**17.4.1** A uniform disc of mass $m$ and radius $r$ rolls without slip at constant rate. What is the total kinetic energy of the disk?

16.4.1


Rolling rate = constant = $\omega$

$\vec{V}_c = \vec{\omega} \times \vec{r} \Rightarrow \vec{V}_c = -\omega\hat{K} \times r\hat{J} \Rightarrow V_c = r\omega\hat{i}$

$K.E = \frac{1}{2} m \vec{V}_c \cdot \vec{V}_c + \frac{1}{2} I (\vec{\omega} \cdot \vec{\omega})$

$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2} \frac{mr^2}{2} \omega^2$

$K.E = \frac{3}{4} m r^2 \omega^2$

**17.4.2** A non-uniform disc of mass $m$ and radius $r$ rolls without slip at constant rate. The center of mass is located at a distance $\frac{r}{2}$ from the center of the disc. What is the total kinetic energy of the disc when the center of mass is directly above the center of the disc?


$$I = m \times \frac{r^2}{4}$$

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = -\omega\hat{K} \times \frac{3r}{2}\hat{\jmath} = \frac{3\omega r}{2}\hat{\imath}$$

$$T_B = \frac{1}{2}m\left(\vec{V}_{B/A} \cdot \vec{V}_{B/A}\right) + \frac{1}{2}I\omega^2$$

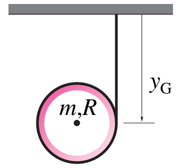
$$= \frac{1}{2}m\left(\frac{3r}{2}\omega\right)^2 + \frac{1}{2}m\left(\frac{r}{2}\right)^2 \times \omega^2$$

$$T_B = \frac{5}{4}mr^2\omega^2$$

**17.4.3 Falling hoop.** A bicycle rim (no spokes, tube, tire, or hub) is idealized as a hoop with mass $m$ and radius $R$. $G$ is at the center of the hoop. An inextensible string is wrapped around the hoop and attached to the ceiling. The hoop is released from rest at the position shown at $t = 0$.

a) Find $y_G$ at a later time $t$ in terms of any or all of $m$, $R$, $g$, and $t$.

b) Does $G$ move sideways as the hoop falls and unrolls?




Problem 17.3

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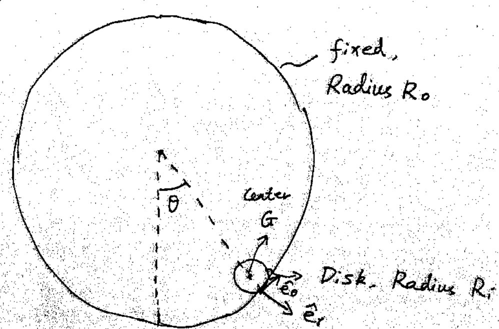
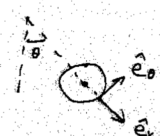



G is doing a circular motion with a Radius $(R_o - R_i)$

$$\vec{V}_G = \dot{\theta}(R_o - R_i)\,\hat{e}_\theta$$

$$\vec{a}_G = \ddot{\theta}(R_o - R_i)\,\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\,\hat{e}_r$$

No slip condition

$\Rightarrow$ Angular velocity of the disk

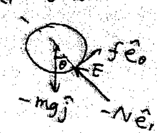
$$\vec{V}_G = -\omega R_i\,\hat{e}_\theta \qquad (\omega > 0 \text{ if counter clockwise})$$

$$\omega = -\frac{R_o - R_i}{R_i}\dot{\theta}$$

Angular acceleration

$$\dot{\omega} = -\frac{R_o - R_i}{R_i}\ddot{\theta}\ , \quad \dot{\vec{\omega}} = \dot{\omega}\hat{k} = -\frac{R_o - R_i}{R_i}\ddot{\theta}\hat{k}$$

Take AMB about the contact point E.




$$\dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I_G\dot{\vec{\omega}}$$

$$= (-R_i\hat{e}_r) \times m\left(\ddot{\theta}(R_o - R_i)\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\hat{e}_r\right) + \frac{1}{2}mR_i^2\left(-\frac{R_o - R_i}{R_i}\ddot{\theta}\right)\hat{k}$$

$$= -m\ddot{\theta}R_i(R_o - R_i)\hat{k} - \frac{1}{2}m(R_o - R_i)R_i\ddot{\theta}\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$= -\frac{3}{2} m R_i (R_o - R_i) \ddot{\theta} \hat{k}$$

$$\Sigma \vec{M}_{/E} = \vec{r}_{G/E} \times (-mg\hat{j})$$

$$= (-R_i \hat{e}_r) \times (-mg\hat{j}) = mg R_i \sin\theta \hat{k}$$

AMB : $\Sigma \vec{M}_{/E} = \dot{\vec{H}}_{/E} \Rightarrow mgR_i \sin\theta \hat{k} = -\frac{3}{2} m R_i (R_o - R_i) \ddot{\theta} \hat{k}$

$$\Rightarrow \frac{3}{2} m R_i (R_o - R_i) \ddot{\theta} + mg R_i \sin\theta = 0$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{2g}{3(R_o - R_i)} \sin\theta = 0}$$

If the disk radius gets arbitrarily small, then $R_i = 0$

The equation of motion becomes

$$\ddot{\theta} + \frac{2g}{3R_o} \sin\theta = 0$$

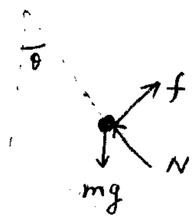
This equation does not reduce to the simple pendulum equation, which should be $\ddot{\theta} + \frac{g}{R_o} \sin\theta = 0$

Reason: As the radius of the disk, $R_i$, goes to 0, our problem is NOT the same as a simple pendulum.
To enforce no slip condition, there must be friction acting the the object. However, for a simple pendulum, the reaction force should be in normal direction.
The difference can be illustated in the FBD's below

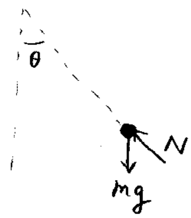
Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

[redacted]

Our problem when $R_i \to 0$ :



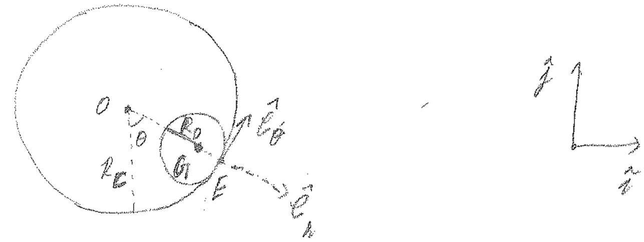

Simple pendulum.




The existence of friction force in our problem makes it different from simple pendulum problem

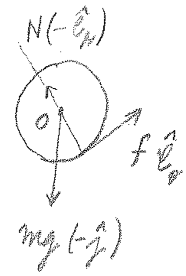
14.56

(a)




(b) During rolling, there is no kinetic friction but there is static friction.

FBD:




(c) LMB: $mg(-\hat{j}) + f(\hat{e}_\theta) + N(-\hat{e}_n) = m\vec{a}_G$

AMB: $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I\dot{\omega}\hat{k}$

(d) Kinematics: G is moving in a circle of radius $(R_C - R_D)$

$$\vec{V}_G = \dot{\theta}(R_C - R_D)\hat{e}_\theta \qquad \text{— (i)}$$

$$\Rightarrow \vec{a}_G = \ddot{\theta}(R_C - R_D)\hat{e}_\theta - \dot{\theta}^2(R_C - R_D)\hat{e}_n \qquad \text{— (ii)}$$

(Remember: $\dot{\theta} \neq \omega$. Here, $\omega$ refers to rotation of the disc.)

Rolling w/o slip :

$$\vec{V}_G = -\omega(\hat{k}) \times R_D(\hat{e}_r) = -\omega R_D \hat{e}_\theta$$

Using (i) : $\omega = -\dfrac{R_C - R_D}{R_D}\dot{\theta}$

---

(e) Now substitute $\vec{a}_G$ from (ii) in AMB,

& $\vec{r}_{G/E} = R_D(-\hat{e}_R)$

& $\sum M_{/E} = \vec{r}_{G/E} \times mg(-\hat{j}) = mg R_D \sin\theta \, \hat{k}$

(after some algebra ....)

$$\boxed{\ddot{\theta} + \frac{2g}{3(R_c - R_D)}\sin\theta = 0}$$

(f) If $R_D \to 0$, $\ddot{\theta} = -\dfrac{2}{3}\dfrac{g}{R_c}\sin\theta$

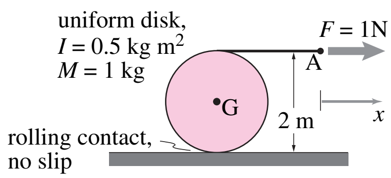
which is not the same as $\ddot{\theta} = -\dfrac{g}{R_c}\sin\theta$, the equation for simple pendulum.

This is because even at small $R_c$, there is rolling! The function for $\ddot{\theta}$ will not abruptly jump by continuously varying $R_D$.

**17.4.4** A uniform disk with radius $R$ and mass $m$ has a string wrapped around it. The string is pulled with a force $F$. The disk rolls without slipping.

a) What is the angular acceleration of the disk, $\vec{\boldsymbol{\alpha}}_{Disk} = -\ddot{\theta}\hat{\boldsymbol{k}}$? Make any reasonable assumptions you need that are consistent with the figure information and the laws of mechanics. State your assumptions.

b) Find the acceleration of the point A in the figure.

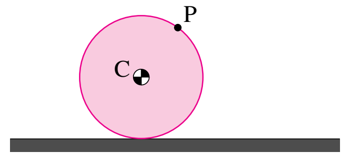


Problem 17.4

**Answer:**
(a) $\vec{\boldsymbol{\alpha}}_{Disk} = -\frac{4}{3}\ \text{rad/s}^2\hat{\boldsymbol{k}}$.
(b) $\vec{\boldsymbol{a}}_A = \frac{8}{3}\ \text{m/s}^2\hat{\boldsymbol{\imath}}$.

*Disclaimer:* *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

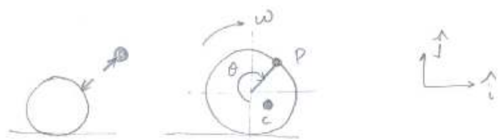
**17.4.5** If a pebble is stuck to the edge of the wheel in problem **??**, what is the maximum speed of the pebble during the motion? When is the force on the pebble from the wheel maximum? Draw a good FBD including the force due to gravity.




Problem 17.5

(4)

16.4.5




$\vec{r}_c = R\theta\,\hat{i} + R\,\hat{j}$

$\vec{V}_c = R\dot{\theta}\,\hat{i}$ $\qquad \vec{\omega} = -\dot{\theta}\,\hat{k}$

$\qquad\qquad\qquad \dot{\vec{\omega}} = -\ddot{\theta}\,\hat{k}$

$\vec{r}_P = \vec{r}_c + \vec{r}_{P/C} = R\theta\,\hat{i} + R\,\hat{j} + (-R\cos\theta\,\hat{j} - R\sin\theta\,\hat{i})$

$= R(\theta - \sin\theta)\,\hat{i} + R(1-\cos\theta)\,\hat{j}$

$\vec{V}_P = R\left[(\dot{\theta} - \cos\theta\,\dot{\theta})\,\hat{i} + \sin\theta\,\dot{\theta}\,\hat{j}\right]$

$V_P = R\dot{\theta}\left[(1-\cos\theta)\,\hat{i} + \sin\theta\,\hat{j}\right]$

If $\theta = 90^\circ \Rightarrow V_P = R\dot{\theta}\,(\hat{i} + \hat{j})$

$\theta = 180^\circ \rightarrow V_P = 2R\dot{\theta}\,\hat{i}$

$|\vec{V}_P|_{max} = 2R\dot{\theta}$

$\vec{a}_P = \left.\dfrac{dv_P}{dt}\right|_{\ddot{\theta}=0} = R\dot{\theta}^2(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$|a_P| = R\dot{\theta}^2\sqrt{\sin^2\theta + \cos^2\theta} = R\dot{\theta}^2$

when $\theta = \pi$

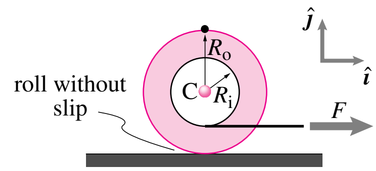
$\vec{a}_P = -R\dot{\theta}^2\cos\theta\,\hat{j} - g\,\hat{j}$

**17.4.6 Spool Rolling without Slip and Pulled by a Cord.** The light-weight spool is nearly empty but a lead ball with mass $m$ has been placed at its center. A force $F$ is applied in the horizontal direction to the cord wound around the wheel. Dimensions are as marked. Coordinate directions are as marked.

a) What is the acceleration of the center of the spool?

