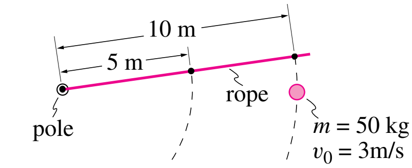
17.5.3 A narrow pole is in the middle of a pond with a 10 m rope tied to it. A frictionless ice skater of mass 50 kg and speed 3 m/s grabs the rope. The rope slowly wraps around the pole. What is the speed of the skater when the rope is 5 m long? (A tricky question.)



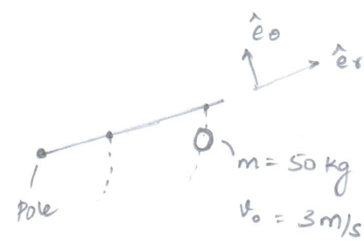
Problem 17.3

Angular momentum about 'O' is conserved

$H_o = \vec{r} \times m\vec{V}$

$= r\hat{e}_r \times mV_o\hat{e}_\theta$

$= mrV_o\hat{K}$



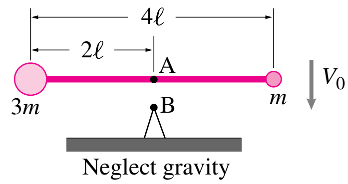
$H_o^+ = \vec{r} \times m\vec{V} = mrV_o$

$\frac{mr}{2} V\hat{K} = rmV_o\hat{K}$

$V = 2V_o$

Speed of skater at 5 m distance is 6 m/s.

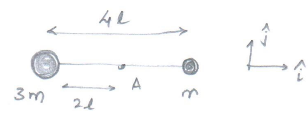
17.5.4 The masses m and $3m$ are joined by a light-weight bar of length 4ℓ. If point A in the center of the bar strikes fixed point B vertically with velocity V_0, and is not permitted to rebound, find $\dot{\theta}$ of the system immediately after impact.



Problem 17.4: Neglect gravity!

16.5.4

Before collision



$L = 4mV_0$

$\vec{H}^- = \vec{H}^+$

$\vec{H}^+ = \vec{r} \times m v^+ + I^{cm} \omega^+$

$\vec{H}^+ = 2l\,\hat{i} \times m v\,\hat{j} + 2l(-\hat{i}) \times (3m) v^+ \hat{j}$

$= 2l\,\hat{i} \times m(2l\omega)\hat{j} + 2l(-\hat{i}) \times 3m(-2l\omega\hat{j})$

$= 4l^2 m\omega\,\hat{k} + 12ml^2\omega\,\hat{k}$

$\vec{H}^- = 16ml^2\omega\,\hat{k}$

$V_m = \omega\hat{k} \times 2l\,\hat{i}$

$V_M = \omega\hat{k} \times 2l(-\hat{i})$

$V_m = 2l\omega\,\hat{j}$

$V_M = -2l\omega\,\hat{j}$

$H^-_{/A} = I^{cm}\omega^-\hat{k} + r \times m v^-$

$= 2l\,\hat{i} \times m\,v_0(-\hat{j}) + (-2l)\,\hat{i} \times 3m(-v_0\hat{j})$

$= -2mlv_0\hat{k} + 6mlv_0\hat{k}$

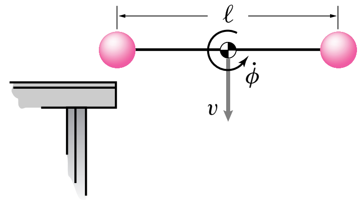
$\therefore \quad 4mlv_0\hat{k} = 16ml^2\omega\,\hat{k}$

$\omega = \dfrac{v_0}{4l}$

$\vec{\omega} = \dfrac{v_0}{4l}\hat{k}$

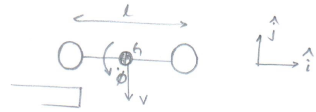
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.5.5 Two equal masses each of mass m are joined by a massless rigid rod of length ℓ. The assembly strikes the edge of a table as shown in the figure, when the center of mass is moving downward with a linear velocity v and the system is rotating with angular velocity $\dot\phi$ in the counter-clockwise sense. The impact is 'elastic'. Find the immediate subsequent motion of the system, assuming that no energy is lost during the impact and that there is no gravity. Show that there is an interchange of translational and rotational kinetic energy.

