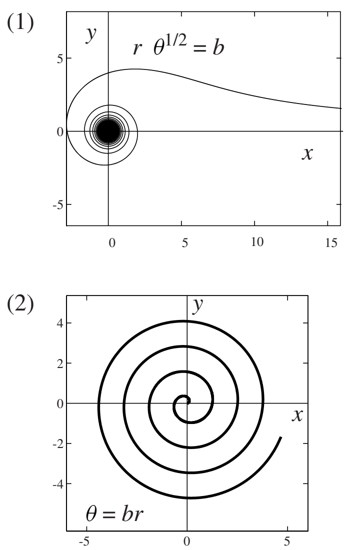
18.1 Polar coordinates and path coordinates

18.1.1 A particle moves along the two paths (1) and (2) as shown.

a) In each case, determine the velocity of the particle in terms of b, θ, and $\dot{\theta}$. *

b) Find the x and y coordinates of the path as functions of b and r or b and θ. *



Problem 18.1.1

18.1.2 For the particle path (1) in problem 18.1.1, find the acceleration of the particle in terms of b, θ, and $\dot{\theta}$.

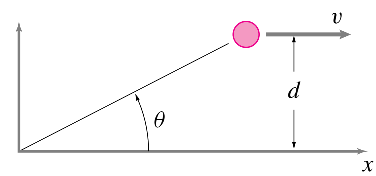
18.1.3 For the particle path (2) in problem 18.1.1, find the acceleration of the particle in terms of b, θ, and $\dot{\theta}$.

18.1.4 A body moves with constant velocity V in a straight line parallel to and at a distance d from the x-axis.

a) Calculate $\dot{\theta}$ in terms of V, d, and θ.

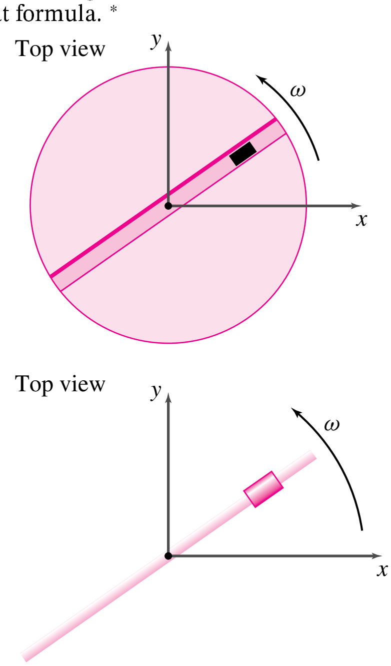
b) Calculate the $\hat{\boldsymbol{e}}_\theta$ component of acceleration.

Carefully define, using a sketch and/or words, any variables, coordinate systems, and reference frames you use. Express your answers using any convenient coordinate system (just make sure its orientation has been clearly defined).



Problem 18.1.4

18.1.5 Picking apart the polar coordinate formula for velocity. This problem concerns a small mass m that sits in a slot in a turntable. Alternatively you can think of a small bead that slides on a rod. The mass always stays in the slot (or on the rod). Assume the mass is a little bug that can walk as it pleases on the rod (or in the slot) and you control how the turntable/rod rotates. Name two situations in which one of the terms is zero but the other is not in the two term polar coordinate formula for velocity, $\dot{R}\hat{\boldsymbol{e}}_R + R\dot{\theta}\hat{\boldsymbol{e}}_\theta$. You should thus gain some insight into the meaning of each of the two terms in that formula. *



Problem 18.1.5

18.1.6 Picking apart the polar coordinate formula for acceleration. Reconsider the configurations in problem 18.1.5. This time, name four situations in which all of the terms, but one, in the four term polar coordinate formula for acceleration, $\vec{\boldsymbol{a}} = (\ddot{R} - R\dot{\theta}^2)\hat{\boldsymbol{e}}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\boldsymbol{e}}_\theta$, are zero. Each situation should pick out a different term. You should thus gain some insight into the meaning of each of the four terms in that formula. *

18.1.7 The two differential equations:

$$\begin{aligned} \ddot{R} - R\dot{\theta}^2 &= 0 \\ 2\dot{R}\dot{\theta} + R\ddot{\theta} &= 0 \end{aligned}$$

have the general solution

$$\begin{aligned} R &= \sqrt{d^2 + (v(t - t_0))^2} \\ \theta &= \theta_0 + \tan^{-1}(v(t - t_0)/d) \end{aligned}$$

where θ_0, d, t_0, and v are arbitrary constants. This solution could be checked by plugging back into the differential equations — you need not do this (tedious) substitution. The solution describes a curve in the plane. That is, if for a range of values of t, the values of R and θ were calculated and then plotted using polar coordinates, a curve would be drawn. *What can you say about the shape of this curve?*

[Hints:

a) Actually make a plot using some random values of the constants and see what the plot looks like.

b) Write the equation $\mathbf{F} = m\mathbf{a}$ for a particle in polar coordinates and think of a force that would be relevant to this problem.

c) The answer is something simple.]

18.1.8 A car driver on a very boring highway is carefully monitoring her speed. Over a one hour period, the car travels on a curve with constant radius of curvature, $\rho = 25$ mi, and its speed increases uniformly from 50 mph to 60 mph. What is the acceleration of the car half-way through this one hour period, in path coordinates?

18.1.9 Find expressions for $\hat{\boldsymbol{e}}_t$, a_t, a_n, $\hat{\boldsymbol{e}}_n$, and the radius of curvature ρ, at any position (or time) on the given particle paths for

a) problem 12.1.11, *

b) problem 12.1.12,

c) problem 18.2.4,

d) problem 12.1.14,

e) problem 12.1.13, and

f) problem 12.1.10.

18.1.10 A particle travels at non-constant speed on an elliptical path given by $y^2 = b^2(1 - \frac{x^2}{a^2})$. Carefully sketch the ellipse for particular values of a and b. For various positions of the particle on the path, sketch the position vector $\vec{\boldsymbol{r}}(t)$; the polar coordinate basis vectors $\hat{\boldsymbol{e}}_r$ and $\hat{\boldsymbol{e}}_\theta$; and the path coordinate basis vectors $\hat{\boldsymbol{e}}_n$ and $\hat{\boldsymbol{e}}_t$. At what points on the path are $\hat{\boldsymbol{e}}_r$ and $\hat{\boldsymbol{e}}_n$ parallel(or $\hat{\boldsymbol{e}}_\theta$ and $\hat{\boldsymbol{e}}_t$ parallel)?