CHAPTER 18

Kinematics using time-varying basis vectors

Here is a second approach to the kinematics of particle motion. Now, instead of using constant base vectors, we use time-varying base vectors. The discussion of polar coordinates started in Chapter 15 is completed here. Path coordinates, where one base vector is parallel to the velocity and the others orthogonal to that, are introduced. The challenging kinematics topic of relative motion is introduced in two stages: first using rotating base vectors connected to a moving rigid object and then using the more abstract notation associated with frame-dependent differentiation and the famous "five term acceleration formula."

Many parts of practical machines and structures move in ways that can be idealized as straight-line motion (Chapter 14) or circular motion (Chapter 15). But often an engineer must analyze parts with more general motions as we began to study in Chapter 17.

In principle one can study all motions of all things using one fixed, say xyz, coordinate system. If one knows the x, y, and z coordinates of all points at all times then one can evaluate the linear momentum, the angular momentum, and their rates of change. In this way one can do all of mechanics. But when a machine has various parts, each moving relative to the other, it turns out it is helpful to make use of additional base vectors besides those fixed to a Newtonian ("fixed") reference frame. That is, the formulas for velocity and acceleration are in some senses simplified (or clarified) by using moving base vectors. Most often these moving base vectors move with the moving parts.

You have seen the time-varying base vectors $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$, the polar coordinate base vectors used to describe circular motion. These are the ideas on which we build here. Altogether we discuss 4 approaches that use time varying base vectors:

1. Polar coordinates are generalized to include more than just circular motion;
2. Path coordinates and path base vectors;
3. General rotating base vectors and coordinate systems with an origin that moves are introduced; and
4. Formulas are presented for differentiation 'in' moving frames that don't depend on any particular choice of base vectors.

The basic idea is to try to use coordinate systems that most simply describe the motions of interest, even if these coordinate systems are superficially confusing because they rotate and move. Time-varying base vectors are difficult at first. Like many shortcuts, they have a cost. In the end, however, they can aid intuition and simplify calculations.