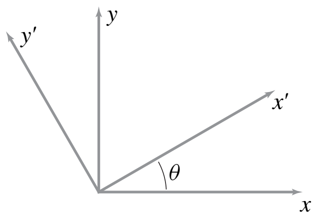
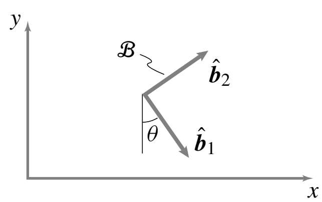
18.2 Rotating reference frames

18.2.1 Express the basis vectors $(\hat{\boldsymbol{\imath}}', \hat{\boldsymbol{\jmath}}')$ associated with axes x' and y' in terms of the standard basis $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$ for $\theta = 30^\circ$.



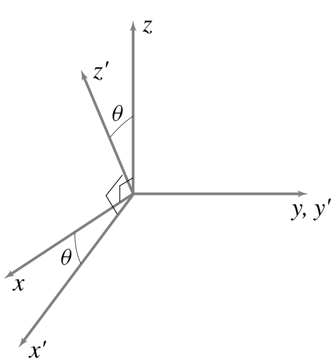
Problem 18.2.1

18.2.2 Body frames are frames of reference attached to a body in motion. The orientation of a coordinate system attached to a body frame $\mathcal{B}$ is shown in the figure at some instant of interest. For $\theta = 60^\circ$, express the basis vectors $(\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2)$ in terms of the standard basis $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}})$.



Problem 18.2.2

18.2.3 Find the components of (a) $\vec{\boldsymbol{v}} = 3\,\text{m/s}\hat{\boldsymbol{\imath}} + 2\,\text{m/s}\hat{\boldsymbol{\jmath}} + 4\,\text{m/s}\hat{\boldsymbol{k}}$ in the rotated basis $(\hat{\boldsymbol{\imath}}', \hat{\boldsymbol{\jmath}}', \hat{\boldsymbol{k}}')$ and (b) $\vec{\boldsymbol{a}} = -0.5\,\text{m/s}^2\hat{\boldsymbol{\jmath}}' + 3.0\,\text{m/s}^2\hat{\boldsymbol{k}}'$ in the standard basis $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}})$. ($y$ and y' are in the same direction and x' and z' are in the xz plane.)



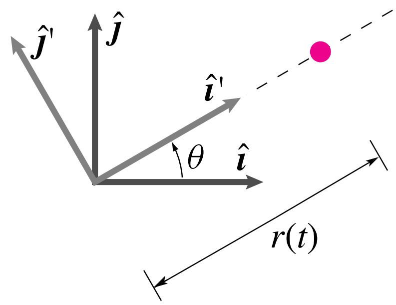
Problem 18.2.3

18.2.4 A particle travels in a straight line in the xy-plane parallel to the x-axis at a distance $y = \ell$ in the positive x direction. The position of the particle is denoted by $\vec{\boldsymbol{r}}(t)$. The angle of $\vec{\boldsymbol{r}}$ measured positive counter-clockwise from the x axis is decreasing at a constant rate with magnitude ω. If the particle starts on the y axis at $\theta = \pi/2$, what is $\vec{\boldsymbol{r}}(t)$ in cartesian coordinates?

18.2.5 Given that $\vec{\boldsymbol{r}}(t) = ct^2\hat{\boldsymbol{\imath}}'$ and that $\theta(t) = d\sin(\lambda t)$, find $\vec{\boldsymbol{v}}(t)$

a) in terms of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$,

b) in terms of $\hat{\boldsymbol{\imath}}'$ and $\hat{\boldsymbol{\jmath}}'$.



Problem 18.2.5

18.2.6 A bug walks on a turntable. In polar coordinates, the position of the bug is given by $\vec{\boldsymbol{R}} = R\hat{\boldsymbol{e}}_r$, where the origin of this coordinate system is at the center of the turntable. The $(\hat{\boldsymbol{e}}_r, \hat{\boldsymbol{e}}_\theta)$ coordinate system is attached to the turntable and, hence, rotates with the turntable. The kinematical quantities describing the bug's motion are $\dot{R} = -\dot{R}_0$, $\ddot{R} = 0$, $\dot{\theta} = \omega_0$, and $\ddot{\theta} = 0$. A fixed coordinate system Oxy has origin O at the center of the turntable. As the bug walks through the center of the turntable:

a) What is its speed?

b) What is its acceleration?

c) What is the radius of the osculating circle (i.e., what is the radius of curvature of the bug's path?).