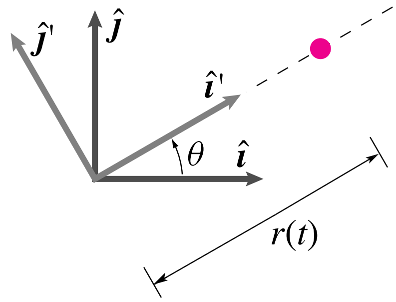
18.2.5 Given that $\vec{\boldsymbol{r}}(t) = ct^2\hat{\boldsymbol{\imath}}'$ and that $\theta(t) = d\sin(\lambda t)$, find $\vec{\boldsymbol{v}}(t)$

a) in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$,

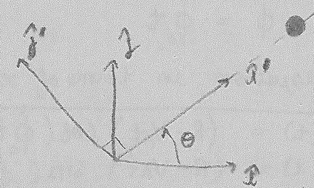
b) in terms of $\hat{\boldsymbol{\imath}}'$ and $\hat{\boldsymbol{\jmath}}'$.



Problem 18.5

2

Q15.15



b) $\vec{r}(t) = ct^2\,\hat{\imath}'$

$\vec{v}(t) = \dot{\vec{r}}(t) = 2ct\,\hat{\imath}' + ct^2\,\dot{\hat{\imath}}'$

$= 2ct\,\hat{\imath}' + ct^2\,\dot{\theta}\,\hat{\jmath}'$

$\boxed{\vec{V} = 2ct\,\hat{\imath}' + d\lambda ct^2\cos(\lambda t)\,\hat{\jmath}'}$ —— 1

a) now $\hat{\imath}' = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\jmath}' = -\sin\theta\,\hat{\imath} \quad \cos\theta\,\hat{\jmath}$

using these in above I get

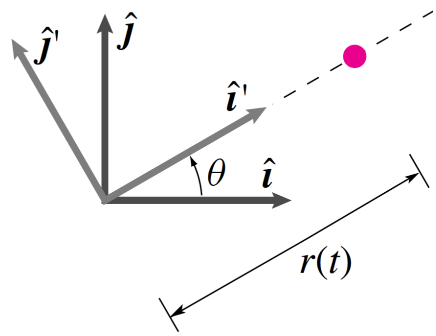
$\boxed{\vec{V} = (2ct\cos\theta - d\lambda ct^2\sin\theta)\,\hat{\imath} + (2ct\sin\theta + d\lambda ct^2\cos\theta)}$

$\boxed{\text{where } \theta = d\sin(\lambda t)}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.2.5

Friday, April 26, 2019 3:33 PM



Given $\vec{r}(t) = ct^2\,\hat{i}'$ find $\vec{v}(t)$ a) in $\hat{i}, \hat{j}$

$\theta(t) = d\sin(\lambda t)$ b) in $\hat{i}', \hat{j}'$

b) $\dot{\theta}(t) = \lambda d\cos(\lambda t)$

$\vec{r}(t) = ct^2\,\hat{i}'$

$\vec{v}(t) = \dot{\vec{r}}(t) = 2ct\,\hat{i}' + ct^2\,\dfrac{d\hat{i}'}{dt}$

$= 2ct\,\hat{i}' + ct^2\cdot\dot{\theta}\,\hat{j}'$

$= 2ct\,\hat{i}' + \lambda d\,ct^2\cdot\cos(\lambda t)\,\hat{j}'$

a) $\hat{i}' = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{j}' = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

substitute into part a)

$\vec{v}(t) = 2ct\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + \lambda d\,ct^2\cos(\lambda t)\,(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

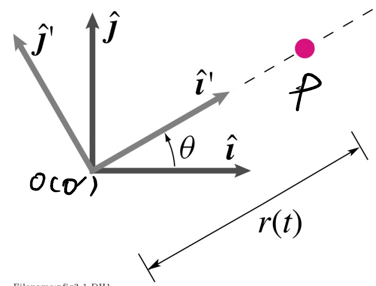
$= (2ct\cos\theta - \lambda d\,ct^2\cos(\lambda t)\sin\theta)\,\hat{i} + (2ct\sin\theta + \lambda d\,ct^2\cos(\lambda t)\cos\theta)\,\hat{j}$

18.2.5 Given that $\vec{r}(t) = ct^2\hat{\imath}'$ and that $\theta(t) = d\,\sin(\lambda t)$, find $\vec{v}(t)$

a) in terms of $\hat{\imath}$ and $\hat{\jmath}$,

b) in terms of $\hat{\imath}'$ and $\hat{\jmath}'$.

edited from Shuwen Lu
Katherine Cao



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Problem 18.2.5

Given $\vec{r}(t) = ct^2\hat{\imath}'$, particle P
$\theta(t) = d\sin(\lambda t)$
fix frame $\mathcal{F}$ with $\hat{\imath}, \hat{\jmath}$
rotating frame B with $\hat{\imath}', \hat{\jmath}'$
Find $\vec{v}(t)$

(a) in fix frame using $\hat{\imath}$ $\hat{\jmath}$

$$\vec{v}(t) = \dot{\vec{r}}(t) = \frac{d(ct^2\hat{\imath}')}{dt}$$
$$= 2ct\,\hat{\imath}' + ct^2\frac{d\hat{\imath}'}{dt}$$
$$= 2ct\,\hat{\imath}' + ct^2\dot{\theta}(t)\,\hat{\jmath}'$$
$$\dot{\theta}(t) = \lambda d\cos(\lambda t)$$
$$\Rightarrow \vec{v}(t) = 2ct\,\hat{\imath}' + \lambda dct^2\cos(\lambda t)\,\hat{\jmath}' \quad (*)$$

express $\hat{\imath}', \hat{\jmath}'$ usin $\hat{\imath}, \hat{\jmath}$

$$\hat{\imath}' = \cos(\theta(t))\,\hat{\imath} + \sin(\theta(t))\,\hat{\jmath}$$
$$\hat{\jmath}' = -\sin(\theta(t))\,\hat{\imath} + \cos(\theta(t))\,\hat{\jmath}$$

Substitute into (*)

$$\boxed{\begin{aligned}\vec{v}(t) = {}&\left[2ct\cos(\theta(t)) - \lambda dct^2\cos(\lambda t)\sin(\theta(t))\right]\hat{\imath}\\ &+ \left[2ct\sin(\theta(t)) + \lambda dct^2\cos(\lambda t)\cos(\theta(t))\right]\hat{\jmath}\end{aligned}}$$

(b) in fix frame using $\hat{\imath}'$ $\hat{\jmath}'$

already solved in (a) as (*)

$$\boxed{\vec{v}(t) = 2ct\,\hat{\imath}' + \lambda dct^2\cos(\lambda t)\,\hat{\jmath}'}$$

Note: (*) is not particle P's velocity relative to rotating frame B ($\vec{v}_{P/B}$)

$$\vec{v}_{P/B} = \frac{d(ct^2)}{dt}\,\hat{\imath}' = 2ct\,\hat{\imath}'$$