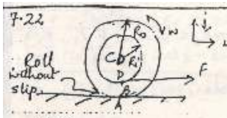
b) What is the horizontal force of the ground on the spool?



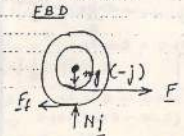

Problem 17.6

**Answer:**
(a) $\boldsymbol{a}_C = (F/m)(1 - R_i/R_O)\hat{\boldsymbol{\imath}}$.
(b) $(-R_i/R_O)F\hat{\boldsymbol{\imath}}$.

7-22




FBD




$\underline{v}_C = v_c\,\underline{i}$, $\underline{a}_C = a_c\,\underline{i}$, $\underline{\omega} = -\omega\underline{k}$

$\underline{v}_A = \underline{v}_B = 0$, because point A coincides with point B at this instant, and point A is a stationary point on the ground, the wheel doesn't slip

(contd)

$\Rightarrow \underline{v}_B = \underline{v}_C + \underline{v}_{B/C}$ ... $= 0$

$v_c\underline{i} - \omega\underline{k} \times R_o\underline{j} = 0,$

$v_c\underline{i} - R_o\omega\underline{i} = 0$

$\Rightarrow v_c = R_o\omega$

Taking the time derivative, $a_c = R_o\dot{\omega}$

(a) AMB : $\sum \underline{M}_{/B} = \dot{\underline{H}}_{/B}$

$\sum \underline{M}_{/B} = \underline{r}_{Q/B} \times \underline{F} = (R_o - R_i)\underline{j} \times F\underline{i}$

$= -F(R_o - R_i)\underline{k}$ (1)

$\dot{\underline{H}}_{/B} = \cancel{[I^{cm}]}^{\,0}\cdot\dot{\underline{\omega}} + \underline{\omega}\times\cancel{[I^{cm}]\cdot\underline{\omega}}^{\,0,\text{ since 2-D}} + \underline{r}_{c/B}\times m\underline{a}_c$

$= R_o\underline{j} \times m a_c\underline{i} = -R_o m a_c\underline{k}$ (2)

From (1) and (2) :

$-F(R_o - R_i)\underline{k} = -R_o m a_c \underline{k}$

$\Rightarrow \boxed{a_c = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)}$ The spool moves to the right.

Horizontal force of ground on spool?

(b) LMB : $\sum \underline{F} = m\underline{a}_c$

$F\underline{i} + F_f(-\underline{i}) + N\underline{j} - mg\underline{j} = m a_c\underline{i}$

$\{\}\cdot\underline{j} \Rightarrow N = mg$

$\{\}\cdot\underline{i} \Rightarrow F - F_f = m a_c$

We know $a_c$ from part (a), and

$F_f = \mu N = \mu m g$

$\Rightarrow F_f = F - m a_c = F - m\frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)$

$= F\left(\frac{R_i}{R_o}\right)$.

$\underline{F}_f = F\frac{R_i}{R_o}(-\underline{i}) = -F\frac{R_i}{R_o}\underline{i}$

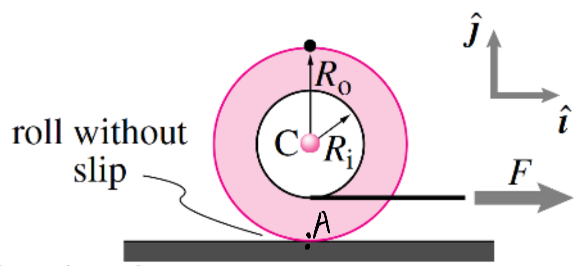
$\boxed{\underline{F}_f = -F\frac{R_i}{R_o}\underline{i}}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

# 16.4.6

Saturday, April 20, 2019 8:23 PM

Spool Rolling without slip and pulled by a cord

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Problem 16.4.6

A'

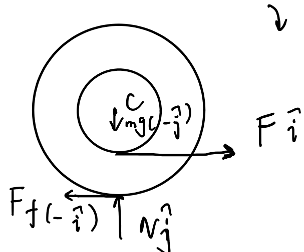
A light - weight spool with center C

A lead ball with mass m at C

A force F is in $\hat{\imath}$ direction

$\circlearrowright \omega$

FBD




$\vec{v}_c = v_c\,\hat{\imath}$

$\vec{a}_c = a_c\,\hat{\imath}$

$\omega = -\omega\,\hat{k}$

$\vec{v}_A = \vec{v}_{A'} = 0$ (no slip)

$\vec{v}_c = \vec{v}_A + \vec{\omega} \times \vec{v}_{c/A}$

$= \vec{0} + (-\omega\,\hat{k}) \times R_o\,\hat{j}$

$= R_o\omega\,\hat{\imath} \Rightarrow v_c = R_o\,\omega$

Take derivative,

$\vec{a}_c = R_o \dot{\omega}$

(a) Find the acceleration of C

AMB/A : $\sum M_{/A} = \dot{H}_{/A}$

$\Rightarrow (R_o - R_i)\hat{j} \times F\hat{i} = I^{cm} \cdot \dot{\vec{\omega}} + \vec{r}_{c/A} \times m\vec{a}_c$

$-(R_o - R_i) F\hat{k} = R_o\hat{j} \times m a_c \hat{i} = -R_o m a_c \hat{k}$

$\therefore \quad a_c = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)$

$\vec{a}_c = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)\hat{i}$

(b) Find $F_f$

LMB : $\sum \vec{F} = m\vec{a}_c$

$\{F\hat{i} + F_f(-\hat{i}) + N\hat{j} - mg\hat{j} = m a_c \hat{i}\}$

$\{\}\cdot\hat{i} \Rightarrow F - F_f = m a_c$

We know $a_c$ from part (a)

$\Rightarrow \quad F_f = F - m a_c = F\frac{R_i}{R_o}$

$\vec{F}_f = -F\frac{R_i}{R_o}\hat{i}$

*Disclaimer:* *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

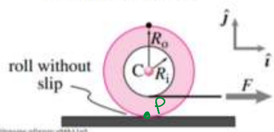
# #58 Solution Michelle deSouza

Saturday, April 24, 2021 10:07 PM

**17.4.6 Spool Rolling without Slip and Pulled by a Cord.** The light-weight spool is nearly empty but a lead ball with mass $m$ has been placed at its center. A force $F$ is applied in the horizontal direction to the cord wound around the wheel. Dimensions are as marked. Coordinate directions are as marked.

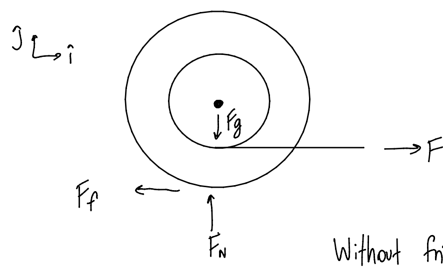
a) What is the acceleration of the center of the spool? *

b) What is the horizontal force of the ground on the spool? *




Problem 17.4.6

FBD:

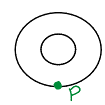



Without friction, the spool would be sliding in the $+\hat{\imath}$ direction. Friction force opposes motion and acts in the $-\hat{\imath}$ direction.

## a) Find $\vec{a}_c$ (c is also the center of mass)

Since the spool is rolling, we know $\vec{a}_c = a_c\hat{\imath}$

To find $\vec{a}_c$ in terms of only known F, take AMB about point P




$$\sum \vec{M}_{/P} = \dot{\vec{H}}_{/P}$$

0, massless spool

$$(R_o - R_i)\hat{j} \times F\hat{\imath} = R_o\hat{j} \times m a_c\hat{\imath} + \cancel{I^c}\ddot{\theta}\hat{k}$$

$$-F(R_o - R_i)\hat{k} = -m a_c R_o\hat{k}$$

$$\{\} \cdot \hat{k} \quad \frac{F}{m}\frac{R_o - R_i}{R_o} = a_c$$

## b) Find $F_f$

LMB: $\sum \vec{F} = -F_f\hat{\imath} + F\hat{\imath} - mg\hat{j} + F_N\hat{j} = m\underline{\vec{a}_c}$ plug in from a)

$$\{\} \cdot \hat{\imath} \quad -F_f + F = F\left(\frac{R_o - R_i}{R_o}\right)$$

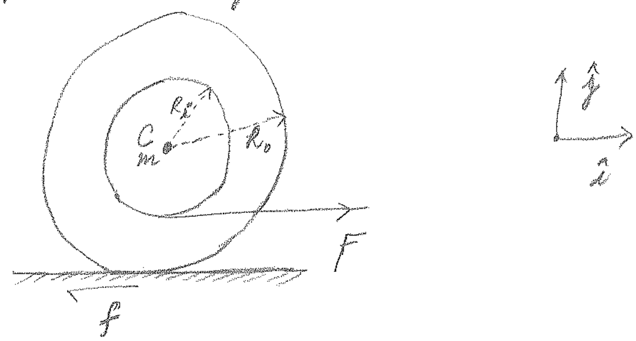
$$F_f = F\left(1 - \frac{R_o - R_i}{R_o}\right)$$

$$F_f = \frac{R_i}{R_o}F$$

Note: This problem is made simpler by modeling a point mass instead of a rigid object with distributed mass → no rolling kinematics necessary.

*Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.39 Spool rolling w/o slip




* We need to determine the acceleration of the center of mass $\vec{a}_{cm}$, and the horizontal force $\vec{f}$ acting on the lowest pt. as shown.

* Newton's II Law immediately yields:

LMB: $F\hat{\imath} - f\hat{\imath} = m\vec{a}_{cm}$ — (i)

& AMB: $R_i(-\hat{j}) \times F(\hat{\imath}) + R_o(-\hat{j}) \times f(-\hat{\imath})$

$= I^c \vec{\alpha}$ — (ii)

* Since the spool is "massless", $I^c \to 0$

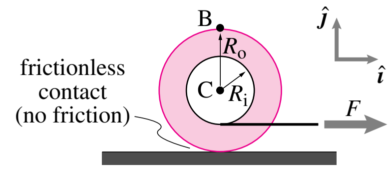
$\Rightarrow$ (ii) yields $R_i F = R_o f$

Substituting in (i) $\boxed{\vec{a}_{cm} = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)\hat{\imath}}$

Also, $\boxed{-\left(\frac{R_i}{R_o}\right)F\hat{\imath} = \vec{f}}$

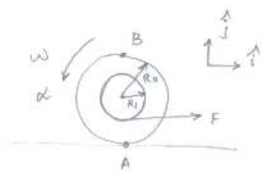
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.7** **A film spool** is placed on a very slippery table. Assume that the film and reel (together) have mass distributed the same as a uniform disk of radius $R_i$. What, in terms of $R_i$, $R_o$, $m$, $g$, $\hat{\imath}$, $\hat{\jmath}$, and $F$ are the accelerations of points C and B at the instant shown (the start of motion)?