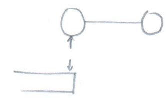


Problem 17.5

16.5.5



$\vec H^- = \vec H^+$



$I^{cm}\dot\phi\hat K + \frac{\ell}{2}\hat\imath \times mv.-\hat\jmath + \frac{\ell}{2}(-\hat\imath)\times mv(-\hat\jmath)$

$\vec H^- = I^{cm}\dot\phi\hat K$

$m(\underline{v}_i^+ + (v+\frac{\dot\phi\ell}{2})\hat\jmath) = P\hat\jmath$

$I^{cm}(\omega^+ - \omega^-) = r_{/G}\times P\hat\jmath$

$2m(\frac{\ell}{2})^2[\omega^+ - \dot\phi\hat K] = \frac{\ell}{2}(-\hat\imath)\times P\hat\jmath$

$\frac{m\ell^2}{2}(\omega^+ - \dot\phi\hat K) = \frac{P\ell}{2}\,{-\hat K} = -2m(v+\frac{\dot\phi\ell}{2})\frac{\ell}{2}\hat K$

$\omega^+ - \dot\phi\hat K = -\frac{(2v+\dot\phi\ell)}{\ell}\hat K$

$\underline{\omega}^+ = -\frac{2v}{\ell}\hat K$

$m(\underline{v}_i^+ - \underline{v}_i^-) = P_j$

$v_i^- = v_{cm} + \dot\phi\hat K\times\frac{\ell}{2}(-\hat\imath)$

$= v(-\hat\imath) - \frac{\dot\phi\ell}{2}\hat\jmath = -(v+\frac{\dot\phi\ell}{2})\hat\jmath$

$\{(\underline{v}_t^+ - \underline{v}_i^+) = -e(\underline{v}_t^- - \underline{v}_i^-)\}\cdot\hat\jmath$

$(0 - v_i^+\cdot\hat\jmath) = -[0 - v_i^-\cdot\hat\jmath]$

$-v_i^+\hat\jmath = v_i^-\cdot\hat\jmath = -(v+\frac{\dot\phi\ell}{2})\hat\jmath$

$\therefore \underline{v}_i^+ = (v+\frac{\dot\phi\ell}{2})\hat\jmath$

$KE^{rotation\,+} = \frac12 I\omega^{+2}$

$= \frac12\frac{m\ell^2}{2}\cdot\frac{4v^2}{\ell^2} = \frac12 mv^2 + \frac12 mv^2 =$ K.E Translation$^-$

$KE^- = \frac12 m(v+\frac{\dot\phi\ell}{2})^2 + \frac12 m(v-\frac{\dot\phi\ell}{2})^2$

$= \frac12 mv^2 + \frac12 mv^2 + \frac12 m(\frac{\dot\phi\ell}{2})^2 + \frac12 m(\frac{\dot\phi\ell}{2})^2$

$=$ K.E Trans$^-$ $+ \frac{m\ell^2}{4}\dot\phi^2$

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$$= K.E^{Trans^-} + \frac{1}{2}\left(\frac{ml^2}{2}\right)\dot{\phi}^2$$

$$= K.E^{Trans^-} + K.E^{rot^-}$$

$K.E^{Trans^+}$

$$V_2^+ = v_1^+ + (\underline{\omega} \times l\hat{i})$$

$$v_2^+ = \left(v + \frac{\dot{\phi}l}{2}\right)\hat{j} + \left[\frac{-2v}{l}\hat{k} \times l\hat{i}\right]$$

$$= \left(v + \dot{\phi}l/2\right)\hat{j} - 2v\hat{j}$$

$$\underline{v}_2^+ = \left(-v + \frac{\dot{\phi}l}{2}\right)\hat{j}$$

$$K.E^+ = \frac{1}{2}m\left[\left(v + \frac{\dot{\phi}l}{2}\right)\hat{j}\right]^2 + \frac{1}{2}m\left[\left(-v + \dot{\phi}l/_2\right)\hat{j}\right]$$

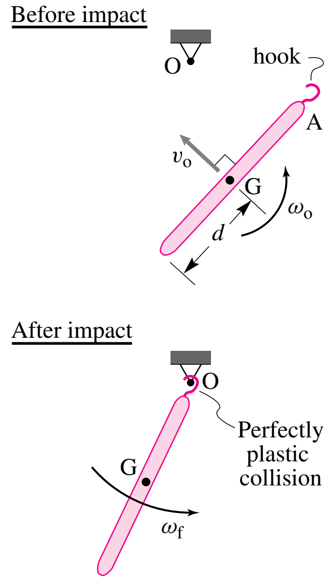
$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m\dot{\phi}^2\frac{l^2}{2}$$

17.5.6 In the *absence of gravity*, a thin rod of mass m and length ℓ is initially tumbling with constant angular speed ω_o, in the counter-clockwise direction, while its mass center has constant speed v_o, directed as shown below. The end A then makes a perfectly plastic collision with a rigid peg O (via a hook). The velocity of the mass center happens to be perpendicular to the rod just before impact.

a) What is the angular speed ω_f immediately after impact?

