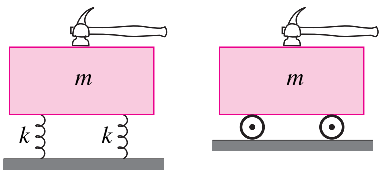
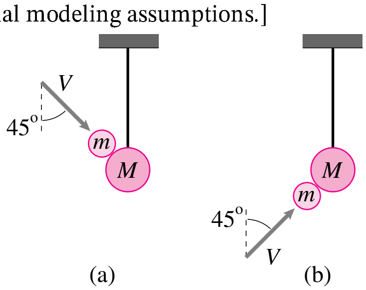
17.5 Collisions

17.5.1 The two blocks shown in the figure are identical except that one rests on two springs while the other one sits on two massless wheels. Draw free-body diagrams of each mass as each is struck by a hammer. Here we are interested in the free-body diagrams only during collision. Therefore, ignore all forces that are much smaller than the *impulsive* forces. State in words why the forces you choose to show should not be ignored during the collision.



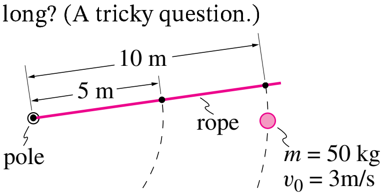
Problem 17.5.1

17.5.2 These problems concern two colliding masses. In (a) the smaller mass hits the hanging mass from above at an angle 45° with the vertical. In (b) the second case the smaller mass hits the hanging mass from below at the same angle. Assuming perfectly elastic impact between the balls, find the velocity of the hanging mass just after the collision. [Note, these problems are *not* well posed and can only be solved if you make additional modeling assumptions.]



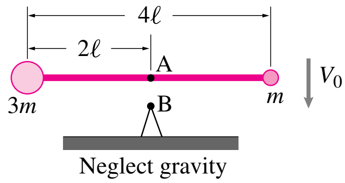
Problem 17.5.2

17.5.3 A narrow pole is in the middle of a pond with a 10 m rope tied to it. A frictionless ice skater of mass 50 kg and speed 3 m/s grabs the rope. The rope slowly wraps around the pole. What is the speed of the skater when the rope is 5 m long? (A tricky question.)



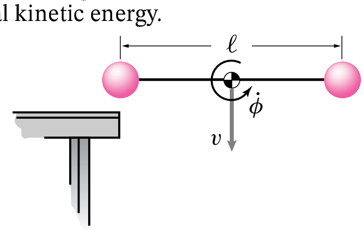
Problem 17.5.3

17.5.4 The masses m and $3m$ are joined by a light-weight bar of length 4ℓ. If point A in the center of the bar strikes fixed point B vertically with velocity V_0, and is not permitted to rebound, find $\dot{\theta}$ of the system immediately after impact.



Problem 17.5.4: Neglect gravity!

17.5.5 Two equal masses each of mass m are joined by a massless rigid rod of length ℓ. The assembly strikes the edge of a table as shown in the figure, when the center of mass is moving downward with a linear velocity v and the system is rotating with angular velocity $\dot{\phi}$ in the counter-clockwise sense. The impact is 'elastic'. Find the immediate subsequent motion of the system, assuming that no energy is lost during the impact and that there is no gravity. Show that there is an interchange of translational and rotational kinetic energy.



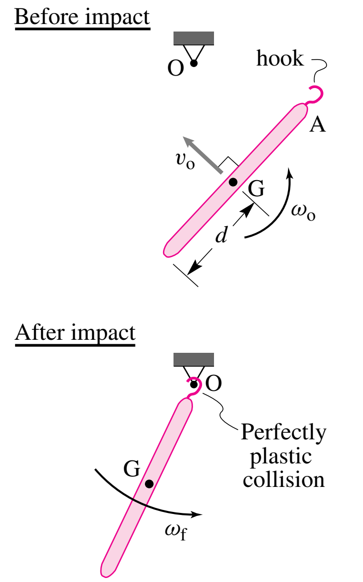
Problem 17.5.5

17.5.6 In the *absence of gravity*, a thin rod of mass m and length ℓ is initially tumbling with constant angular speed ω_o, in the counter-clockwise direction, while its mass center has constant speed v_o, directed as shown below. The end A then makes a perfectly plastic collision with a rigid peg O (via a hook). The velocity of the mass center happens to be perpendicular to the rod just before impact.

a) What is the angular speed ω_f immediately after impact?

b) What is the angular speed 10 seconds after impact? Why?

c) What is the loss in energy in the plastic collision?