Problem 17.7

16.4.7




$$\sum \vec{F} = m\vec{a}_{cm}$$

$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A}$$

LMB

$$F\hat{\imath} = m\vec{a}_{cm}$$

$$= m\ddot{x}\hat{\imath} + m\ddot{y}\hat{\jmath}$$

$$\ddot{x} = \frac{F}{m} \quad \therefore \quad \vec{a}_{cm} = \frac{F}{m}\hat{\imath}$$

AMB /c

$$\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$$

$$-R_i\hat{\jmath} \times F\hat{\imath} = \vec{r}_{cm} \times m a_{cm} + I^{cm}\dot{\vec{\omega}}\hat{k} + \vec{\omega} \times I\vec{\omega}$$

$$R_i F\hat{k} = I^{cm}\dot{\omega}\hat{k} = \frac{mR_i^2}{2}\cdot\dot{\omega}\hat{k}$$

$$\dot{\vec{\omega}} = \frac{2F}{mR_i}\hat{k}$$

$$F = \frac{mR_i\dot{\omega}}{2}$$

$$\Rightarrow \quad \vec{a}_{cm} = \frac{mR_i\dot{\vec{\omega}}}{2m} = \frac{R_i\dot{\vec{\omega}}}{2}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\vec{a}_B = \vec{a}_c + \vec{a}_{B/c} = \vec{a}_{cm} + \left[\dot{\vec{\omega}} \times \vec{R}_o\right] + \vec{\omega} \times (\vec{\omega} \times \vec{R}_o)$$

$$= \vec{a}_{cm} + \frac{2}{mR_i} F\hat{k} \times R_o\hat{j} + \omega\hat{k} \times (\omega\hat{k} \times R_o\hat{j})$$

$$= \frac{F}{m}\hat{i} + \frac{2R_oF}{mR_i}(-\hat{i}) - \omega^2 R_o\hat{j}$$

$$\vec{a}_B = \frac{F}{m}\left(1 - \frac{2R_o}{R_i}\right)\hat{i} - \omega^2 R_o\hat{j}$$

$$\vec{a}_B = \frac{F}{m}\left(1 - \frac{2R_o}{R_i}\right)\hat{i}$$

$$\dot{\omega} = \frac{2F}{mR_i}$$

$$\omega = \frac{2F}{mR_i}t + C_1$$

$$\omega = \frac{2F}{mR_i}t$$

$$t = 0, \ \omega = 0$$

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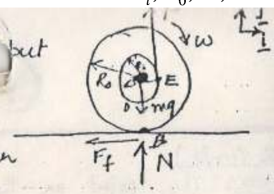
**17.4.8 Again, Spool Rolling without Slip and Pulled by a Cord.** Reconsider the spool from problem 17.4.6. This time, a force $F$ is applied vertically to the cord wound around the wheel. In this case, what is the acceleration of the center of the spool? Is it possible to pull the cord at some angle between horizontal and vertical so that the angular acceleration of the spool or the acceleration of the center of mass is zero? If so, find the angle in terms of $R_i$, $R_0$, $m$, and $F$.

#5

7.23 Same as 7.[illegible] but F acts vertically as shown.




From 7.22, we obtain $v_c = R_o\omega$ and $a_c = R_o\dot{\omega}$

(a) AMB : $\Sigma \underline{M}_{/B} = \dot{\underline{H}}_{/B}$

$\Sigma \underline{M}_{/B} = \underline{r}_{E/B} \times \underline{F} = (R_i\underline{i} + R_o\underline{j}) \times F\underline{j}$

$= R_i F \underline{k}$ —(1)

$\dot{\underline{H}}_{/B} = \cancel{[I^{cm}]\cdot\dot{\underline{\omega}}}^{\,0} + \cancel{\underline{\omega}\times[I^{cm}]\cdot\underline{\omega}}^{\,0} + \underline{r}_{c/B} \times m\underline{a}_c$

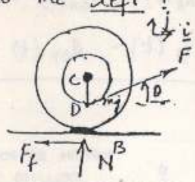
$= R_o\underline{j} \times m a_c \underline{i} = -R_o m a_c \underline{k}$ —(2)

From (1) and (2):

$a_c = -\frac{F}{m}\left(\frac{R_i}{R_o}\right)$

$\boxed{a_c = -\frac{F}{m}\left(\frac{R_i}{R_o}\right)}$ The spool moves to the left!

(b)




AMB : $\Sigma \underline{M}_{/B} = \dot{\underline{H}}_{/B}$

$\Sigma \underline{M}_{/B} = \underline{r}_{D/B} \times \underline{F}$

$= (R_o - R_i)\underline{j} \times (F\cos\theta)\underline{i}$

$= -(R_o - R_i)(F\cos\theta)\underline{k}$ —(1)

$\dot{\underline{H}}_{/B} = \cancel{[I^{cm}]\cdot\dot{\underline{\omega}}}^{\,0} + \cancel{\underline{\omega}\times[I^{cm}]\cdot\underline{\omega}}^{\,0} + \underline{r}_{c/B} \times m\underline{a}_c$

$\dot{\underline{H}}_{/B} = -R_o m a_c \underline{k}$ —(2)

$\therefore\ -(R_o - R_i)(F\cos\theta)\underline{k} = -R_o m a_c \underline{k}$

$\Rightarrow \cos\theta = \frac{R_o}{(R_o - R_i)} \cdot \frac{1}{F} m a_c$

$\Rightarrow \theta = \cos^{-1}\left[\left(\frac{R_o}{R_o - R_i}\right)\frac{1}{F} m a_c\right]$

if $a_c = 0$ then $\theta = \cos^{-1}(0) \Rightarrow \theta = \pi/2, 3\pi/2, 5\pi/2$, etc.

#6 (cont.)

It is not possible to p[illegible] the cord at some angle between [illegible]izontal and vertical in order for the accln of center of mass to be zero.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

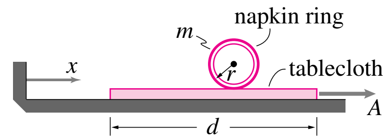
**17.4.9** A napkin ring lies on a black velvet tablecloth. The thin ring (of mass $m$, radius $r$) rolls without slip as a mischievous child pulls the tablecloth (mass $M$) out with acceleration $\vec{a}_A$. The ring starts at the right end ($x = d$). You can make a reasonable physical model of this situation with an empty soda can and a piece of paper on a flat table.

a) What is the ring's acceleration as the tablecloth is being withdrawn?

b) How far has the tablecloth moved to the right from its starting point $x = 0$ when the ring rolls off its left-hand end?

c) Clearly describe the subsequent motion of the ring. Which way does it end up rolling at what speed?

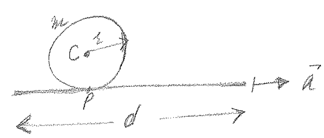
d) Would your answer to the previous question be different if the ring slipped on the cloth as the cloth was being pulled out?




Problem 17.9

14.4.2 "Napkin ring" problem.

(a) The lowest pt on the ring will have the same acceleration as the cloth.




$\alpha = \dot\omega$

$$\vec{a}_p = a\hat{i} = a_{cm}\hat{i} + \alpha\hat{k} \times r(-\hat{j}) \quad \text{— (i)}$$

Since friction acts at the lowest pt, then

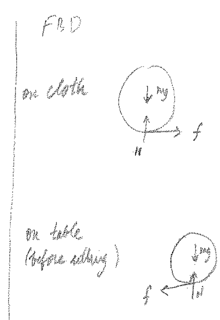
$$\vec{M}_p = \dot{\vec{H}}_{/p} = r\hat{j} \times m a_{cm}\hat{i} + I^c_{zz}\alpha\hat{k} = \vec{0}$$

$$\Rightarrow \quad m a_{cm} r = I^c_{zz}\alpha$$

$$\cancel{m} a_{cm} \cancel{r} = \cancel{m} r^{\cancel{2}} \alpha$$

$$\Rightarrow \quad \boxed{a_{cm} = r\alpha}$$

Using (i) $\quad \boxed{a_{cm} = \dfrac{a}{2}}$




(b) The ring moves 'd' in time 't' relative to the cloth with acceleration $a/2$

$$\Rightarrow \quad d = \frac{1}{2}\frac{a}{2}t^2 \quad \Rightarrow \quad t = 2\sqrt{d/a}$$

In the same time, the cloth moves $\frac{1}{2}a\left(2\sqrt{d/a}\right)^2 = \boxed{2d}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(C) When the ring leaves the cloth, the lowest pt has velocity $\vec{V}_p = 2\sqrt{ad}\,\hat{\imath}$ $(v = v_0 + at)$

Now two cases are possible:

(i) Table is smooth: In this all velocities (angular, linear) are conserved. The ring coasts with the same velocity it has when it leaves the cloth.

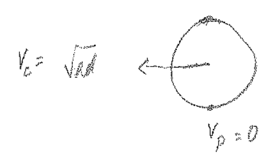
(ii) Table has friction: In this case, friction will act until lowest pt. comes to rest. Since lowest pt. is moving to the right — friction will be in the ~~m~~ $-\hat{\imath}$ direction. Friction will cause a clockwise torque about C.M.

Assuming friction ($\mu$), one can calculate the time '$t_R$' when lowest pt will come to rest: $t_R = \dfrac{2\sqrt{ad}}{\mu g}$ $(v = at)$

Now turn attention to the center-of-mass: As it leaves the cloth, the center-of-mass will have velocity $v_c = \sqrt{ad}\ \hat{i}$.

On the table, because of friction, it will deaccelerate. By the time the lowest pt. comes to rest (in time '$t_s$'), the car will have velocity $\sqrt{ad} - \frac{2\sqrt{ad}}{\mu g}\mu g = -\sqrt{ad}\ \hat{i}$

At $t_s$, we have a situation like this:

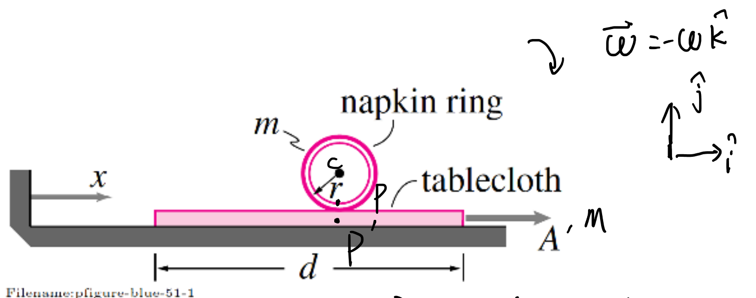



Now on there is no work from friction. So it stays rolling like above. (Obviously $v_c = \omega R$).

(d) Of course, if there is sliding initially on the cloth, we don't expect the same $v_c$ as above. But it remains ~~as~~ true the ring will roll in the same direction.

# 16.4.9

Saturday, April 20, 2019 21:15




Problem 16.4.9

P: contact point

Given:

ring mass m, radius r

tablecloth mass M

ring starts at x=d without slip

as tablecloth moves with acceleration A.

$\vec{\omega} = -\omega\hat{k}$

$\vec{a}_c = a_c\hat{i}$

(a) Find the ring's acceleration $\vec{\alpha} = \dot{\vec{\omega}} = -\dot{\omega}\hat{k} = -\alpha\hat{k}$

$\vec{a}_P = \vec{a}_c + \dot{\vec{\omega}} \times \vec{r}_{P/c} - \omega^2 \vec{r}_{P/c}$

$\vec{a}_{P'} = \vec{A} = A\hat{i}$

no slip $\Rightarrow \{\vec{a}_P\}\cdot\hat{i} = \{\vec{a}_{P'}\}\cdot\hat{i} = A$

$\Rightarrow \quad a_c - \alpha r = A \quad ①$

AMB/P: $\vec{M}_{/P} = \dot{\vec{H}}_{/P}$

$\vec{0} = r\hat{j} \times m a_c\hat{i} + I^c\alpha\hat{k} \quad ②$

$I^c = mr^2 \quad ③$

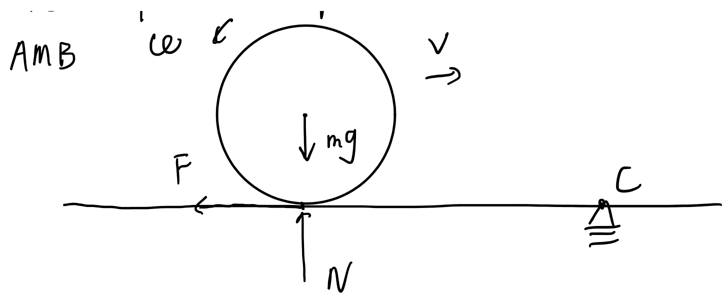
① ② ③ $\Rightarrow \quad a_c = \frac{A}{2}$

(b) How far has the tablecloth moved?
(from $x=0$ to when the ring rolls off)
Assume the whole process lasts for $t$.
Relative distance of motion $= d$

$$X_{cloth} - X_{ring} = d$$
$$X_{cloth} = \frac{1}{2} A t^2$$
$$X_{ring} = \frac{1}{2} a_c t^2$$
$$\Rightarrow X_{ring} = 2d$$

(c) Clearly describe the subsequent motion of ring
Which way does it end up rolling at what speed?
When the ring leaves the cloth, the contact point has velocity $\vec{v}_p = 2\sqrt{Ad}\ \hat{i}$
Now two cases are possible:
(i) Table is frictionless: then all velocities (angular, linear) are conserved.
(ii) Table has friction: friction will act until the ring stops moving at all. The linear velocity and angular velocity of the ring while rolling are not compatible with each other. The ring will slip at the very beginning. In fact, the angular momentum provided by the friction and its actual angular momentum $I^c \omega$ are in the opposite directions, and they are incompatible with the rolling condition. Thus the ring will stop in the end.

The equation explanations are as follows:




$\vec{H}_{/C} = \vec{0}$ at all $t$.