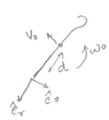
b) What is the angular speed 10 seconds after impact? Why?

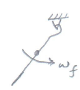
c) What is the loss in energy in the plastic collision?



Problem 17.6

16.5.6





$$\underline{H}^- = \underline{H}^+$$

$$r \times m v_{cm} + I^{cm} \underline{\omega}^- \hat{k} = I^{cm} \underline{\omega}^+ \hat{k} + r_{cm} \times m v_{cm}$$

$$d\hat{e}_r \times m v_o(-\hat{e}_\theta) + \frac{m\ell^2}{12}\omega_o \hat{k} = \frac{m\ell^2}{12}\omega^+ \hat{k} + d\hat{e}_r \times m(\underline{\omega}^+ \times d\hat{e}_r)$$

$$\Rightarrow \{\,\} \cdot \hat{k}$$

$$-m d v_o \hat{k} + \frac{m\ell^2}{12}\omega_o \hat{k} = \frac{m\ell^2}{12}\omega^+ \hat{k} + d\hat{e}_r \times m(d\omega^+ \hat{e}_\theta) = \frac{m\ell^2}{12}\omega^+ \hat{k} + m d^2 \omega^+ \hat{k}$$

$$\frac{m\ell^2}{12}\omega_o - m d v_o = \left(\frac{m\ell^2}{12} + \frac{m\ell^2}{4}\right)\omega^+ = \frac{1}{2} m\ell^2 \omega^+$$

$$\omega^+ = \frac{3}{\ell^2}\left[\frac{\ell^2}{12}\omega_o - \frac{\ell}{2} v_o\right]$$

$$\omega^+ = \frac{1}{4}\left(\omega_o - \frac{3v_o}{d}\right)$$

$$\omega^+_{10s} = \omega^+ = \text{constant}$$

$$KE^+ = \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{1}{3} m\ell^2\right)\left[\frac{1}{4}\left(\omega_o - \frac{3v_o}{d}\right)\right]^2$$

$$KE^- = \frac{1}{2} m v_o^2 + \frac{1}{2} I \omega_o^2 = \frac{1}{2} m v_o^2 + \frac{1}{2}\left(\frac{1}{12} m\ell^2\right)\omega_o^2$$

$$E_{lost} = KE^- - KE^+$$

$$= \frac{1}{2} m v_o^2 + \frac{m\ell^2\omega_o^2}{32} - \frac{3 m v_o^2}{8} + \frac{m}{8}\ell\omega_o v_o$$

$$= \frac{1}{8} m v_o^2 + \frac{m\ell^2}{32}\omega_o^2 + \frac{m\ell\omega_o v_o}{8} = \frac{m}{8}\left(v_o^2 + \frac{(\ell\omega_o)^2}{4} + 2\frac{(\ell\omega_o)}{2} v_o\right)$$

$$= \frac{m}{8}\left(v_o + \frac{\ell\omega_o}{2}\right)^2$$

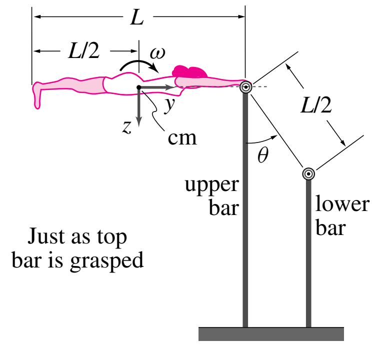
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.5.7 A gymnast of mass m and extended height L is performing on the uneven parallel bars. About the x, y, z axes which pass through her center of mass, her radii of gyration are k_x, k_y, and k_z, respectively. Just before she grasps the top bar, her fully extended body is horizontal and rotating with angular rate ω; her center of mass is then stationary. Neglect any friction between the bar and her hands and assume that she remains rigid throughout the entire stunt.

a) What is the gymnast's rotation rate just *after* she grasps the bar? State clearly any approximations/assumptions that you make.

b) Calculate the linear speed with which her hips (CM) strike the lower bar. State all assumptions/approximations.

c) Describe (in words, no equations please) her motion immediately after her hips strike the lower bar if she releases her hands just prior to this impact.



