π s in a positive sense about the z-axis. Its surface is always in the xy-plane. At time $t = 0$, the engraved line is aligned with the x-axis, the bug is at the origin and headed towards the positive x direction.

a) What is the x component of the bug's velocity at $t = \pi$ s?

b) What is the y component of the bug's velocity at $t = \pi$ seconds?

c) What is the y component of the bug's acceleration at $t = \pi$?

d) What is the radius of curvature of the bug's path at $t = 0$?

18.3.2 Actual path of bug trying to walk a straight line. A straight line is inscribed on a horizontal turntable. The line goes through the center. Let ϕ be the angle of rotation of the turntable which spins at constant rate $\dot{\phi}_0$. A bug starts on the outside edge of the turntable of radius R and walks towards the center, passes through it, and continues to the opposite edge of the turntable. The bug walks at a constant speed v_A, as measured by how far her feet move per step, on the line inscribed on the table. Ignore gravity.

a) **Picture.** Make an accurate drawing of the bug's path as seen in the room (which is not rotating with the turntable). In order to make this plot, you will need to assume values of v_A and $\dot{\phi}_0$ and initial values of R and ϕ. You will need to write a parametric equation for the path in terms of variables that you can plot (probably x and y coordinates). You will also need to pick a range of times. Your plot should include the instant at which the bug walks through the origin. Make sure your x and y-axes are drawn to the same scale. A computer plot would be nice.

b) Calculate the radius of curvature of the bug's path as it goes through the origin.

c) Accurately draw (say, on the computer) the osculating circle when the bug is at the origin on the picture you drew for (a) above.

d) **Force.** What is the force on the bug's feet from the turntable when she starts her trip? Draw this force as an arrow on your picture of the bug's path.

e) **Force.** What is the force on the bug's feet when she is in the middle of the turntable? Draw this force as an arrow on your picture of the bug's path.

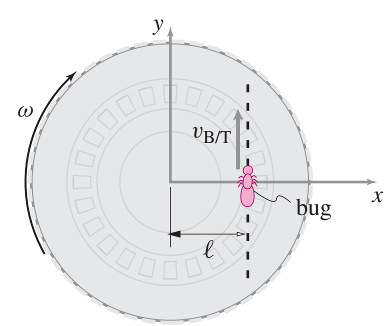
18.3.3 A small bug is crawling on a straight line scratched on an old record. The scratch is a distance $\ell = 6$ cm from the center of the turntable. The turntable is turning clockwise at a constant angular rate $\omega = 2$ rad/s. The bug is walking, relative to the turntable, at a constant rate $v_{B/T} = 12$ cm/s, straight along the scratch in the y-direction. At the instant of interest, everything is aligned as shown in the figure. The bug has a mass $m_B = 1$ gram.

a) What is the bug's velocity?

b) What is the bug's acceleration?

c) What is the sum of all forces acting on the bug?

d) Sketch the path of the bug in the neighborhood of its location at the time of interest (indicate the direction the bug is moving on this path).

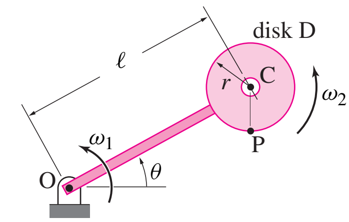


Problem 18.3.3

18.3.4 Arm OC rotates with constant rate ω_1. Disc D of radius r rotates about point C at constant rate ω_2 measured with respect to the arm OC. What are the absolute velocity and acceleration of point P on the disc, $\vec{v}_P$ and $\vec{a}_P$? (To do this problem will require defining a moving frame of reference. More than one choice is possible.) *

a) Pick a suitable moving frame and do the problem.

b) Pick another suitable moving frame and redo the problem. Make sure the answers are the same.

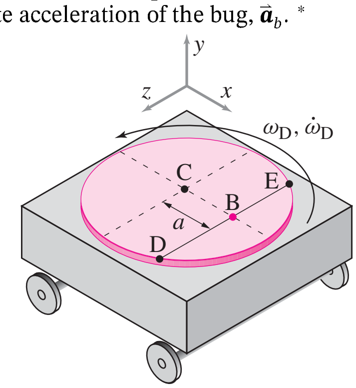


Problem 18.3.4

18.3.5 For the configuration in problem 18.3.4 what is the absolute angular velocity of the disk, D? *

18.3.6 For the configuration in problem 18.3.4, taking ω_2 to be the angular velocity of disk $\mathcal{D}$ relative to the rod, what is the absolute angular acceleration of the disk, D? What is the absolute angular acceleration of the disk if ω_1 and ω_2 are not constant?

18.3.7 A turntable oscillates with displacement $\vec{x}_C(t) = A\sin(st)\hat{\imath}$. The disc of the turntable rotates with angular speed and acceleration ω_D and $\dot{\omega}_D$. A small bug walks along line DE with velocity v_b relative to the turntable. At the instant shown, the turntable is at its maximum amplitude $x = A$, the line DE is currently aligned with the z-axis, and the bug is passing through point B on line DE. Point B is a distance a from the center of the turntable, point C. Find the absolute acceleration of the bug, $\vec{a}_b$. *



Problem 18.3.7

18.3.8 A small 0.1 kg toy train engine is going *clockwise* at a constant rate (relative to the track) of 2 m/s on a circular track of radius 1 m. The track itself is on a turntable $\mathcal{B}$ that is rotating *counterclockwise* at a constant rate of 1 rad/s. The dimensions are as shown. At the instant of interest the train is pointing due south ($-\mathbf{j}$) and is *at* the center O of the turntable.

a) What is the velocity of the train relative to the turntable $\vec{v}_{/\mathcal{B}}$?