$\Rightarrow \vec{r} \times m\vec{v} - I\vec{\omega} = 0$ at all $t$. ①

rolling condition: $v = -r\omega$

$v + r\omega = 0$ ②

① ② $\Rightarrow$ $v = \omega = 0$

(d) Answer to (c) different if there is slip?
No. the ring will still stop in the end.
The reason is the same as above.

HW11-P59-17.4.9

Wednesday, April 21, 2021 2:09 PM

Solution: Michael Zakavorotny

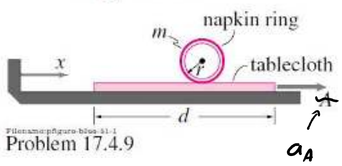
**17.4.9** A napkin ring lies on a thick velvet tablecloth. The thin ring (of mass $m$, radius $r$) rolls without slip as a mischievous child pulls the tablecloth (mass $M$) out with acceleration $a_A$. The ring starts at the right end ($x = d$). You can make a reasonable physical model of this situation with an empty soda can and a piece of paper on a flat table.

a) What is the ring's acceleration as the tablecloth is being withdrawn?

b) How far has the tablecloth moved to the right from its starting point $x = 0$ when the ring rolls off its left-hand end?

c) Clearly describe the subsequent motion of the ring. Which way does it end up rolling at what speed?

d) Would your answer to the previous question be different if the ring slipped on the cloth as the cloth was being pulled out?



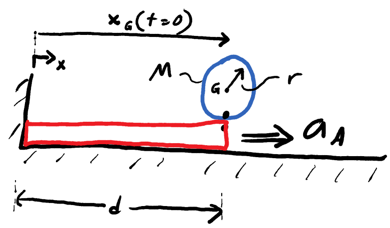

Problem 17.4.9

Given: Ring has $m$, $r$

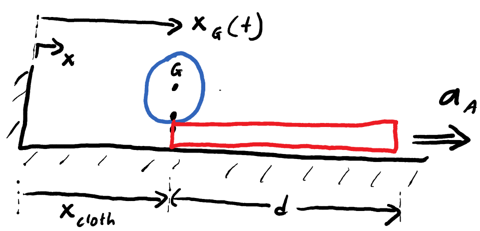
Table cloth has $M$, pulled at acceleration $a_A$

Ring starts at $x = d$

At $t = 0$

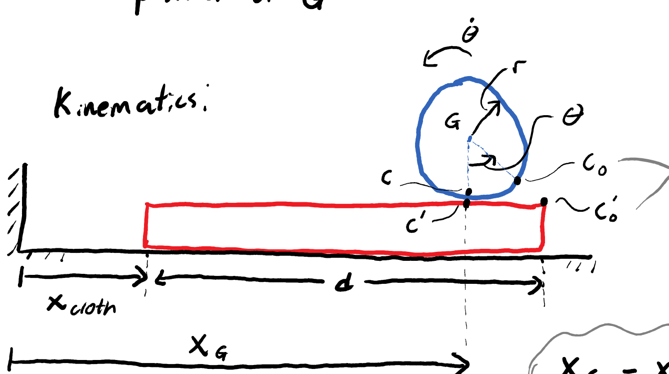



At a later time




Let $x_G$ track the position of G

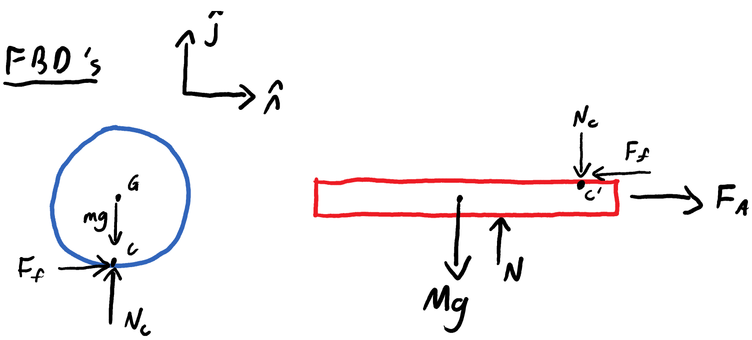
Kinematics:




$C_0$ and $C_0'$ are initially coincident at $t = 0$

$$\boxed{x_G = x_{cloth} + d - r\theta}$$

FBD's




a) Find ring's acceleration when being pulled

Consider AMB of ring about point C

$$\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$$

$$\{\ \vec{0} = \vec{r}_{G/c} \times m\vec{a}_{G/F} + I^G \ddot{\theta}\hat{k}\ \}$$

$$\vec{r}_{G/c} = r\hat{\jmath} \qquad \vec{a}_{G/F} = a_G\hat{\imath}$$

$$\{\}\cdot\hat{k}: \quad -rma_G + I^G\ddot{\theta} = 0$$

$$a_G = \frac{I^G}{mr}\ddot{\theta} \qquad (1)$$

From kinematics of the system

$$x_G = x_{cloth} + d - r\theta$$

$$\dot{x}_G = \dot{x}_{cloth} - r\dot{\theta}$$

$$\ddot{x}_G = a_G = \underbrace{\ddot{x}_{cloth}}_{= a_A} - r\ddot{\theta}$$

$$\ddot{\theta} = \frac{a_A - a_G}{r} \quad (2)$$

Plug (2) into (1) :

$$a_G = \frac{I^G}{mr} \cdot \frac{a_A - a_G}{r}$$

$$a_G \left(1 + \frac{I^G}{mr^2}\right) = \frac{I^G}{mr^2} \cdot a_A$$

$$\boxed{a_G = \frac{I^G}{mr^2 + I^G} \cdot a_A}$$

Hold even if $a_A = a_A(t)$

We know $I^G = mr^2$ for a thin ring

$$a_G = \frac{mr^2}{mr^2 + mr^2} \cdot a_A \Rightarrow \boxed{a_G = \frac{a_A}{2}}$$

b) Find distance moved by ring by the time at which it rolls off left end of cloth

This occurs when $x_G = x_{cloth}$

From kinematics: $x_G = x_{cloth} + d - r\theta$

From AMB $\overset{(1)}{:}$ $a_G = \frac{I^G}{mr}\ddot{\theta} \Rightarrow \{\ddot{x}_G = r\ddot{\theta}\}$

$\int \{\}\, dt \Rightarrow \{\dot{x}_G = r\dot{\theta}\} + \cancel{c_1}^{\,0}$, since $\dot{x}_G(0)=0$ when $\dot{\theta}(0)=0$

$\int \{\}\, dt \Rightarrow x_G = r\theta + c_2$ when $\theta = 0$, $x_G = d$ so, $c_2 = d$

$x_G = r\theta + d$

$r\theta = x_G - d$

Combine kinematics and AMB:

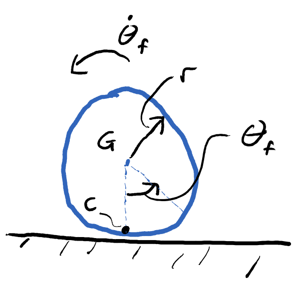
$x_G = x_{cloth} + d - (x_G - d)$

$2x_G - x_{cloth} = 2d$

Apply $x_G = x_{cloth}$

$$\boxed{x_{cloth} = 2d}$$

c) Describe ring's motion after cloth has been pulled




From kinematics, we now have

$V_{G,f} = -r\dot{\theta}_f$ (3) for no-slip rolling on the table

( f = final state)

Again, from AMB:

$\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

We can use the same AMB formula as a) and b) because we only looked at FBD of ring, and it does not depend on whether ring is actually rolling, or whether the ring is on a moving cloth or stationary table. We therefore obtain $\boxed{\ddot{x}_{G,f} = r\ddot{\theta}_f}$, which can be integrated to obtain $\boxed{\dot{x}_{G,f} = r\dot{\theta}_f}$. However, now kinematics tells us that $\boxed{\dot{x}_{G,f} = -r\dot{\theta}_f}$ (since ring is rolling on a stationary surface). Combining:

$$\dot{x}_{G,f} = r\dot{\theta}_f = -r\dot{\theta}_f \Rightarrow \boxed{\dot{\theta}_f = 0,\ v_{G,f} = 0}$$

Initially, there must be some slip between point C on the ring and the surface, because C moved at same velocity as point C′ on the cloth. Eventually, AMB necessitates that the ring must come to rest.

d) Would c) be different if ring slipped on cloth?

$\boxed{\text{No}}$, the ring would still come to rest because this reasoning does not depend on the kinematics of the ring when it is on the cloth.

**17.4.10** A block of mass $M$ is supported by two rollers (uniform cylinders) each of mass $m$ and radius $r$. They roll without slip on the block and the ground. A force $F$ is applied in the horizontal direction to the right, as shown in the figure. Given $F$, $m$, $r$, and $M$, find:

a) the acceleration of the block,

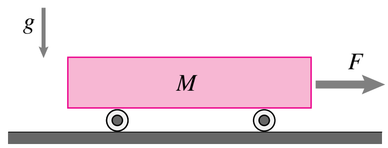
b) the acceleration of the center of mass of this block/roller system,

c) the reaction at the wheel bases,

d) the force of the right wheel on the block,

e) the acceleration of the wheel centers, and

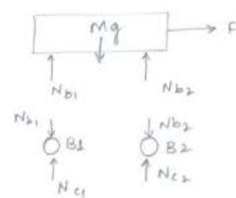
f) the angular acceleration of the wheels.




Problem 17.10

7

16·4·10




$$\sum \vec{F} = m\vec{a}_b$$

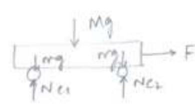
$$F\hat{i} - Mg\hat{j} + N_{b_1}\hat{j} + N_{b_2}\hat{j} = M\vec{a}_b$$

$$\vec{a}_b = a_x\hat{i} + a_y\hat{j}$$

a) $F = Ma_x \therefore a_x = \dfrac{F}{M}$

$$\therefore \vec{a}_b = \frac{F}{M}\hat{i}$$

b) $\sum \vec{F} = m\vec{a}_{cm}$




$$F\hat{i} + N_{c_1}\hat{j} + N_{c_2}\hat{j} \underbrace{- Mg\hat{j} - 2mg\hat{j}} = (M+2m)\,\vec{a}_{cm}$$

$$N_{c_1} + N_{c_2} = Mg + 2mg$$

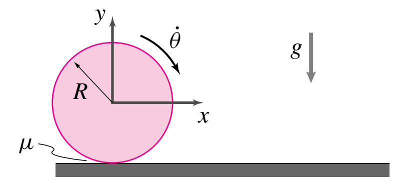
$$\vec{a}_{cm} = \frac{F}{M+2m}\hat{i}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.11 Dropped spinning disk. 2-D** . A uniform disk of radius $R$ and mass $m$ is gently dropped onto a surface and doesn't bounce. When it is released it is spinning clockwise at the rate $\dot{\theta}_0$. The disk skids for a while and then is eventually rolling.

a) What is the speed of the center of the disk when the disk is eventually rolling (answer in terms of $g$, $\mu$, $R$, $\dot{\theta}_0$, and $m$)?

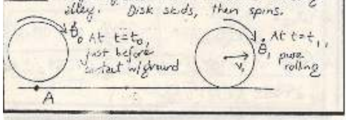
b) In the transition from slipping to rolling, energy is lost to friction. How does the amount lost depend on the coefficient of friction (and other parameters)? How does this loss make or not make sense in the limit as $\mu \to 0$ and the dissipation rate $\to$ zero?



Problem 17.11

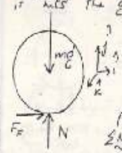
**Answer:**
(a) speed $v = \frac{R\dot{\theta}}{3}$.
(b) The energy lost to friction is $E_{fric} = \frac{mR^2\dot{\theta}_0^2}{6}$. The energy lost to friction is independent of $\mu$ for $\mu > 0$. Thus, the energy lost to friction is constant for given $m$, $R$, and $\dot{\theta}_0$. As $\mu \to 0$, the transition time to rolling $\to \infty$. It is not true, however, that the energy lost to friction $\to 0$ as $\mu \to 0$. Since the energy lost is constant for any $\mu > 0$, the disk will slip for longer and longer times so that the distance of slip goes to infinity. The dissipation rate $\to 0$ since the constant energy is divided by increasing transition time. The energy lost is zero only for $\mu = 0$.

7.80 Maybe you've seen this at the bowling alley. Disk skids, then spins.




7.80 continued p.3

FBD of disk at any time after it hits the ground.




Note that angular momentum $\underset{\sim}{H}$ about the point A on the ground is conserved between $t_0$ and $t_1$. (verify by confirming that $\sum \underset{\sim}{M}_{/A}$ always equals zero $\Rightarrow$ $\dot{\underset{\sim}{H}}_{/A}$ is also always zero.