Problem 17.7

16.5.7

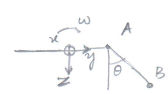
Angular momentum is conserved during impact.

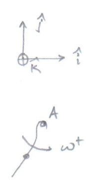
$\therefore \underline{H}^-_{/A} = \underline{H}^+_{/A}$

$$\underline{H}^-_{/A} = \frac{L}{2}(-\hat{i}) \times m\left[-\omega\hat{k} \times \frac{L}{2}(-\hat{i})\right] + I^{cm}(-\omega)\hat{k}$$

$$= \frac{mL^2}{4}\omega(-\hat{i}) \times (\hat{j}) - m k_x^2 \omega \hat{k}$$

$$= -\left(\frac{mL^2}{4}\omega + m k_x^2 \omega\right)\hat{k}$$





$$\underline{H}^+_{A} = I^A_{zz}\,\omega^+\hat{k} = \left[m k_x^2 + m\left(\frac{L}{2}\right)^2\right]\omega^+\hat{k}$$

$$\underline{\omega}^+ = -\omega\hat{k} \Rightarrow \omega^+ = -\omega$$

b)

$$\underline{v}_{cm} = (\underline{\omega} \times \underline{r})$$

$$= \left(\omega^+\hat{k} \times \frac{L}{2}\hat{\lambda}\right)$$

$$= \left(\frac{\omega^+ L}{2}\right)\hat{n}$$

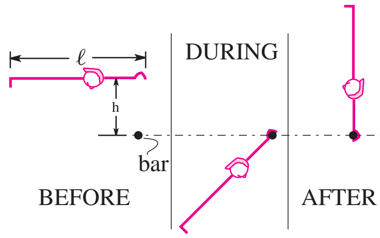
$$\hat{n} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$v_{cm} = \left(\frac{\omega^+ L}{2}\right)\cos\theta\,\hat{i} + \left(\frac{\omega^+ L}{2}\right)\sin\theta\,\hat{j}$$

c) If the hands are released prior to the impact between the centre of mass and the lower bar, gymnast swings about her c.m as she initially has ω+ rotation rate.

Disclaimer: **We have lost track of the many people who wrote these solutions.** *They are not official and have not been reviewed by the authors for correctness.*

17.5.8 An acrobat modeled as a rigid body with uniform rigid mass m of length l. She falls without rotation in the position shown from height h where she was stationary. She then grabs a bar with a firm but slippery grip. What is h so that after the subsequent motion the acrobat ends up in a stationary handstand? [Hint: What quantities are preserved in what parts of the motion?]



Problem 17.8

Answer:
$h = 2L/3$

7.54 Height h from which an acrobat must fall in order to execute a hand stand from a parallel bar? Model acrobat as bar

L
ACROBAT
h
mass, m
B, Bar

Energy Conserved in Fall

$$E_{P_1} + E_{K_1} = E_{P_2} + E_{K_2}$$

energy before fall = energy before impact

$$mgh = \frac{1}{2} m v_0^2$$

no rotational k.E.

Vel. of CM before impact $\boxed{V_0 = \sqrt{2gh}}$

$\underline{V}_0 = \sqrt{2gh}\ \hat{J}$

G
B, Bar
large impulsive force from Bar at impact

Angular Momentum Conserved About B through Impact

$$\underline{H}_{0/B} = \underline{H}_{1/B}$$

$$\left\{ \underline{r}_{G/B} \times m \underline{V}_0 + [I^{cm}] \cancel{\underline{\omega}_0} = \underline{r}_{G/B} \times m \underline{V}_1 + [I^{cm}] \underline{\omega}_1 \right\} \cdot \hat{k}$$

$$\frac{L}{2} m V_0 + 0 = \frac{L}{2} m \omega_1 \frac{L}{2} + \frac{mL^2}{12} \omega_1$$

where $\underline{V}_1 = \underline{\omega} \times \underline{r}_{G/B}$

$$\omega_1 = \frac{V_0 L}{2}\left(\frac{3}{L^2}\right) = \frac{3V_0}{2L} = \boxed{\frac{3\sqrt{2gh}}{2L} = \omega_1}$$