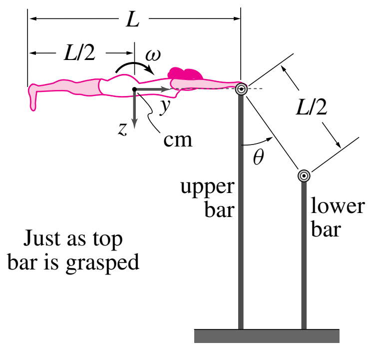
Problem 17.5.6

17.5.7 A gymnast of mass m and extended height L is performing on the uneven parallel bars. About the x, y, z axes which pass through her center of mass, her radii of gyration are k_x, k_y, and k_z, respectively. Just before she grasps the top bar, her fully extended body is horizontal and rotating with angular rate ω; her center of mass is then stationary. Neglect any friction between the bar and her hands and assume that she remains rigid throughout the entire stunt.

a) What is the gymnast's rotation rate just *after* she grasps the bar? State clearly any approximations/assumptions that you make.

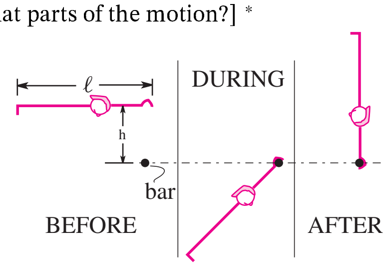
b) Calculate the linear speed with which her hips (CM) strike the lower bar. State all assumptions/approximations.

c) Describe (in words, no equations please) her motion immediately after her hips strike the lower bar if she releases her hands just prior to this impact.



Problem 17.5.7

17.5.8 An acrobat modeled as a rigid body with uniform rigid mass m of length l. She falls without rotation in the position shown from height h where she was stationary. She then grabs a bar with a firm but slippery grip. What is h so that after the subsequent motion the acrobat ends up in a stationary handstand? [Hint: What quantities are preserved in what parts of the motion?] *

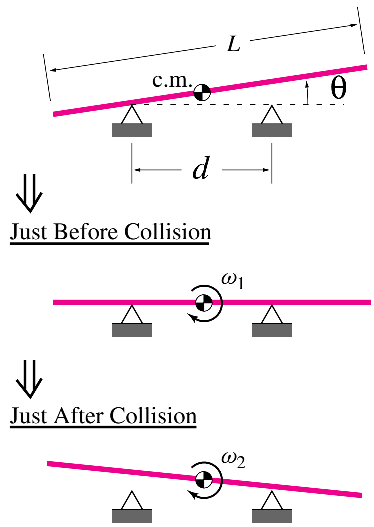


Problem 17.5.8

17.5.9 A crude see-saw is built with two supports separated by distance d about which a rigid plank (mass m, length L) can pivot smoothly. The plank is placed symmetrically, so that its center of mass is midway between the supports when the plank is at rest.

a) While the left end is resting on the left support, the right end of the plank is lifted to an angle θ and released. At what angular velocity ω_1 will the plank strike the right hand support?

b) Following the impact, the left end of the plank can pivot purely about the right end if d/L is properly chosen and the right end does not bounce. Find ω_2 under these circumstances.



Problem 17.5.9

17.5.10 Baseball bat. In order to convey the ideas without making the calculation too complicated, some of the simplifying assumptions here are highly approximate. Assume that a bat is a uniform rigid stick with length L and mass m_s. The motion of the bat is a pivoting about one end held firmly in place with hands that rotate but do not move. The swinging of the bat occurs by the application of a constant torque M_s at the hands over an angle of $\theta = \pi/2$ until the point of impact with the ball. The ball has mass m_b and arrives perpendicular to the bat at an absolute speed v_b at a point a distance ℓ from the hands. The collision between the bat and the ball is completely elastic.

a) To maximize the speed v_{hit} of the hit ball *How heavy should a baseball bat be? Where should the ball hit the bat?* Here are some hints for one way to approach the problem.

   - Find the angular velocity of the bat just before collision by drawing a FBD of the bat etc.
   - Find the total energy of the ball and bat system just before the collision.
   - Draw a FBD of the ball and of the bat during the collision (with this model there is an impulse at the hands on the bat). Call the magnitude of the impulse of the ball on the bat (and vice versa) $\int F dt$.
   - Use various momentum equations to find the angular velocity of the bat and velocity of the ball just after the collision in terms of $\int F dt$ and other quantities above. Use these to find the energy of the system just after collision.
   - Solve for the value of $\int F dt$ that conserves energy. As a check you should see if this also predicts that the relative separation speed of the ball and bat (at the impact point) is the same as the relative approach speed (it should be).
   - You now can calculate v_{hit} in terms of m_b, m_s, M_s, L, ℓ, and v_b.
   - Find the maximum of the above expression by varying m_s and ℓ. Pick numbers for the fixed quantities if you like.

b) Can you explain in words what is wrong with a bat that is too light or too heavy?

c) Which aspects of the model above do you think lead to the biggest errors in predicting what a real ball player should pick for a bat and place on the bat to hit the ball?

d) Describe as clearly as possible a different model of a baseball swing that you think would give a more accurate prediction. (You need not do the calculation).