$\Rightarrow \underset{\sim}{H}_{/A}(t=t_0) = I_{zz}^{cm}\,\underset{\sim}{\omega}_0 = -\frac{1}{2}mR^2\dot{\theta}_0\hat{k}$

$\Rightarrow \underset{\sim}{H}_{/A}(t=t_1) = I_{zz}^{cm}\,\underset{\sim}{\omega}_1 + \underset{\sim}{r}_{cm/A} \times m\underset{\sim}{v}_{cm}$

(note $\underset{\sim}{v}_{cm} = R\dot{\theta}\hat{i}$) $\Rightarrow \underset{\sim}{H}_{/A}(t=t_1) = -\frac{1}{2}mR^2\dot{\theta}_1\hat{k} + R\hat{j} \times mR\dot{\theta}_1\hat{i} = -\frac{3}{2}mR^2\dot{\theta}_1\hat{k}$

$\Rightarrow (-\hat{k})\cdot(\underset{\sim}{H}_{/A}(t=t_0) = \underset{\sim}{H}_{/A}(t=t_1)) =$

$\frac{1}{2}mR^2\dot{\theta}_0 = \frac{3}{2}mR^2\dot{\theta}_1 \Rightarrow \dot{\theta}_1 = \frac{1}{3}\dot{\theta}_0$

thus $V_{cm} = R\dot{\theta}_1 = \boxed{\frac{1}{3}R\dot{\theta}_0 = V_{cm}}$

b) Find energy lost to friction.
Let $E_{K_0}$ = kinetic energy of disk at $t=t_0$
Let $E_{K_1}$ = " " " " " $t=t_1$

then Energy lost $= E_{K_0} - E_{K_1}$

$= \left(\frac{1}{2}I_{zz}^{cm}\dot{\theta}_0^2\right) - \left(\frac{1}{2}I_{zz}^{cm}\dot{\theta}_1^2 + \frac{1}{2}mV_{cm_1}^2\right)$

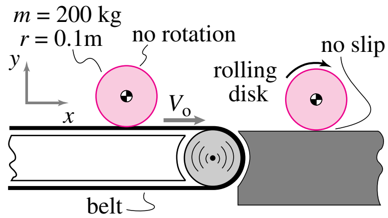
$= \frac{1}{4}mR_0^2\dot{\theta}_0 - \frac{1}{4}mR_0^2\left(\frac{1}{3}\dot{\theta}_0\right)^2 - \frac{1}{2}m\left(\frac{R\dot{\theta}_0}{3}\right)^2$

$= \boxed{\frac{1}{6}MR_0^2\dot{\theta}_0^2 = \text{energy lost (does not depend on } \mu\text{!)}}$

In a bowling alley, you can see that it slides and rolls for a long way (perhaps all the way) down the alley before it reaches the state of pure rolling. $\mu$ is low. On a rough surface, $\mu$ is higher and the transition takes place more quickly. But, as long as $\mu \neq 0$, this transition will occur eventually.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

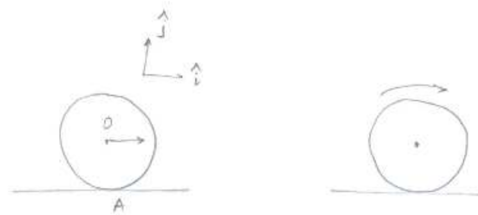
**17.4.12 Disk on a conveyor belt.** A uniform metal cylinder with mass of 200 kg is carried on a conveyer belt which moves at $V_0 = 3\,\mathrm{m/s}$. The disk is not rotating when on the belt. The disk is delivered to a flat hard platform where it slides for a while and ends up rolling. How fast is it moving (i.e. what is the speed of the center of mass) when it eventually rolls?




Problem 17.12

**Answer:**
$V = 2\,\mathrm{m/s}$.

16.4.12




Just before rolling

$$^1\underline{H}_{/A} = \underline{r}_{O/A} \times m\,\underline{v}_{cm} \longrightarrow (1)$$

During rolling

$$^2\underline{H}_{/A} = \underline{r}_{O/A} \times m\,\underline{v}_f + \underline{\underline{I}}^{cm} \cdot \underline{\omega}_f \longrightarrow (2)$$

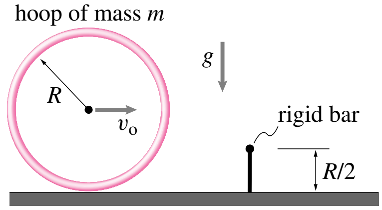
Equating (1) and (2)

$$-mRv_0 = -Rmv_f - \frac{mr^2}{2} \times \frac{v_f}{r}$$

$$-v_0 = -v_f - \frac{v_f}{2} \Rightarrow v_0 = \frac{3v_f}{2} \Rightarrow v_f = 2\,m/s$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.13** A rigid hoop with radius $R$ and mass $m$ is rolling without slip so that its center has translational speed $v_o$. It then hits a narrow bar with height $R/2$. When the hoop hits the bar suddenly it sticks and doesn't slide. It does hinge freely about the bar, however. The gravitational constant is $g$. How big is $v_o$ if the hoop just barely rolls over the bar?

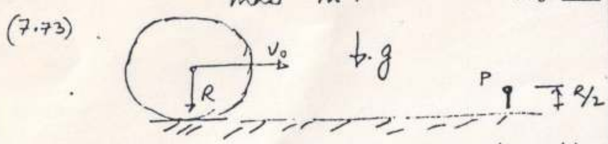



Problem 17.13

**Answer:**
$v_o = 2\frac{\sqrt{2gR}}{3}$.

Page 13.

(7.73) mass m.




The hoop rolls w/o slipping. How big should $v_0$ be such that the hoop barely climbs above bar. Assume it sticks at P and hinges freely at that point

Solution:

Since the force of friction does no work during rolling w/o slip, we use conservation of energy.

Initial energy: $E_P = 0$ (assumed reference position)

$E_K = \frac{1}{2} m v_0^2 + \frac{1}{2} I_{cm} \omega_0^2$

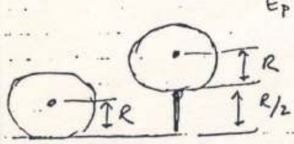
Since it rolls w/o slip $\omega_0 = \frac{v_0}{R}$

$\therefore E_K = \frac{1}{2} m v_0^2 + \frac{1}{2} \underbrace{(m R^2)}_{I_{zz}^{cm} \text{ for a hoop}} \cdot \frac{v_0^2}{R^2}$

$E_K = \boxed{m v_0^2}$

Final energy: $E_K = 0$

$E_P = mg\frac{R}{2}$

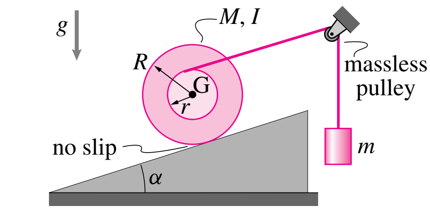



Equating $E_{initial}$ and $E_{final}$.

$\boxed{v_0 = \sqrt{\frac{gR}{2}}}$

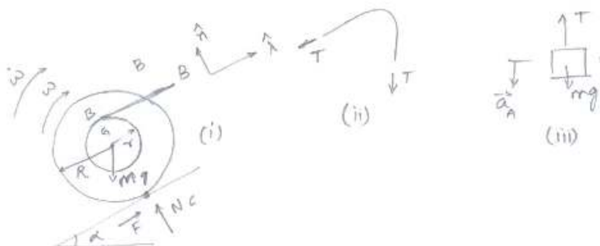
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.15 Spool and mass.** A reel of mass $M$ and moment of inertia $I_{zz}^{cm} = I$ rolls without slipping upwards on an incline with slope-angle $\alpha$. It is pulled up by a string attached to mass $m$ as shown. Find the acceleration of point G in terms of some or all of $M, m, I, R, r, \alpha, g$ and any base vectors you clearly define.




Problem 17.15

16.4.15




$$\sum \underline{M}_{/C} = \dot{H}_{/C}$$

$$R\hat{n} \times mg(-\hat{j}) + (R+r)\hat{n} \times T\hat{\lambda} = \underline{R} \times m\underline{a}_G + \underline{I}^{cm}\dot{\omega}\hat{k}$$

$$mgR\sin\alpha\,\hat{k} - (R+r)T\hat{k} = R\hat{n} \times m(-\dot{\omega}R)\hat{\lambda} + I\dot{\omega}\hat{k}$$

$$(mgR\sin\alpha - (R+r)T)\hat{k} = (mR^2\dot{\omega} + I\dot{\omega})\hat{k} \longrightarrow (1)$$

Now, From the FBD of the mass 'm'.

$$T\hat{j} - mg\hat{j} = m\underline{a}_A$$

$$|a_A| = |a_B|$$

$$\therefore (T - mg) = m(\underline{a}_G + a_{B/G})$$

$$(T - mg) = m(\dot{\omega}R + \dot{\omega}r)$$

$$T = mg - m\dot{\omega}(R+r)$$

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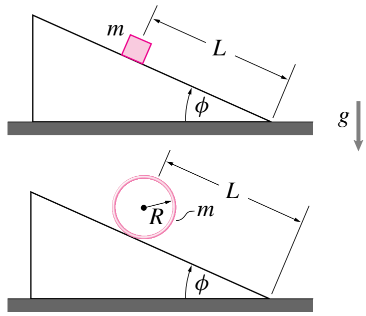
on substiting the value in (I).

$$mgR\sin\alpha - mg(R+r) + m\dot{\omega}(R+r)^2 = (mR^2 + I)\dot{\omega}$$

$$\dot{\omega} = \frac{mg(R\sin\alpha - (R+r))}{(mR^2 + I - m(R+r)^2)}$$

$$\underline{a}_{cm} = \frac{mgR[R\sin\alpha - (R+r)]}{(mR^2 + I - m(R+r)^2)}\hat{\lambda}$$

**17.4.16** Two objects are released on two identical ramps. One is a sliding block (no friction), the other a rolling hoop (no slip). Both have the same mass, $m$, are in the same gravity field and have the same distance to travel. It takes the sliding mass 1 s to reach the bottom of the ramp. How long does it take the hoop? [Useful formula: "$s = \frac{1}{2}at^2$"]




Problem 17.16

16.4.16

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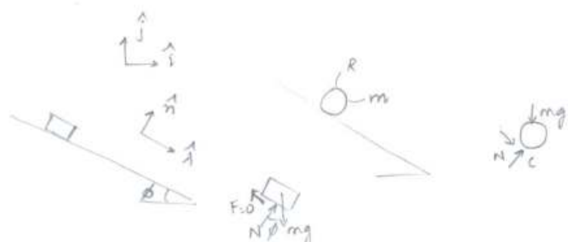


$$\sum \vec{F} = m\vec{a}$$

$$N\hat{n} - mg\hat{j} = ma\hat{\lambda}$$

$$N\hat{n} + mg\sin\phi\,\hat{\lambda} - mg\cos\phi\,\hat{n} = ma\hat{\lambda}$$

$$\hat{j} = -\sin\phi\,\hat{\lambda} + \cos\phi\,\hat{n}$$

$$\{\ \}\cdot\hat{\lambda} \Rightarrow \quad a = g\sin\phi$$

$$\{\ \}\cdot\hat{n} \Rightarrow \quad N = mg\cos\phi$$

$$s = \frac{1}{2}at^2 \Rightarrow \quad L = \frac{1}{2}g\sin\phi$$

$$\vec{a}_G = \dot{\vec{\omega}} \times R\hat{n}$$
$$= -\dot{\omega}\hat{k} \times R\hat{n}$$
$$\vec{a}_G = a_h\hat{\lambda}$$

$$\sum M_{/c} = \dot{\underline{H}}_{/c}$$

$$R\hat{n} \times mg(-\hat{j}) = R\hat{n} \times m\vec{a}_G + I^{cm}(-\dot{\omega})\hat{k}$$

$$-mgR\hat{k}\sin\phi = -ma_hR\hat{k} - mRa\hat{k}$$

$$a_G = \frac{g}{2}\sin\phi$$

$$t^2 \propto \frac{1}{a}$$

$$\frac{t_h^2}{t_b^2} = \frac{a_b}{a_h} \quad \times t_h^2 = \frac{a_b}{a_h}\cdot t_b^2$$

$$= \frac{g\sin\phi}{\frac{g\sin\phi}{2}}$$

$$t_h = \sqrt{2}\,t_b$$

The hoop takes $\sqrt{2}$ times longer than the mass

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

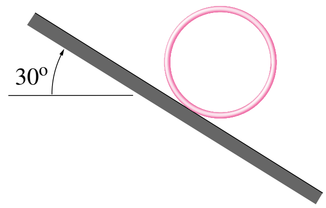
**17.4.17** The hoop is rolled down an incline that is 30° from horizontal. It does not slip. It does not fall over sideways. It is let go from rest at t = 0.

a) At $t = 0^+$ what is the acceleration of the hoop center of mass?

b) At $t = 0^+$ what is the acceleration of the point on the hoop that is on the incline?

c) At $t = 0^+$ what is the acceleration of the point on the hoop that is furthest from the incline?