ang vel. of acrobat after impact

Energy Conserved After Impact Through to Handstand

G
B
ω_1

$$\cancel{E_{P_3}}^{0} + E_{K_3} = E_{P_4} + \cancel{E_{K_4}}^{0}$$

$$0 + \frac{1}{2} m V_1^2 + \frac{1}{2} I_{zz}^{cm} \omega_1^2 = mg\frac{L}{2} + 0$$

or $\frac{1}{2} I_{zz}^{B} \omega_1^2 = mg\frac{L}{2}$ $\qquad I_{zz}^{B} = \frac{mL^2}{3}$

$$\omega_1 = \sqrt{\frac{3g}{L}}$$

$$\frac{9}{4}\frac{2gh}{L^2} = \frac{3g}{L}$$

$$\boxed{h = \frac{2}{3}L}$$

G
B

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.5.8

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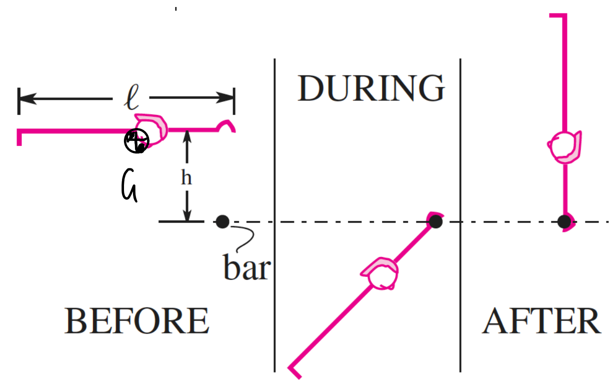

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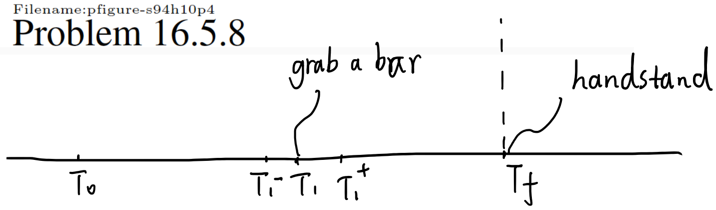
Problem 16.5.8



Given: m, l

Find: h, so that she ends up in a handstand

Before: dropping without rotation.

$T_0 \sim T_1^-$ energy conserves

$$mgh = \frac{1}{2} m v_1^2$$

Collision: $T_1^- \sim T_1^+$ $\vec{\omega} = \omega \hat{k}$

energy does not conserve

since she 'collides' with the bar

but AMB/A conserves. $\vec{H}_{/k}^- = \vec{H}_{/k}^+$ (*)

@ T_1^- : $\vec{H}_{/k}^- = m v_1^- \frac{l}{2} \hat{k}$ (Just before impact)

@ T_1^+ : $\vec{H}_{/k}^+ = I_G \omega^+ \hat{k} + \vec{r}_{G/A} \times m \vec{v}_G^+$ (Just after impact)

$\vec{V}_G^+ = \vec{\omega}^+ \times \underbrace{\vec{r}_{G/A}}_{-\frac{l}{2}\hat{i}}$ plug into above and note *

$\Rightarrow \quad \omega = \frac{6}{7}\frac{\sqrt{2gh}}{l}$ at T_1^+

During : Since the bar is slippery,
$(T_1 \sim T_f)$ energy conserves / no friction

$E_i = E_1^+ = E_f$

$\frac{1}{2} I_G \omega^2 = mg \cdot \frac{l}{2}$

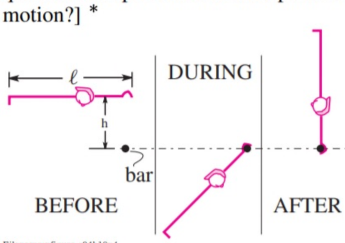
$\Rightarrow \frac{ml^2}{3}\left(\frac{6}{7}\frac{\sqrt{2gh}}{l}\right)^2 = mgl$

$\Rightarrow \quad h = \frac{49}{24} l$

HW 11 P61 17.5.8

Teaya Yang

17.5.8 An acrobat modeled as a rigid body with uniform rigid mass m of length l. She falls without rotation in the position shown from height h where she was stationary. She then grabs a bar with a firm but slippery grip. What is h so that after the subsequent motion the acrobat ends up in a stationary handstand? [Hint: What quantities are preserved in what parts of the motion?] *