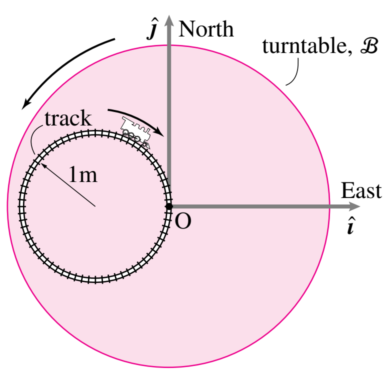
b) What is the absolute velocity of the train $\vec{v}$?

c) What is the acceleration of the train relative to the turntable $\vec{a}_{/\mathcal{B}}$?

d) What is the absolute acceleration of the train $\vec{a}$?

e) What is the total force acting on the train?

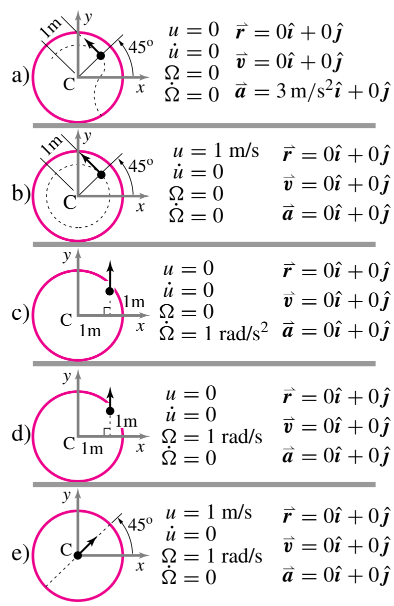
f) Sketch the path of the train for one revolution of the turntable (surprise)?



Problem 18.3.8

18.3.9 A giant bug walks on a horizontal disk. An xyz frame is attached to the disk. The disk is rotating about the z axis (out of the paper) and simultaneously translating with respect to an inertial frame XYZ plane. In each of the cases shown in the figure, determine the total force acting on the bug. In each case, the dotted line is scratched on the disk and is the path the bug follows walking in the direction of the arrow. The location of the bug is marked with a dot. At the instants shown, the xyz coordinate system shown is aligned with the inertial XYZ frame (which is not shown in the pictures). In this problem:

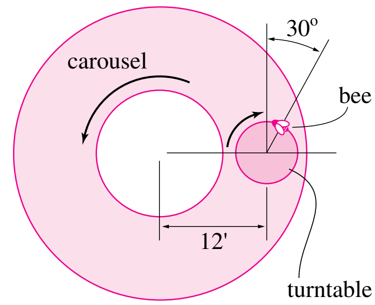
$\vec{r}$ = the position of the center of the disk,
$\vec{v}$ = the velocity of the center of the disk,
$\vec{a}$ = the acceleration of the center of the disk,
Ω the angular velocity of the disk (i.e., $\vec{\Omega} = \Omega\boldsymbol{k}$),
$\dot{\Omega}$ = the angular acceleration of the disk,
u = the speed of the bug traversing the dotted line (arc length on the disk per unit time),
$\dot{u} = du/dt$, and
m = 1 kg = mass of the bug.



Problem 18.3.9

18.3.10 Repeat questions (a)-(f) for the toy train in problem 18.3.8 going *counterclockwise* at constant rate (relative to the track) of 2 m/s on the circular track.

18.3.11 A nostalgic honeybee alights on a phonograph turntable that is being carried by a carnival goer who is riding on a carousel. The situation is sketched below. The carousel has angular velocity of 5 rpm, which is increasing (accelerating) at 10 rev/min^2; the phonograph rotates at a constant 33 1/3 rpm. The honeybee is at the outer edge of the phonograph record in the position shown in the figure; the radius of the record is 7 inches. Calculate the magnitude of the acceleration of the honeybee.



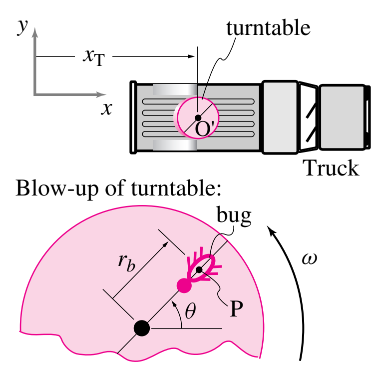
Problem 18.3.11

18.3.12 Consider a turntable on the back of a pick-up truck. A bug walks on a line on the turntable. The line may or may not be drawn through the center of the turntable. The truck may or may not be going at constant rate. The turntable may or may not be spinning, and, if it spins, it may or may not go at a constant rate. The bug may be anywhere on the line and may or may not be walking at a constant rate.

a) **Draw a picture of the situation.** Clearly define all variables you are going to use. What is the moving frame, and what is the reference point on that frame?

b) **One term at a time.** For each term in the five term acceleration formula, find a situation where all but one term is zero. Use the turntable as the moving frame, the bug as the particle of interest. *

c) **Two terms at a time.** (harder) How many situations can you find where a pair of terms is not zero but all other terms are zero? (Don't try to do all 10 cases unless you really think this infamous formula is fun. Try at least one or two.) *



Problem 18.3.12: **Picking apart the five term acceleration formula.**