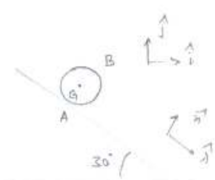
d) After the hoop has descended 2 vertical meters (and traveled an appropriate distance down the incline) what is the acceleration of the point on the hoop that is (at that instant) furthest from the incline?




Problem 17.17

(9)

16.4.17




a) $a_G = ?$ (from previous problem)

$a_G = \frac{g}{2}\sin\phi = \frac{g}{4}\hat{\lambda}$

or

$a_G = \dot{\vec{\omega}} R \hat{\lambda}$

b) $a_A = ?$

$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$

$\vec{a}_G = \dot{\omega} R \hat{\lambda}$

$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G} = \omega^2 R \hat{n}$

$\vec{a}_{A/G} = \dot{\vec{\omega}} \times R(-\hat{n}) + \vec{\omega} \times (\vec{\omega} \times R(-\hat{n}))$

$= -\dot{\vec{\omega}}\hat{k} \times R(-\hat{n}) + (-\omega\hat{k}) \times (-\omega\hat{k} \times -R\hat{n})$

$= \dot{\vec{\omega}} R(\hat{k} \times \hat{n}) - \omega\hat{k} \times \omega R(-\hat{\lambda})$

$\vec{a}_{A/G} = -\dot{\omega} R \hat{\lambda} + \omega^2 R \hat{n}$

c) $\vec{a}_B = ?$

$\vec{a}_B = \vec{a}_G + \vec{a}_{B/G}$

$\vec{a}_{B/G} = -\dot{\omega}\hat{k} \times R(\hat{n}) + [-\omega\hat{k} \times (-\omega\hat{k} \times R\hat{n})]$

$= \dot{\omega} R \hat{\lambda} - \omega^2 R \hat{n}$

Now,

$\vec{a}_B = 2\dot{\omega} R \hat{\lambda} - \omega^2 R \hat{n}$

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d)

As $a_G = \dot{\omega} R = \frac{g}{2} \sin 30^\circ = \frac{g}{4}$

$\dot{\omega} = \frac{g}{4R} \Rightarrow \omega = \frac{gt}{4R} + C_1$ at $t=0$, $\omega = 0$

$C_1 = 0$

$\Rightarrow \omega = \frac{gt}{4R} \Rightarrow \theta = \frac{1}{2} \frac{g}{4R} t^2$

No slip $\Rightarrow$

$x = R\theta \Rightarrow L = R \cdot \frac{1}{2} \frac{g}{4R} t^2$

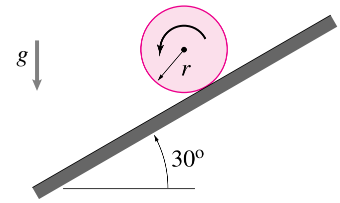
$t = \sqrt{\frac{8L}{g}} = 4\sqrt{\frac{y}{g}}$

$\omega = \frac{g4}{4R}\sqrt{\frac{y}{g}} = \sqrt{\frac{yg}{R^2}} = \frac{\sqrt{2g}}{R}$

$\therefore \vec{a}_B = 2 \cdot \frac{g}{4} \hat{\lambda} - \frac{2g}{R^2} \cdot R \cdot \hat{n}$

$\vec{a}_B = \frac{g}{2} \hat{\lambda} - \frac{2g}{R} \hat{n}$

**17.4.18** A uniform cylinder of mass $m$ and radius $r$ rolls down an incline *without slip*, as shown below. Determine: (a) the angular acceleration $\alpha$ of the disk; (b) the minimum value of the coefficient of friction $\mu$ that will insure no slip.




Problem 17.18

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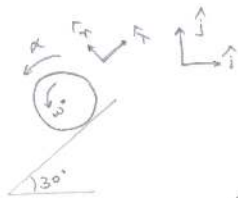
16.4.18





a) $\sum \underline{M}_{/c} = \dot{\underline{H}}_{/c}$

$r\hat{n} \times mg(-\hat{j}) = r\hat{n} \times m\vec{a}_G + I^{cm}\dot{\omega}\hat{k}$

$\sin\alpha \, mgr\hat{k} = mr^2\dot{\omega}\hat{k} + \frac{mr^2}{2}\dot{\omega}\hat{k}$

$\dot{\omega} = \frac{2g\sin\alpha}{3r} \Rightarrow \dot{\omega} = \frac{g}{3r}$

b) Minimum value of co-efficient of friction to ensure no slip

$\sum \vec{F} = m\vec{a}_G$

$N\hat{n} - F\hat{\lambda} - mg\hat{j} = ma_G(-\hat{\lambda})$

$\Rightarrow \{N\hat{n} - F\hat{\lambda} - mg\sin\hat{\lambda} - mg\cos\alpha\,\hat{n} = -m\dot{\omega}r\hat{\lambda}\}$

$\{\}\cdot\hat{n} \Rightarrow N = mg\cos\alpha$, $\{\}\hat{\lambda} \Rightarrow -F - mg\sin\alpha = -m\dot{\omega}r = -\frac{mg}{3}$

$F = \frac{mg}{3} - \frac{mg}{2} \Rightarrow F = -\frac{mg}{6} \Rightarrow F = \mu N$, $\mu = \frac{F}{N}$

$\mu = \frac{-\frac{mg}{6}}{mg\cos\alpha} = \frac{1}{3\sqrt{3}}$

Any $\mu < \frac{1}{3\sqrt{3}}$, at $\theta = 30°$ results in cylinder slip

**17.4.19 Race of rollers.** A uniform disk with mass $M_0$ and radius $R_0$ is allowed to roll down the quite slip-resistant ($\mu = 1$) 30° ramp shown. It is raced against four other objects ($A$, $B$, $C$ and $D$), one at a time. Who wins the races, or are there ties? First try to construct any plausible reasoning. Good answers will be based, at least in part, on careful use of equations of mechanics.

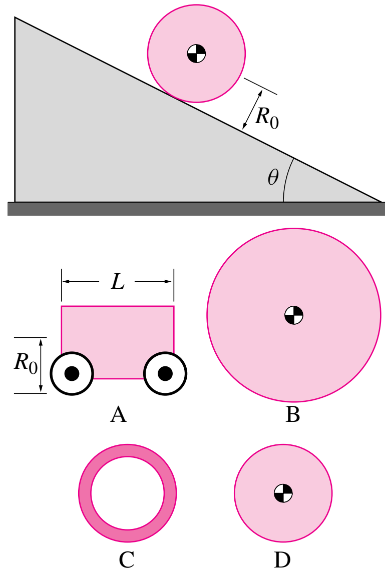
a) Block $A$ has the same mass and has center of mass a distance $R_0$ from the ramp. It rolls on massless wheels with frictionless bearings.

b) Uniform disk $B$ has the same mass ($M_B = M_0$) but twice the radius ($R_B = 2R_0$).

c) Hollow pipe $C$ has the same mass ($M_C = M_0$) and the same radius ($R_C = R_0$).

d) Uniform disk $D$ has the same radius ($R_D = R_0$) but twice the mass ($M_D = 2M_0$).

Can you find a round object which will roll as fast as the block slides? How about a massless cylinder with a point mass in its center? Can you find an object which will go slower than the slowest or faster than the fastest of these objects? What would they be and why? (This problem is harder.)

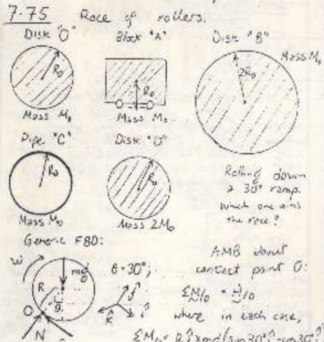



Problem 17.19

**Answer:**
Accelerations of the center of mass, where $\hat{\boldsymbol{\imath}}$ is parallel to the slope and pointing down: (a) $\vec{\boldsymbol{a}}_{cm} = g\sin\theta\hat{\boldsymbol{\imath}}$, (b) $\vec{\boldsymbol{a}}_{cm} = \frac{2}{3}g\sin\theta\hat{\boldsymbol{\imath}}$, (c) $\vec{\boldsymbol{a}}_{cm} = \frac{1}{2}g\sin\theta\hat{\boldsymbol{\imath}}$, (d) $\vec{\boldsymbol{a}}_{cm} = \frac{2}{3}g\sin\theta\hat{\boldsymbol{\imath}}$. So, the block is fastest, all uniform disks are second, and the hollow pipe is third.

Solutions to Problem Set 10 · TAM 203 Su 96
TA: Erich Schneider · July 26, 1996

7.75 Race of rollers.




Rolling down a 30° ramp. Which one wins the race?

$\theta = 30°$;

AMB about contact point O:

$\sum \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O}$

where in each case,

$\sum \underset{\sim}{M}_{/O} = R\hat{\jmath} \times mg(\sin 30°\hat{\imath} - \cos 30°\hat{\jmath})$

$\sum \underset{\sim}{M}_{/O} = -Rmg\sin 30°\hat{k}$

For things that roll: (Disks "O", "B", "D", pipe "C")

$\dot{\underset{\sim}{H}}_{/O} = \underset{\sim}{r}_{cm/O} \times m\underset{\sim}{a}_{cm} + I^{cm}_{33}\dot{\underset{\sim}{\omega}}$ (1)

where $\dot{\underset{\sim}{\omega}} = -\ddot{\theta}\hat{k}$; $\underset{\sim}{a}_{cm} = a_{cm}\hat{\imath} = R\ddot{\theta}\hat{\imath}$

Some reasoning/guesses first:

Note that each term in the AMB is proportional to the mass of the object. This is true as well for the block:

block: $\dot{\underset{\sim}{H}}_{/O} = \underset{\sim}{r}_{cm/O} \times m\underset{\sim}{a}_{cm} + \cancel{I^{cm}_{33}\dot{\underset{\sim}{\omega}}}^{\,0}$ straight line motion

where $\underset{\sim}{a}_{cm} = a_{cm}\hat{\imath}$

Thus, the mass will cancel out ⇒ speed of rolling does not depend on mass, but rather on geometry. Thus, disk "D" and disk "O" will roll at the same rate.

Note from (1) above that $\ddot{\theta} \propto 1/R$; but $a_{cm} = R\ddot{\theta}$, so the acceleration of the center of mass will not depend on the radius of the body. In fact, the first

Note: see addendum on last pg for more of prob. 7.75

7.75 continued p. 2

term in (1), as well as the LHS of the AMB, is the same for each of our 5 objects. The answer depends only on the relative moment of inertia of each of our rolling bodies. Since the moment of inertia of hoop "C" ($= mR^2$) is bigger than the moment of inertia of a disk ($= \frac{1}{2}mR^2$), I would expect that the disks will all accel. at the same rate ($a_{cm}$ does not depend on R or M) which is faster than the rate at which the hoop will accelerate. The box will accelerate fastest of all, since it is undergoing straight line acceleration ($\dot{\omega} \,\&\, \omega = 0$).