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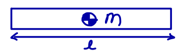
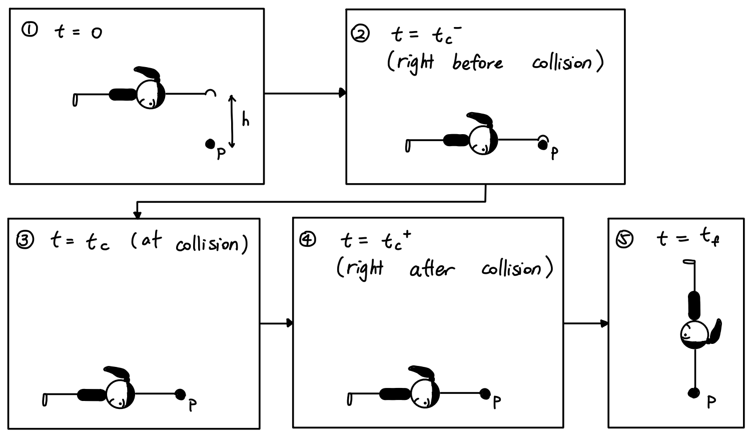
Problem 17.5.8

Given: Person modeled as uniform rigid rod.

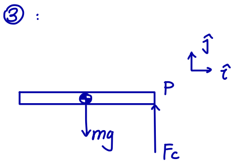


Find: h to end in a handstand.

Snapshots in time:



FBD at ③:



From assumptions for 2D collision (see P894 in Ruina & Pratap):

I. Collision forces are big ($F_c >> mg$)

II. Collisions are quick ($\Delta t << 1$)

Using AMB at ③:

$$\Sigma \vec{M}_{/P} = \dot{\vec{H}}_{/P}$$

$$\Sigma \vec{M}_{/P} = -\frac{\ell}{2}\hat{\imath} \times -mg\hat{\jmath} = \frac{\ell}{2}mg\hat{k} = \dot{\vec{H}}_{/P}$$

But if we integrate from ② to ④:

$$\Delta\vec{H}_{/P} = \int_{t_c^-}^{t_c^+} \dot{\vec{H}}_{/P}\, dt = \dot{\vec{H}}_{/P}\,(\cancel{t_c^+ - t_c^-})^{0}$$ from assumption II.

$\Rightarrow \Delta\vec{H}_{/P} = 0$, $\vec{H}_{/P}$ is conserved during the collision.

Seperating motion into 3 parts:

① → ② : Energy is conserved ; bar in free fall

② → ④ : Angular momentum about point P is conserved ; $\vec{H}_{/P}^{\,-} = \vec{H}_{/P}^{\,+}$

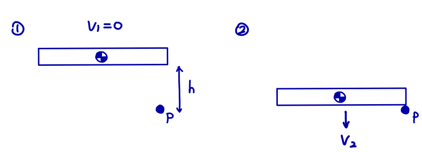
④ → ⑤ : Energy is conserved ; pendulum swinging about point P.

① → ② :

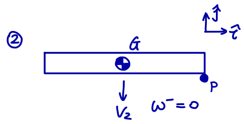
$$E_1 = E_2$$

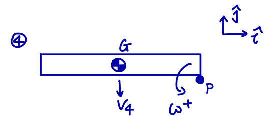
$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2$$

$$v_2 = \sqrt{2gh}$$



② → ④ : $\vec{H}_{/P}^{\,-} = \vec{H}_{/P}^{\,+}$



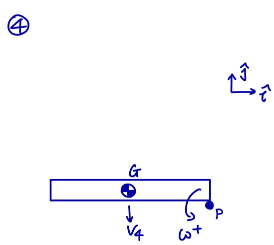


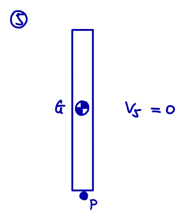
$$\vec{H}_{/P}^{\,-} = \vec{r}_{G/P} \times m\vec{v}_2 + I^G \cancel{\omega^-}^{\,0}\,\hat{k}$$
$$= -\tfrac{\ell}{2}\hat{\imath} \times m(-v_2\hat{\jmath})$$
$$= \tfrac{\ell}{2} m v_2 \hat{k}$$
$$\vec{H}_{/P}^{\,-} = \tfrac{\ell}{2} m\sqrt{2gh}\ \hat{k}$$

$$\vec{H}_{/P}^{\,+} = \vec{r}_{G/P} \times m\vec{v}_4 + I^G \omega^+ \hat{k}$$
$$= -\tfrac{\ell}{2}\hat{\imath} \times m(\omega^+\hat{k} \times \vec{r}_{G/P}) + I^G\omega^+\hat{k}$$
$$= -\tfrac{\ell}{2}\hat{\imath} \times (-m\omega^+\tfrac{\ell}{2}\hat{\jmath}) + I^G\omega^+\hat{k} \qquad I^G = \tfrac{1}{12} m\ell^2$$
$$\vec{H}_{P}^{\,+} = \tfrac{1}{3} m\ell^2\omega^+\hat{k}$$

$$\{\vec{H}_{/P}^{\,-} = \vec{H}_{/P}^{\,+}\}\cdot\hat{k} : \quad \tfrac{\cancel{\ell}}{2}\cancel{m}\sqrt{2gh} = \tfrac{1}{3}\cancel{m}\ell^{\cancel{2}}\omega^+$$
$$\tfrac{3}{2}\sqrt{2gh} = \ell\omega^+$$
$$\omega^+ = \tfrac{3}{2}\frac{\sqrt{2gh}}{\ell}$$

④ → ⑤ :

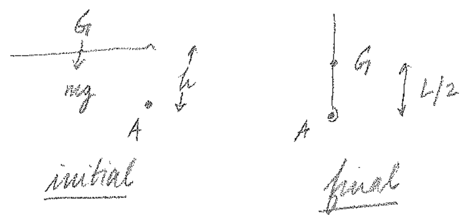




$E_4 = E_5$:

$$\tfrac{1}{2} I^G(\omega^+)^2 + \tfrac{1}{2} m v_4^2 = mg\tfrac{\ell}{2} \qquad \vec{v}_4 = -\omega^+\tfrac{\ell}{2}\hat{\jmath}$$
$$\tfrac{1}{2}\left(\tfrac{9}{2}\tfrac{\cancel{g}h}{\cancel{\ell^2}}\right)\left(\tfrac{1}{12}\cancel{m}\cancel{\ell^2}\right) + \tfrac{1}{2}\cancel{m}\left(\tfrac{9}{8}\cancel{g}h\right) = \cancel{m}\cancel{g}\tfrac{\ell}{2} \qquad = -\tfrac{3}{4}\sqrt{2gh}\,\hat{\jmath}$$
$$\tfrac{9}{48}h + \tfrac{9}{16}h = \tfrac{\ell}{2}$$
$$\tfrac{3}{4}h = \tfrac{\ell}{2}$$
$$\boxed{h = \tfrac{2}{3}\ell}$$

14.65



* NOTE: Cannot simply use conservation of energy because there is loss upon impact. However, angular momentum is conserved during impact.

$$\vec{H}^{-}_{/A} = m\sqrt{2gh}\,\frac{L}{2}\,\hat{k} \qquad \text{(just before impact)}$$

$$\vec{H}^{+}_{/A} = I_A^{zz}\,\omega_1\,\hat{k} \qquad \text{(just after impact)}$$

$$= \frac{mL^2}{3}\,\omega_1\,\hat{k}$$

$$\Rightarrow \quad \text{(just after impact)} \quad \omega_1 = \frac{3}{2}\frac{\sqrt{2gh}}{L}$$

Since the pivot is "slippery", energy is conserved during the swing:

$$E_i = E_f$$

$$\frac{1}{2} I_A^{zz}\omega_1^2 = mg\frac{L}{2}$$

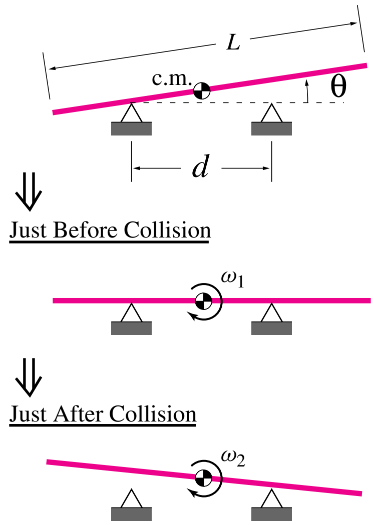
$$\Rightarrow \quad \frac{1}{2}\,\frac{mL^2}{3}\left(\frac{3}{2}\frac{\sqrt{2gh}}{L}\right)^2 = \frac{mgL}{2}$$

$$\boxed{h = \frac{2}{3}L}$$

17.5.9 A crude see-saw is built with two supports separated by distance d about which a rigid plank (mass m, length L) can pivot smoothly. The plank is placed symmetrically, so that its center of mass is midway between the supports when the plank is at rest.