Thus

| Block A | (Disks O, B, D) | Hoop C |
|---|---|---|
| Fastest | same accel. | slowest |

Calculations

Disk O

$-R_0M_0 g\sin 30° = R_0\hat{\jmath} \times M_0(R_0\ddot{\theta}\hat{\imath}) - \frac{1}{2}M_0R_0^2\ddot{\theta}\hat{k}$

$\Rightarrow \ddot{\theta} = \frac{2}{3R_0}g\sin 30° \Rightarrow \boxed{a_{cm} = \frac{2}{3}g\sin 30° \quad \text{Disk "O"}}$

Block "A"

$-R_0M_0 g\sin 30° = R_0\hat{\jmath} \times M_0 a_{cm}\hat{\imath}$

$\Rightarrow \boxed{a_{cm} = g\sin 30° \quad \text{Block "A"}}$

Disk "B"

$-2R_0M_0\sin 30° = 2R_0\hat{\jmath} \times M_0(2R_0\ddot{\theta}\hat{\imath}) - \frac{1}{2}M_0(2R_0)^2\ddot{\theta}\hat{k}$

$\Rightarrow \ddot{\theta} = \frac{1}{3R}g\sin 30° \xRightarrow{a_{cm} = 2R_0\ddot{\theta}} \boxed{a_{cm} = \frac{2}{3}g\sin 30° \quad \text{Disk "B"}}$

Pipe "C"

$-R_0M_0\sin 30° = R_0\hat{\jmath} \times M_0(R_0\ddot{\theta}\hat{\imath}) - M_0R_0^2\ddot{\theta}\hat{k}$

$\Rightarrow \ddot{\theta} = \frac{1}{2R_0}g\sin 30° \Rightarrow \boxed{a_{cm} = \frac{1}{2}g\sin 30° \quad \text{Hoop "C"}}$

Disk "D"

$-R_0(2M_0)\sin 30° = R_0\hat{\jmath} \times (2M_0)R_0\ddot{\theta}\hat{\imath} - \frac{1}{2}(2M_0)R_0^2\ddot{\theta}\hat{k}$

$\Rightarrow \ddot{\theta} = \frac{2}{3R_0}g\sin 30° \Rightarrow \boxed{a_{cm} = \frac{2}{3}g\sin 30° \quad \text{Disk "D"}}$

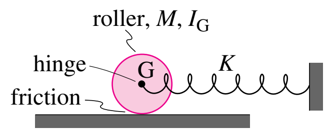
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.20** A roller of mass $M$ and polar moment of inertia about the center of mass $I_G$ is connected to a spring of stiffness $K$ by a frictionless hinge as shown in the figure. Consider two kinds of friction between the roller and the surface it moves on:

1. Perfect slipping (no friction), and
2. Perfect rolling (infinite friction).

a) What is the period of oscillation in the first case?

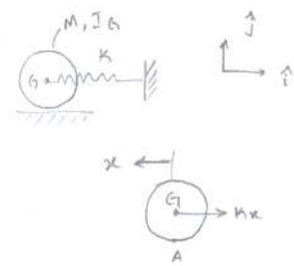
b) What is the period of oscillation in the second case?



Problem 17.20

(12)

16.4.20




a) No friction

$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$r\hat{j} \times Kx\,\hat{i} = r\hat{j} \times m\vec{a}_G$$

$$Krx\,(\hat{K}) = -r\hat{j} \times m\ddot{x}\,\hat{i} = m\ddot{x}r\hat{K}$$

$$m\ddot{x} + Kx = 0 \Rightarrow \quad \omega = \sqrt{\frac{K}{m}}\,, \quad T = 2\pi\sqrt{\frac{m}{K}}$$

b) Perfect rolling

$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$r\hat{j} \times Kx\,\hat{i} = r\hat{j} \times m\ddot{x}(-\hat{i}) + I\dot{\omega}\hat{K}$$

$$Kxr(-\hat{K}) = m\ddot{x}r\hat{K} + I\frac{\ddot{x}}{r}\hat{K}$$

$$\left(m + \frac{I}{r^2}\right)\ddot{x} + Kx = 0$$

$$\omega = \sqrt{\frac{K}{m + \frac{I}{r^2}}}$$

$$T = 2\pi\sqrt{\frac{m + \frac{I}{r^2}}{K}}\,, \quad I = \frac{mr^2}{2}$$

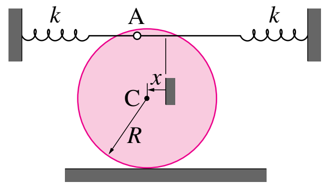
$$T = \left(\sqrt{\tfrac{3}{2}}\right) 2\pi\sqrt{\frac{m}{K}}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.21** A uniform cylinder of mass $m$ and radius $R$ rolls back and forth without slipping through small amplitudes (i.e., the springs attached at point A on the rim act linearly and the vertical change in the height of point A is negligible). The springs, which act both in compression and tension, are unextended when A is directly over C.

a) Determine the differential equation of motion for the cylinder's center.

b) Calculate the natural frequency of the system for small oscillations.




Problem 17.21

(13)

16.4.21

$$\sum \underline{M}_{/c} = \dot{\underline{H}}_{/c}$$

a) $\cos\alpha\, 2R\hat{j} \times 2Kx\hat{i} = R\hat{j} \times m\underline{a}_{cm} + I^{cm}\dot{\omega}\hat{k}$

$-m\dot{\omega}r\hat{i}$

$$\cos\alpha(-4KxR)\hat{k} = m\dot{\omega}R^2\hat{k} + \frac{mR^2}{2}\dot{\omega}\hat{k}$$

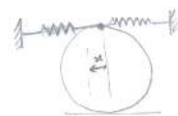
$$= mR\ddot{x}\hat{k} + mR\frac{\ddot{x}}{2}\hat{k}$$

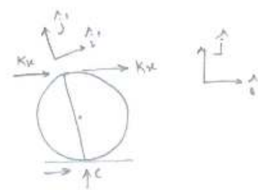
$$(\cos\alpha)(-4KxR\hat{k}) = \frac{3}{2}mR\ddot{x}\hat{k}$$

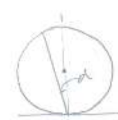
$$\frac{3}{2}m\ddot{x} + 4Kx\cos\alpha = 0$$

$$m\ddot{x} + \frac{8}{3}Kx\cos\alpha = 0$$

$$\omega = \sqrt{\frac{8K\cos\alpha}{3m}}$$





Differential Eq^n

$$3m\ddot{x} + 8K\cos\alpha = 0$$

b) For small oscillations $\cos\alpha = 1$

$$f = \frac{1}{2\pi}\sqrt{\frac{8K}{3m}}$$

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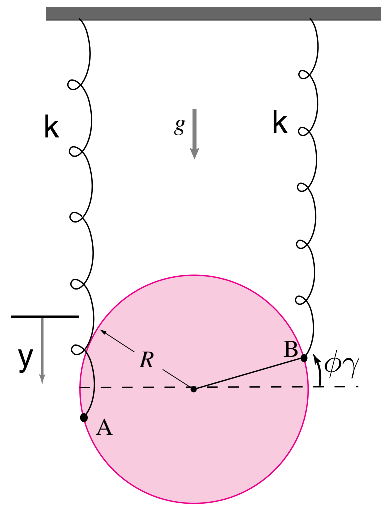
**17.4.22 Hanging disk, 2-D.** A uniform thin disk of radius $R$ and mass $m$ hangs in a gravity field $g$ from a pair of massless springs each with constant $k$. In the static equilibrium configuration the springs are vertical and attached to points A and B on the right and left edges of the disk. In the equilibrium configuration the springs carry the weight, the disk counter-clockwise rotation is $\phi = 0$, and the downwards vertical deflection is $y = 0$. Assume throughout that the center of the disk only moves up and down, and that $\phi$ is small so that the springs may be regarded as vertical when calculating their stretch ($\sin\phi \approx \phi$ and $\cos\phi \approx 1$).

a) Find $\ddot{\phi}$ and $\ddot{y}$ in terms of some or all of $\phi, \dot{\phi}, y, \dot{y}, k, m, R$, and $g$.

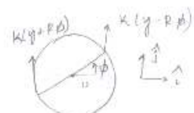
b) Find the natural frequencies of vibration in terms of some or all of $k, m, R$, and $g$.

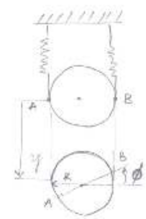



Problem 17.22

14

16.4.22




$\sum M_{/o} = \dot{H}_{/o}$

Let $\phi$ = small

$I\ddot{\phi} = KR(y - R\phi) - KR(y + R\phi) \Rightarrow I\ddot{\phi} = -KR^2\phi - KR^2\phi$

$m\ddot{\phi} + 4K\phi = 0$

$\omega_1 = \sqrt{\frac{4K}{m}}$

Now,

$\sum F = m\ddot{a}$

$[K(y - R\phi) + K(y + R\phi)]\hat{j} = m\ddot{y}(-\hat{j})$

$m\ddot{y} + 2Ky = 0$

$\omega_2 = \sqrt{\frac{2K}{m}}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**17.4.23 A disk rolls in a cylinder.** For all of the problems below, the disk rolls without slip and rocks back and forth due to gravity.

a) **Sketch.** Draw a neat sketch of the disk in the cylinder. The sketch should show all variables, coordinates and dimension used in the problem.

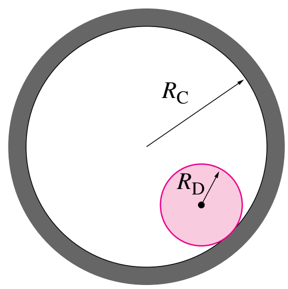
b) **FBD.** Draw a free-body diagram of the disk.

c) **Momentum balance.** Write the equations of linear and angular momentum balance for the disk. Use the point on the cylinder which touches the disk for the angular momentum balance equation. Leave as unknown in these equations variables which you do not know.

d) **Kinematics.** The disk rolling in the cylinder is a *one*-degree-of-freedom system. That is, the values of only *one* coordinate and its derivatives are enough to determine the positions, velocities and accelerations of all points. The angle that the line from the center of the cylinder to the center of the disk makes from the vertical can be used as such a variable. Find all of the velocities and accelerations needed in the momentum balance equation in terms of this variable and its derivative. [Hint: you'll need to think about the rolling contact in order to do this part.]

e) **Equation of motion.** Write the angular momentum balance equation as a single second order differential equation.

f) **Simple pendulum?** Does this equation reduce to the equation for a pendulum with a point mass and length equal to the radius of the cylinder, when the disk radius gets arbitrarily small? Why, or why not?

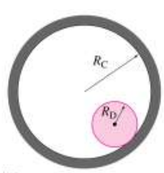



Problem 17.23: A disk rolls without slip inside a bigger cylinder.

16.4.23

Saturday, April 20, 2019 10:38 PM

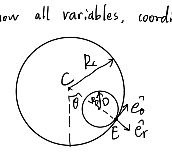
A disk rolls in a cylinder

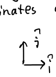



Problem 16.4.23: A disk rolls without slip inside a bigger cylinder.

No slip disk rolls back and forth due to gravity

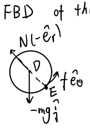
a) sketch the disk in the cylinder.
Show all variables, coordinates and dimension




(b) Draw FBD of the disk

FBD




$e_r = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$e_\theta = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

(c) Find LMB & AMB/$_E$ for the disk

*Disclaimer:* *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

LMB : $\sum \vec{F} = m\vec{a}_c$

$$-mg\hat{j} + f\hat{e}_\theta - N\hat{e}_r = m\vec{a}_c$$

AMB/E : $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E}$

where $\sum \vec{M}_{/E} = R_D\hat{e}_r \times mg\hat{j} = R_D(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \times mg\hat{j}$
$= R_D mg\sin\theta\,\hat{k}$

and $\dot{\vec{H}}_{/E} = \vec{r}_{D/E} \times m\vec{a}_D + I\dot{\omega}\hat{k} = -R_D\hat{e}_r \times m\vec{a}_D + \frac{1}{2}mR_D^2\dot{\omega}\hat{k}$

(d) Kinematics

Find all the velocities and accelerations needed in momentum balance equation in terms of $\theta$.

$$\vec{V}_D = \cancelto{0}{\vec{V}_C} + \dot{\theta}\hat{k} \times (R_C - R_D)\hat{e}_r$$
$$= \dot{\theta}(R_C - R_D)\hat{e}_\theta \qquad (1)$$

Take derivative

$$\vec{a}_D = \ddot{\theta}(R_C - R_D)\hat{e}_\theta - \dot{\theta}^2(R_C - R_D)\hat{e}_r \qquad (2)$$

Note : $\dot{\theta} \neq \omega$, angular velocity of disk,
find $\omega$ as follows

Rolling w/o slip :

$$\vec{V}_E = \vec{0}$$

$$\vec{V}_D = \vec{V}_E + \vec{\omega} \times (-\vec{R}_D\hat{e}_r) = -\omega R_D\hat{e}_\theta \qquad (3)$$

(1) (3) $\Rightarrow$ $\omega = -\dfrac{R_C - R_D}{R_D}\dot{\theta}$

Take derivative $\dot{\omega} = -\dfrac{R_C - R_D}{R_D}\ddot{\theta}$

(e) Equation of motion

Find AMB/E as a 2nd order ODE.