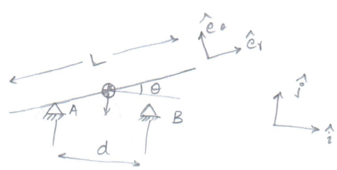
a) While the left end is resting on the left support, the right end of the plank is lifted to an angle θ and released. At what angular velocity ω_1 will the plank strike the right hand support?

b) Following the impact, the left end of the plank can pivot purely about the right end if d/L is properly chosen and the right end does not bounce. Find ω_2 under these circumstances.



Problem 17.9

16.5.9



$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$\sum \underline{M}_{/A} = \frac{d}{2}\hat{e}_r \times mg(-\hat{j})$$

$$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\sum \underline{M}_{/A} = -\frac{mgd}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) \times \hat{\jmath}$$

$$\sum \underline{M}_{/A} = -\frac{mgd}{2}\cos\theta\,\hat{k}$$

$$\dot{\underline{H}}_{/A} = I^{cm}\ddot{\theta}\hat{k} + \underline{r} \times m\underline{a}^{cm}$$

$$= \frac{ml^2}{12}\ddot{\theta}\hat{k} + \frac{d}{2}\hat{e}_r \times \left[\ddot{\theta}\frac{d}{2}\hat{e}_\theta - \dot{\theta}^2\frac{d}{2}\hat{e}_r\right]\cdot m$$

$$= \frac{ml^2}{12}\ddot{\theta}\hat{k} + \frac{md^2}{4}\hat{k}\ddot{\theta}$$

$$\underline{a}^{cm} = (\dot{\underline{\omega}} \times \underline{r}) + [\underline{\omega} \times (\underline{\omega} \times \underline{r})]$$

$$= \ddot{\theta}\hat{k} \times \frac{d}{2}\hat{e}_r + \left(-\dot{\theta}^2\frac{d}{2}\hat{e}_r\right)$$

$$= \ddot{\theta}\frac{d}{2}\hat{e}_\theta - \dot{\theta}^2\frac{d}{2}\hat{e}_r$$

$$m\left(\frac{l^2}{12} + \frac{d^2}{4}\right)\ddot{\theta} = -\frac{mgd}{2}\cos\theta$$

$$\ddot{\theta} = -\frac{gd}{2\left(\frac{l^2}{12} + \frac{d^2}{4}\right)}\cos\theta$$

$$\ddot{\theta} = -\frac{6gd}{L^2 + 3d^2}\cos\theta$$

on integration

$$\frac{1}{2}\dot{\theta}^2\Big|_0^{\dot{\theta}_0} = -\frac{6gd}{L^2 + 3d^2}\sin\theta\Big|_\theta^0$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\dot{\theta} = \frac{\sqrt{2 \times 6 gd \sin\theta}}{\sqrt{L^2 + 3d^2}}$$

$$\dot{\theta} = \sqrt{\frac{12 gd}{L^2 + 3d^2} \sin\theta}$$

using Principle of work and energy

$$U = mg\left[\frac{d}{2}\sin\theta\right]$$

$$\cancel{T_1}^{0} + U = T_2$$

$$U = T_2$$

$$T_2 = \frac{1}{2} m v^2 + \frac{1}{2} I^{cm} \omega^2$$

$$T_2 = \frac{1}{2} m (\underline{\omega} \times \underline{r})^2 + \frac{1}{2} \frac{m l^2}{12} \omega^2$$

$$T_2 = \frac{1}{2} m \left(\dot{\theta}^2 \frac{d^2}{4}\right) + \frac{1}{2} \frac{m l^2}{12} \omega^2$$

$$T_2 = \frac{m\omega^2 d^2}{8} + \frac{m l^2 \omega^2}{24}$$

since $U = T_2$

$$mgd\sin\theta = \frac{m\dot{\theta}^2 d^2}{4} + \frac{m\dot{\theta}^2 l^2}{12}$$

$$\dot{\theta}^2 = \left(\frac{12 gd}{3d^2 + L^2}\right)\sin\theta$$