Substitute $\vec{a}_D$, $\dot{\omega}$ into AMB/E.

$$mg R_D \sin\theta \, \hat{k} = -mR_D \ddot{\theta} (R_C - R_D) \hat{k} + \frac{1}{2} m R_D^2 \left(-\frac{R_C - R_D}{R_D}\right) \ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} + \frac{2g}{3(R_C - R_D)} \sin\theta = 0 \qquad (*)$$

(f) Simple pendulum?

Does $(*)$ reduce to the equation for a pendulum with a point mass and length equal to the cylinder, when $R_D \to 0$?

If $R_D \to 0$, $(*)$ becomes $\ddot{\theta} = -\frac{2}{3}\frac{g}{R_C}\sin\theta$

which is different from simple pendulum equation

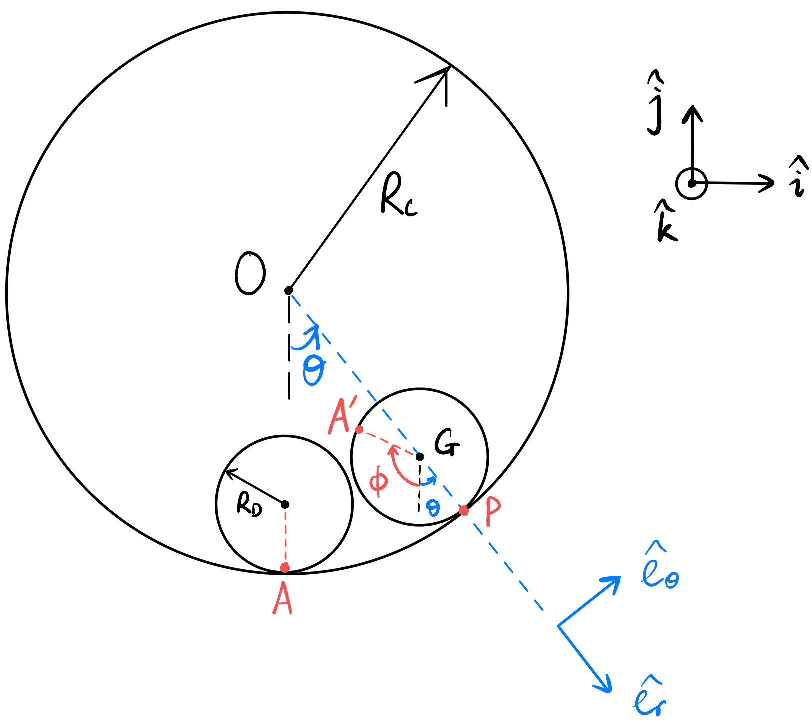
$$\ddot{\theta} = -\frac{g}{R_C}\sin\theta$$

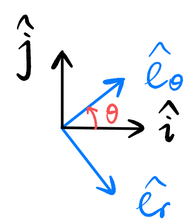
MAE 2030 HW P60 (17.4.23) Solution

Duan Li

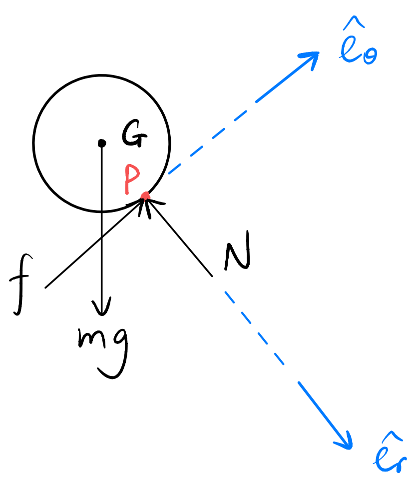
Given : $R_D$ , $R_c$ , no slipping , gravity

a) Sketch




b) FBD



c) Find LMB and $AMB_{/P}$

LMB

$$m\vec{a}_G = \sum \vec{F}$$

$$\boxed{m\vec{a}_G = f\hat{e}_\theta - mg\hat{j} - N\hat{e}_r}$$

$AMB_{/P}$

$$\dot{\vec{H}}_{/P} = \sum \vec{M}_{/P}$$

$$m\vec{r}_{G/P} \times \vec{a}_G + I^G(-\ddot{\phi})\hat{k} = \vec{r}_{G/P} \times (-mg\hat{j})$$

where

$$\vec{r}_{G/P} = -R_D\hat{e}_r$$

$$\vec{a}_G = (R_c - R_D)\ddot{\theta}\hat{e}_\theta - (R_c - R_D)\dot{\theta}^2\hat{e}_r$$

$$I^G = \frac{1}{2}mR_D^2 \quad \text{for a disk}$$

Simplify

$$\boxed{-mR_D(R_c - R_D)\ddot{\theta}\hat{k} - \frac{1}{2}mR_D^2\ddot{\phi}\hat{k} = mgR_D\sin\theta\hat{k}} \quad (1)$$

d) Find $\ddot{\phi}$ in terms of $\theta$

Method 1

Rolling w/o slipping $\rightarrow$ $\vec{v}_P = \vec{0}$

$$
\begin{aligned}
\vec{v}_P &= \vec{v}_G + \vec{v}_{P/G} \\
&= \dot{\theta}\,\hat{k} \times (R_c - R_D)\,\hat{e}_r + (-\dot{\phi})\,\hat{k} \times R_D\,\hat{e}_r \\
&= \vec{0}
\end{aligned}
$$

$$\dot{\phi} = \frac{R_c - R_D}{R_D}\dot{\theta} \longrightarrow \boxed{\ddot{\phi} = \frac{R_c - R_D}{R_D}\ddot{\theta}} \qquad (2)$$

Method 2

Rolling w/o slipping $\rightarrow$ Curve length $|\overset{\frown}{AP}| = |\overset{\frown}{A'P}|$

$$R_c\theta = R_D(\phi + \theta)$$

$$\phi = \frac{R_c - R_D}{R_D}\theta \longrightarrow \boxed{\ddot{\phi} = \frac{R_c - R_D}{R_D}\ddot{\theta}}$$

e) Find EoM ( AMB as a 2nd-order ODE )

Combine Eqn. (1) and (2)

$$-mR_D(R_c-R_D)\ddot{\theta}\,\hat{k}-\frac{1}{2}mR_D^{\cancel{2}}\frac{R_c-R_D}{\cancel{R_D}}\ddot{\theta}\,\hat{k}=mgR_D\sin\theta\,\hat{k}$$

$$-\frac{3}{2}mR_D(R_c-R_D)\ddot{\theta}\,\hat{k}=mgR_D\sin\theta\,\hat{k}$$

$$\boxed{\ddot{\theta}=-\frac{2}{3}\frac{g}{R_c-R_D}\sin\theta}$$

f) Compare w. a simple pendulum

As $R_D\to 0$, $\ddot{\theta}=-\frac{2}{3}\frac{g}{R_c}\sin\theta$

Simple pendulum w. length $R_c$: $\ddot{\theta}=-\frac{g}{R_c}\sin\theta$

The EoM does $\boxed{\text{NOT}}$ reduce to that for a simple pendulum w. length $R_c$.

Comments:

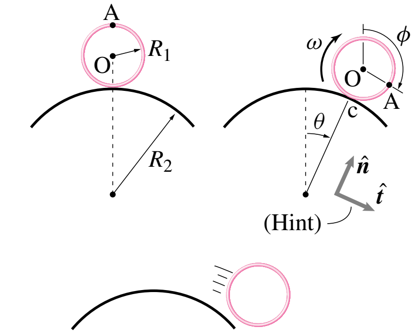
① As $R_D\to 0$, $\dot{\phi}\to\infty$, i.e. the rotation of the disk doesn't vanish.

② If the disk has mass distribution of just a point mass at its center, then $I^G=0$, and the EoM would be the same as that for a simple pendulum w. length $(R_c-R_D)$.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

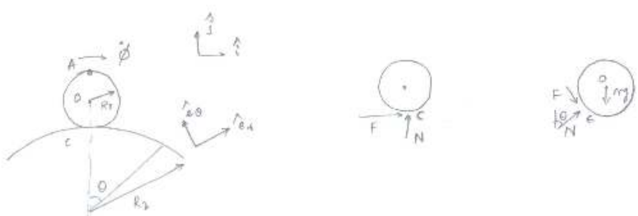
**17.4.24** A uniform hoop of radius $R_1$ and mass $m$ rolls from rest down a semi-circular track of radius $R_2$ as shown. Assume that no slipping occurs. At what angle $\theta$ does the hoop leave the track and what is its angular velocity $\omega$ and the linear velocity $\vec{v}$ of its center of mass at that instant? If the hoop slides down the track without friction, so that it does not rotate, will it leave at a smaller or larger angle $\theta$ than if it rolls without slip (as above)? Give a qualitative argument to justify your answer.

HINT: Here is a geometric relationship between angle $\phi$ hoop turns through and angle $\theta$ subtended by its center when no slipping occurs: $\phi = [(R_1 + R_2)/R_2]\theta$. (You may or may not need to use this hint.)




Problem 17.24

16.4.24




'O' - goes in circles with radius $(R_1+R_2)$

$$\underline{V}_o = \underline{\omega} \times (R_1+R_2)\hat{e}_r$$

$$= \dot{\theta}(-\hat{K}) \times (R_1+R_2)\hat{e}_r$$

$$\underline{V}_o = \dot{\theta}(R_1+R_2)(-\hat{e}_\theta)$$

$$\underline{a}_o = \underline{\omega} \times \underline{\omega} \times (R_1+R_2)\hat{e}_r + \dot{\underline{\omega}} \times (R_1+R_2)\hat{e}_r$$

$$= -\dot{\theta}\hat{K} \times (-\dot{\theta}\hat{K} \times (R_1+R_2)\hat{e}_r) - \ddot{\theta}\hat{K} \times (R_1+R_2)\hat{e}_r$$

$$= \dot{\theta}\hat{K} \times (R_1+R_2)\dot{\theta}\hat{e}_\theta - (R_1+R_2)\ddot{\theta}\hat{e}_\theta$$

$$= -(R_1+R_2)\dot{\theta}^2\hat{e}_r - (R_1+R_2)\ddot{\theta}\hat{e}_\theta$$

$$\hat{J} = \cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\sum \underline{F} = m\underline{a}$$

$$\left\{ -F\hat{e}_\theta + N\hat{e}_r - mg\hat{j} = m\left[-(R_1+R_2)(\dot{\theta}^2\hat{e}_r + \ddot{\theta}\hat{e}_\theta)\right]\right\}$$

$$\{\ \} \cdot \hat{e}_r \Rightarrow \quad N - mg\cos\theta = -m(R_1+R_2)\dot{\theta}^2$$

Hoop looses contact $\Rightarrow N = 0$

$$\Rightarrow \quad \dot{\theta}^2 = \frac{g\cos\theta}{(R_1+R_2)}$$

Energy Balance

$$E_1 = E_2$$

$$E_1 = P.E = mg(R_1+R_2)$$

$$E_2 = K.E + P.E = \frac{1}{2}mv_o^2 + mg(R_1+R_2)\cos\theta + \frac{1}{2}I\omega^2$$

No slip

$$V_o = \underline{\omega} \times R_1\hat{j}$$

$$= -\dot{\phi}\hat{k} \times R\hat{e}_r$$

$$V_o = \dot{\phi}R_1(-\hat{e}_\theta)$$

$$\Rightarrow \dot{\phi} = \left(1 + \frac{R_2}{R_1}\right)\dot{\theta}$$

$$\Rightarrow mg(R_1+R_2) = \frac{1}{2}m(\dot{\phi}R_1)^2 + mg(R_1+R_2)\cos\theta + \frac{1}{2}mR_1^2\dot{\phi}^2$$

$$= \frac{2}{2}m\left(1+\frac{R_2}{R_1}\right)^2\dot{\theta}^2R_1^2 + mg(R_1+R_2)\cos\theta$$

$$\Rightarrow mg(R_1+R_2)(1-\cos\theta) = \frac{2}{2}m(R_1+R_2)^2\dot{\theta}^2$$

$$g(1-\cos\theta) = (R_1+R_2)\frac{g\cos\theta}{(R_1+R_2)}$$

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$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = 60^\circ$

Angular velocity at that instant,

$\dot{\phi} = \frac{R_1 + R_2}{R_1} \cdot \sqrt{\frac{g \cos\theta}{R_1 + R_2}}$

$\underline{\dot{\phi}} = \underline{\omega} = \frac{1}{R_1}\sqrt{\frac{g(R_1+R_2)}{2}} \; (-\hat{k})$

$\underline{v} = \sqrt{\frac{g(R_1+R_2)}{2}} \; (-\hat{e}_\theta)$